Midterm Exam 1, Semester II, 1447

Department of Mathematics, College of Science, KSU

Course: Math 481 Maximum Marks: 25 Duration: 1.5 Hours

Question 1 [4 points]

Determine whether each function is Riemann integrable on $[0,1]$. If it is integrable, compute its integral.

1.
$$f(x) = \begin{cases} x, & x \in [0, \frac{1}{2}), \\ 1-2x, & x \in [\frac{1}{2}, 1] \end{cases}$$

2.
$$f(x) = \begin{cases} \frac{1}{2}, & x \in \mathbb{Q}, \\ 0, & x \notin \mathbb{Q} \end{cases}$$

Question 2 [4 points]

Express the following limits as definite integrals and evaluate them:

1. $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{1}{n+2k}$

2. $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \dfrac{k}{n^2+3k^2}$

Question 3 [8 points]

Let $f_n : [0,1] \to \mathbb{R}$ be defined by

$$f_n(x) = \begin{cases} n^2 x, & x \in [0, 1/n], \\ n^2(x - 2/n), & x \in (1/n, 2/n], \\ 0, & x \in (2/n, 1]. \end{cases}$$

1. Show that $f_n(x) \to 0$ for all $x \in [0,1]$.

2. Compute $\displaystyle\int_0^1 f_n(x)\,dx$.

3. Determine whether $f_n \to 0$ uniformly on $[0, 1]$.

Question 4 [9 points]

For each sequence of functions below, determine whether the convergence is **pointwise** and/or **uniform** on $[0, 1]$.

1. $f_n(x) = \dfrac{x}{nx + 1}$
2. $f_n(x) = \dfrac{nx^3}{1 + nx}$
3. $f_n(x) = x^n(1 - x)$

End of Exam