King Saud University
Department of Mathematics
1st Semester 1435-1436 H

MATH 244 (Linear Algebra)
2nd Midterm Exam
Duration: 105 Minutes

Student's Name	**Student's ID**	**Group No.**

Question No.	**I**	**II**	**III**	**IV**	**Total**
Mark					

[**I**] Determine whether the following is **True** or **False**. [3 *Points*]

(1) If A is an invertible matrix, then $\text{adj}(A)$ is also invertible. ( )

(2) The distance $\mathbf{d}(\mathbf{u}, \mathbf{v})$ between $\mathbf{u} = (2, -1, 3)$ and $\mathbf{v} = (1, 3, 4)$ equals $\sqrt{6}$. ( )

(3) The set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$, where $\mathbf{u} = (1, 1, -1)$, $\mathbf{v} = (-1, 2, 1)$ and $\mathbf{w} = (2, 0, 1)$, is an orthogonal set. ( )

(4) $(-1, 3, 2)$ can be written as a linear combination of $(2, 0, 1)$ and $(0, 2, 4)$. ( )

(5) $W = \{(x, y) : xy = 0\}$ is a subspace of $\mathbb{R}^2$. ( )

(6) $S = \{(1, 1, 0), (1, -1, 1), (0, 1, -1)\}$ spans $\mathbb{R}^3$. ( )

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[II] Choose the correct answer. [5 *Points*]

(1) If A, B and C are 2×2 matrices with $\det(A) = 3$, $\det(B) = 4$ and $\det(C) = 5$ then $\det\left(3A^{-1}B^{T}C^{2}\right)$ equals

(a) 75 **(b)** 100 **(c)** 300 **(d)** None of the previous

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(2) If $\|\mathbf{u} + \mathbf{v}\| = 5$ and $\|\mathbf{u} - \mathbf{v}\| = 1$, then the dot product $\mathbf{u} \cdot \mathbf{v}$ equals

(a) 6 **(b)** 4 **(c)** 1 **(d)** None of the previous

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(3) For $\mathbf{u} = (-3, 1, 1, 0)$ and $\mathbf{v} = (4, 7, -3, 5)$, the vector $\mathbf{x}$ that satisfies the equation $5\mathbf{x} - 2\mathbf{v} = \mathbf{u}$ is

(a) $\mathbf{x} = (5, 15, -5, 10)$ **(b)** $\mathbf{x} = (1, 3, -1, 2)$ **(c)** $\mathbf{x} = (-2, 9, -1, 5)$ **(d)** None of the previous

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(4) The angle θ between $\mathbf{u} = (1, 1, 1)$ and $\mathbf{v} = (2, 0, 1)$ is

(a) $\frac{\pi}{4}$ **(b)** $\frac{\pi}{2}$ **(c)** $\cos^{-1}\sqrt{\frac{3}{5}}$ **(d)** None of the previous

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(5) The solution space of the system $A\mathbf{x} = \mathbf{0}$, where $A = \begin{bmatrix} 1 & -1 & 3 \\ -3 & 6 & 9 \\ -2 & 2 & -6 \end{bmatrix}$, is

(a) a line through the origin **(b)** a plane through the origin **(c)** the origin **(d)** None of the previous

OVER

[III] [6 *Points*]

(a) Let $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 1 & 0 & -1 & 3 \\ 0 & 2 & 1 & 4 \\ -1 & 2 & 0 & 1 \end{bmatrix}$. Compute $\det(A)$ using **row reduction**

(b) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & -2 \\ 3 & 1 & 0 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Solve the system $A\mathbf{x} = \mathbf{b}$ just for x_3 using **Cramer's Rule**.

OVER

[IV] [6*Points*]

Let $V = \{(x, y) \in \mathbb{R}^2 : x \neq 0\}$ with the following addition and scalar multiplication on $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$:

$$\begin{aligned} \mathbf{u} + \mathbf{v} &= (u_1 v_1, u_2 + v_2 - 1) \\ k\mathbf{u} &= (ku_1, ku_2) \end{aligned}$$

(a) Find the object $\mathbf{0} \in V$ such that $\mathbf{u} + \mathbf{0} = \mathbf{u}$ for all $\mathbf{u} \in V$.

(b) If $\mathbf{u} \in V$. Find the object $-\mathbf{u} \in V$ such that $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

(c) Show that V is not a vector space.

Good Luck