College of Science
Department of Mathematics

Second Midterm Exam

Thursday, December 14, 2017	Math 473	Academic year 1438-39H
1:00 - 2:30 pm	Introduction to Differential Geometry	First Semester

Student's Name	
ID number	
Section No.	
Classroom No.	
Teacher's Name	**Dr Nasser Bin Turki**
Roll Number	

25

Instructions:

- Your student identity card must be visible on your desk during the entire examination

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1. Let $\alpha : I \mapsto \mathbb{R}^3$ be unit speed curve whose torsion $\tau(t) = c$, where c is constant. Show that the curve α is Bertrand curve. [3 marks]

2. Let $\alpha : (-\frac{\pi}{2}, \frac{\pi}{2}) \mapsto \mathbb{R}^2$ be a curve given by $\alpha(t) = (2t + \sin 2t, 1 + \cos 2t)$, where $t \in (-\frac{\pi}{2}, \frac{\pi}{2})$. Find the involute curve of α. [3 marks]

3.
Let $X(u,v) = \left(u+v, u-v, \frac{u^2+v^2}{2}\right)$.

(a) Show that X defines a regular surface patch.

(b) Calculate the coefficients E, F, G of the first fundamental form for this surface.

(c) Write down an integral which gives the length of the curve $\gamma_1(t) = X(t, 1)$ on this surface from $t = 1$ to $t = 2$. You do not need to evaluate this integral.

(d) Is X true map. **Why**.

(e) Calculate the coefficients e, f, g of the second fundamental form for this surface.

[10 marks]

4. For the surface $X : \mathbb{R}^2 \mapsto \mathbb{R}^3$ given by $X(u,v) = (u, v, u^2 + v^2)$. Let $\alpha(t) = X(\cos t, \sin t)$ be a curve on the surface X. Find a unit normal vector to the surface X at a point $X(u,v)$. Find the geodesic curvature κ_g, normal curvature κ_n and geodesic torsion κ_t? Is $\alpha(t)$ principal curve? **Why**?

[9 marks]