College of Science
Department of Mathematics

King Saud University

First Midterm Exam

Thursday, November 2, 2017	Math 473	Academic year 1438-39H
1:00 - 2:30 pm	Introduction to Differential Geometry	First Semester

Student's Name		
ID number		
Section No.		
Classroom No.		
Teacher's Name	**Dr Nasser Bin Turki**	25
Roll Number		

Instructions:

- Your student identity card must be visible on your desk during the entire examination.

1. Let $\alpha : \mathbb{R} \mapsto \mathbb{R}^3$ be given by

$$\alpha(t) = \left(\frac{\sqrt{2}}{3}t^3, t^2 + 2t, t^2 - 2t\right).$$

[14 marks]

(a) Compute the Velocity and the speed of α. Show that α is a regular space curve.

(b) Compute the unit tangent T.

(c) Compute the vector $\alpha' \times \alpha''$.

(d) Compute the unit binormal B.

(e) Compute the curvature κ and the torsion τ of α. Show that the curvature κ and the torsion τ of the curve α coincide: $\kappa(t) = \tau(t)$ for all $t \in \mathbb{R}$.

(f) Find the Serret-Frenet basis (Frame) of α.

2. Let $\alpha : I \mapsto \mathbb{R}^3$ be given by

$$\alpha(t) = \left(e^t \cos t, e^t \sin t, e^t\right).$$

[5 marks]

(a) Reparametrise the curve α by arc-length.

(b) Find the equation of the Normal plane of α at $\alpha(0)$.

3. Let $\alpha : I \mapsto \mathbb{R}^3$ be a regular parametrised space curve with $\kappa(t) \neq 0$, for all $t \in I$. Show that α is a Helix if and only if $\frac{\tau(t)}{\kappa(t)} = c$, where c is constant. [6 marks]