King Saud University
Department of Mathematics
Second Semester 1435-1436 H

MATH 244 (Linear Algebra)
Second Midterm Exam
Duration: 105 Minutes

Student's Name	Student's ID	Group No.	Lecturer's Name

Question No.	I	II	III	IV	Total
Mark					

[**I**] Determine whether the following is **True** or **False**. [3 *Points*]

(1) If A and B are $n \times n$ matrices, then $\det(A - B) = \det(A) - \det(B)$. (   )

(2) If C and D are 2×2 matrices with $\det(C) = 5$ and $\det(D) = -1$, then $\det(4DC) = -80$. (   )

(3) If $\mathbf{u} = (4, 3)$ and $\mathbf{v} = (2, -5)$, then $|\mathbf{u} \bullet \mathbf{v}| \leq 2\|\mathbf{u}\|$. (   )

(4) The set $\{(1, 2, -1), (-1, 2, 3), (-1, 1, 1)\}$ is orthogonal. (   )

(5) $S = \{(2, 3, 1), (1, 0, 1), (0, 4, 1)\}$ spans $\mathbb{R}^3$. (   )

(6) The set $\{(1, 1), (3, 5), (4, 2)\}$ is linearly independent in $\mathbb{R}^2$. (   )

OVER

[II] Choose the correct answer. [5 *Points*]

(1) If $A^{-1} = \begin{bmatrix} 1 & 2 & 2 \\ 1 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}^T$, then $adj(A)$ equals

(a) $7A^{-1}$ **(b)** $\frac{1}{7}A^{-1}$ **(c)** $\begin{bmatrix} -1 & 3 & 1 \\ 2 & -6 & 5 \\ 2 & 1 & -2 \end{bmatrix}$ **(d)** None of the previous

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(2) The angle θ between $\mathbf{u} = (1, -1, 0)$ and $\mathbf{v} = (1, 0, 0)$ satisfies

(a) $\cos\theta = 0$ **(b)** $\cos\theta = \frac{1}{\sqrt{2}}$ **(c)** $\cos\theta = \frac{1}{2}$ **(d)** None of the previous

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(3) If $\|\mathbf{u}\| = 3$, $\|\mathbf{u} + \mathbf{w}\| = 6$ and the distance $d(\mathbf{u}, \mathbf{w}) = 2$. Then $\|\mathbf{w}\|$ equals

(a) 1 **(b)** 8 **(c)** $\sqrt{11}$ **(d)** None of the previous

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(4) The solution space of $\begin{bmatrix} 1 & 2 & 0 \\ 3 & 6 & 1 \\ -2 & -4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ is

(a) The origin $\{\mathbf{0}\}$ **(b)** A line through the origin **(c)** A plane through the origin **(d)** None of the previous

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(5) Which of the following is a linear combination of $\mathbf{v}_1 = (1, 1, 2)$, $\mathbf{v}_2 = (1, 0, 1)$ and $\mathbf{v}_3 = (2, 1, 3)$?

(a) $(3, 1, -1)$ **(b)** $(2, 4, 6)$ **(c)** $(2, 0, 1)$ **(d)** None of the previous

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OVER

[III] [6 *Points*]

(a) **Show** that $\{A \in M_{22} : A = A^T\}$ is a subspace of M_{22}.

(b) **Prove** that $S = \{\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3\}$ is a basis for P_2 where $\mathbf{p}_1 = 3 + x$, $\mathbf{p}_2 = 2 - x + x^2$, $\mathbf{p}_3 = 1 - x^2$ and **Find** the coordinate vector $(\mathbf{q})_S$ for $\mathbf{q} = 7 - 2x - 3x^2$

OVER

[IV] [6 *Points*]

Let $V = \{(x, 2) \in \mathbb{R}^2, x \neq 0\}$ with the following addition and scalar multiplication on $\mathbf{u} = (x, 2) \in V$ and $\mathbf{v} = (y, 2) \in V$

$$\begin{aligned} \mathbf{u} + \mathbf{v} &= (xy, 2) \\ k\mathbf{u} &= (kx, 2) \end{aligned}$$

(a) Compute $(1, 2) + (-3, 2)$ and $4(-2, 2)$

(b) Find the object $\mathbf{0} \in V$ such that $\mathbf{u} + \mathbf{0} = \mathbf{u}$ for all $\mathbf{u} \in V$

(c) If $\mathbf{u} \in V$. Find the object $-\mathbf{u} \in V$ such that $-\mathbf{u} + \mathbf{u} = \mathbf{0}$

(d) Show that V is not a vector space

Good Luck