King Saud University
Department of Mathematics
Second Semester 1435-1436 H

MATH 244 (Linear Algebra)
First Midterm Exam
Duration: 105 Minutes

Student's Name	Student's ID	Group No.	Lecturer's Name

Question No.	I	II	III	IV	Total
Mark					

[**I**] Determine whether the following is **True** or **False**. [3 *Points*]

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(1) $(2, 1, -1)$ is a solution of the following linear system ( )

$$\begin{aligned} x + 2y + 5z &= -1 \\ 3x - y + z &= 3 \end{aligned}$$

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(2) If A is an $n \times n$ matrix, then $tr(A) = tr\left(A^T\right)$. ( )

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(3) For $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, D^3 is invertible. ( )

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(4) If B and C are invertible $n \times n$ matrices, then $B + C$ is invertible. ( )

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(5) If A and B are $n \times n$ matrices that commute, then $A^2B = BA^2$. ( )

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(6) If A is a square matrix with two identical columns, then $\det(A) = 0$. ( )

OVER

[II] Choose the correct answer. [5 *Points*]

(1) If $(I+A)^T = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 5 & 0 & 3 \end{bmatrix}$, then $\det(A)$ equals

(a) 7 **(b)** 8 **(c)** 20 **(d)** None of the previous

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(2) If $B^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ and $p(x) = x^2 + 1$, then $p(B)$ equals

(a) $\begin{bmatrix} 32 & -12 \\ -18 & 8 \end{bmatrix}$ **(b)** $\begin{bmatrix} 32 & -11 \\ -17 & 8 \end{bmatrix}$ **(c)** $\begin{bmatrix} 26 & 4 \\ 9 & 2 \end{bmatrix}$ **(d)** None of the previous

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(3) The values of k for which the matrix $\begin{bmatrix} 2 & k+2 \\ k^2 & -1 \end{bmatrix}$ is symmetric are

(a) $k = 2, -1$ **(b)** $k = 1, -2$ **(c)** $k = 0, 1$ **(d)** None of the previous

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(4) For $\begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 5 \\ 3 & 0 & -1 \end{bmatrix}$, the cofactor C_{23} equals

(a) -6 **(b)** 6 **(c)** 30 **(d)** None of the previous

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(5) For any $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, the system

$$\begin{aligned} 3x - 2y &= b_1 \\ 4x + y &= b_2 \end{aligned}$$

has

(a) No solution **(b)** a unique solution **(c)** Infinitely many solutions **(d)** None of the previous

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OVER

[III] Let $A = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 2 & -2 & 2 & 0 \\ 1 & 0 & 3 & 0 \\ -2 & 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$. **Find** the following [7 *Points*]

(a) $\det(A)$ using **Row Reduction**

(b) $(2B)^{-1}$

(c) The matrix X for which $XB = \begin{bmatrix} 4 & 2 & -2 \\ -2 & -4 & 0 \end{bmatrix}$

OVER

[IV] [5 *Points*]

(a) **Solve** the following system

$$\begin{aligned} x + y + 2z &= 0 \\ 2x + y + z &= 1 \\ 3x + 2y + 5z &= 2 \end{aligned}$$

(b) **Can** the coefficient matrix of the previous system be written as a product of elementary matrices? **Justify** your answer.

Good Luck