Math 203 **Second Midterm Examination** **Summer time** **ه 1439-1438** **Time: 90 minute**	Department mathematics College of Science جامعة الملك سعود King Saud University

Solve the following questions

Q1: Evaluate the integral $\int_0^{\sqrt{2}} \int_{y^2}^{2} y^3 e^{x^3}\, dx\, dy$ **[5]**

Q2: Use polar coordinates to evaluate the integral $\iint_R \left(x^2+2y^2\right) dA$ **where the region R is defined by** $R=\left\{(x,y)\in R^2,\ y\geq 0,\ x\geq 0,\ 1\leq x^2+y^2\leq 2\right\}$. **[5]**

Q3: Find the area of the surface S where S is the part of $z = xy$ **that is inside the cylinder** $x^2+y^2=2$ **[5]**

Q4: Use cylindrical coordinates to evaluate the integral

$$\int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{4-x^2-y^2}} z\, dz\, dx\, dy$$ **[5]**

Q5: Find the volume of the solid bounded by the graphs of the equations: **[5]**
$x+y+z=1,\ \ x=0,\ y=0,\ z=0.$

End of the questions.

M - 203

II Mid-term Exam. (Summer Term 1438/39)

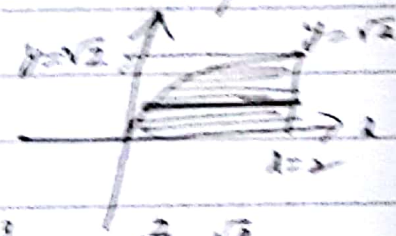
Time: 90 Minutes Max. Marks: 25

Q#1) Evaluate the Integral $\int_0^{\sqrt{2}}\int_{y^2}^{2} y^3 e^{x^3}\,dx\,dy$.

[Marks: 5]

Soln. Given: $y^2 \le x \le 2$, $0 \le y \le \sqrt{2}$ } Horizontal strip.

We change it to vertical strip



$0 \le y \le \sqrt{x}$

$0 \le x \le 2$

Hence, we change $\int_0^{\sqrt{2}}\int_{y^2}^{2} y^3 e^{x^3}\,dx\,dy = \int_0^2\int_0^{\sqrt{x}} y^3 e^{x^3}\,dy\,dx$

3

$$= \int_0^2 \left[\frac{y^4}{4}\right]_0^{\sqrt{x}} e^{x^3}\,dx$$

$$= \frac{1}{4}\int_0^2 x^2 e^{x^3}\,dx$$

Let $x^3 = u$

$3x^2\,dx = du$

$$= \frac{1}{12}\left(e^8 - 1\right)$$

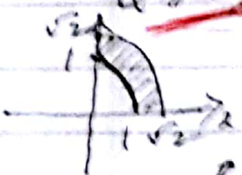
2

$= \frac{1}{4}\cdot\frac{1}{3}\int e^u\,du$

$= \frac{1}{12} e^u = \frac{1}{12}\left[e^{x^3}\right]_0^2$

$= \frac{1}{12}\left(e^8-1\right)$

Q#2) Use polar coordinates to evaluate the integral



$\iint_R (x^2+2y^2)\,dA$ where the region R is defined by

$R = \{(x,y)\in\mathbb{R}^2,\ y \ge 0,\ x \ge 0,\ 1 \le x^2+y^2 \le 2\}$

Soln. $\iint_R (x^2+2y^2)\,dA = \int_0^{\pi/2}\int_1^{\sqrt{2}} r^2(1+\sin^2\theta)\,r\,dr\,d\theta$

3

$= \int_0^{\pi/2}\left[\frac{r^4}{4}\right]_1^{\sqrt{2}}(1+\sin^2\theta)\,d\theta = \frac{3}{4}\int_0^{\pi/2}(1+\sin^2\theta)\,d\theta$

$= \frac{3}{4}\int_0^{\pi/2}\left[1+\frac{1}{2}(1-\cos 2\theta)\right]d\theta = \frac{3}{4}\left[\theta+\frac{1}{2}\theta-\frac{1}{4}\sin 2\theta\right]_0^{\pi/2} = \frac{9\pi}{16}$

2

Q#3) Find the area of the surface S where S is the part of $z = xy$ that is inside the cylinder $x^2+y^2=2$.

[Marks: 5]

Sol. we have $z = xy = f(x,y)$

$\therefore f_x(x,y) = y$ and $f_y(x,y) = x$ (1)

$\therefore S.A = \iint_R \sqrt{1+x^2+y^2}\, dA$ (1)

$= \int_0^{2\pi}\int_0^{\sqrt{2}} \sqrt{1+r^2}\, r\, dr\, d\theta$ (2)

Put $1+r^2 = u \Rightarrow$ $2r\,dr = du$

$= \frac{1}{2}\int \sqrt{u}\, du = \frac{1}{2}\cdot\frac{2}{3}u^{3/2} = \frac{1}{3}(1+r^2)^{3/2}\Big]_0^{\sqrt{2}} = \frac{1}{3}\left(3^{3/2}-1\right)$

$= 2\pi \cdot \frac{1}{3}\left(3^{3/2}-1\right)$

$= \frac{2}{3}\pi\left(3^{3/2}-1\right)$

$= \frac{2}{3}\pi\left(\sqrt{27}-1\right)$ (1)

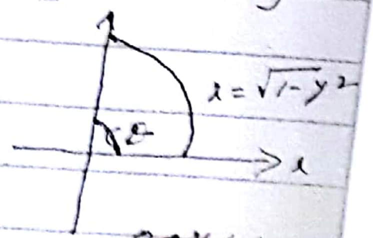
$\simeq 8.79$

Q#4) Use cylindrical coordinates to evaluate the integral.

$$\int_0^1\int_0^{\sqrt{1-y^2}}\int_0^{\sqrt{4-x^2-y^2}} z\, dz\, dx\, dy$$

[Marks: 5]

Sol. $I = \int_0^{\pi/2}\int_0^1\int_0^{\sqrt{4-r^2}} r\, z\, dz\, dr\, d\theta$ (4)



$0 \le r \le 1$

$0 \le \theta \le \pi/2$

$= \int_0^{\pi/2}\int_0^1 r\left[\frac{z^2}{2}\right]_0^{\sqrt{4-r^2}} dr\, d\theta$

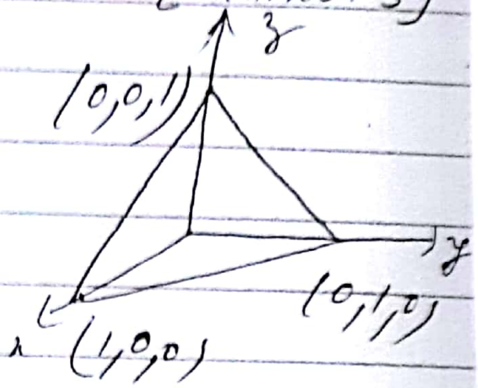
$= \frac{1}{2}\int_0^{\pi/2}\int_0^1 r(4-r^2)\, dr\, d\theta$

$= \frac{1}{2}\int_0^{\pi/2}\int_0^1 (4r - r^3)\, dr\, d\theta = \frac{1}{2}\int_0^{\pi/2}\left[\frac{4r^2}{2} - \frac{r^4}{4}\right]_0^1 d\theta$

$= \frac{1}{2}\int_0^{\pi/2}\left(2-\frac{1}{4}\right)d\theta = \frac{1}{2}\left(\frac{7}{4}\right)\left(\frac{\pi}{2}\right) = \frac{7}{16}\pi$ (1)

Q # 3) Find the volume of the solid bounded by the graphs of the equations: $x+y+z=1$, $x=0$, $y=0$, z [Marks: 5]

Sol. Volume $V = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz\,dy\,dx$ (3)



$= \int_0^1 \int_0^{1-x} (1-x-y)\,dy\,dx$

$= \int_0^1 \left[y - xy - \frac{y^2}{2} \right]_0^{1-x} dx$

$= \int_0^1 \left[(1-x) - x(1-x) - \frac{(1-x)^2}{2} \right] dx$

$= \int_0^1 (1-x)\left[1 - x - \frac{1}{2}(1-x) \right] dx$

$= \int_0^1 \left[(1-x)\frac{1}{2}(1-x) \right] dx = \frac{1}{2}\int_0^1 (1-x)^2 dx$

$= -\frac{1}{2}\left[\frac{(1-x)^3}{3} \right]_0^1 = \frac{1}{6}$

(2)