King Saud University Department of Mathematics October 13, 2025

Math 481 Midterm 1

Time: 90 minutes (Total: 25 points)

Number of questions: 4

1. Decide, with justification, whether each function below is Riemann integrable on $[0,1]$. Use either the ε–criterion (upper/lower sums). If integrable, compute its integral.

   (a) [4]

   $$f(x) = \begin{cases} 1, & x \in \mathbb{Q}, \\ 0, & x \notin \mathbb{Q}, \end{cases} \qquad x \in [0,1].$$

   (b) [4]

   $$f(x) = \begin{cases} x, & x \in \mathbb{Q}, \\ 0, & x \notin \mathbb{Q}, \end{cases} \qquad x \in [0,1].$$

2. Evaluate the following limits. State clearly any theorems you use. [4]

   $$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n+k}$$

3. Let $f : [a,b] \to \mathbb{R}$ be continuous and $f(x) \geq 0$ on $[a,b]$. If $\displaystyle\int_a^b f(x)\,dx = 0$, prove that $f(x) = 0$ for all $x \in [a,b]$. [5]

4. For each integral, determine whether it converges. If it converges, evaluate it; if it diverges, state "diverges" and justify.

   (a) $\displaystyle\int_0^{\infty} \frac{dx}{\sqrt{x}\,(x+1)}$ [4]

   (b) $\displaystyle\int_0^{\infty} \frac{|\sin x|}{x}\,dx$ [4]