King Saud University Department of Mathematics

1st Semester 1435-1436 H

MATH 244 (Linear Algebra)
1st Midterm Exam
Duration: 105 Minutes

| Student's Name | Student's ID | Group No. |
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| Question No. | I | II | III | IV | Total |
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| Mark |  |  |  |  |  |

[I] Determine whether the following is True or False. [3 Points]
(1) The following system of equations is linear

$$
\begin{aligned}
x-y+z & =5 \\
x+2 y z & =0
\end{aligned}
$$

(2) The reduced row echelon form of the matrix $\left[\begin{array}{cc}1 & 2 \\ -3 & 4\end{array}\right]$ is $I_{2}$.
$\qquad$
(3) The matrix $\left[\begin{array}{ccc}0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -3\end{array}\right]$ is elementary.
$\qquad$
(4) The matrix $\left[\begin{array}{ccc}1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & -1\end{array}\right]$ is invertible.
$\qquad$
(5) If $A$ and $B$ are $n \times n$ matrices, then $(A+B)^{2}=A^{2}+2 A B+B^{2}$.
(6) If $A$ is an invertible symmetric matrix, then $A^{-1}$ is symmetric.
[II] Choose the correct answer. [6 Points]
(1) If $A, B$ and $C$ are $n \times n$ matrices, then $A C-\left(C^{T} B\right)^{T}$ equals
(a) $(A-B) C^{T}$
(b) $\left(A-B^{T}\right) C$
(c) $(A C-B C)^{T}$
(d) None of the previous
(2) The values of $a, b$ and $c$ for which $\left[\begin{array}{cc}3 & a \\ 1 & a+b\end{array}\right]=\left[\begin{array}{cc}b & c-2 \\ c+2 & 0\end{array}\right]$ are
(a) $a=1, b=3, c=3$
(b) $a=-3, b=3, c=-1$
(c) $a=0, b=3, c=2$
(d) None of the previous
(3) For $A=\left[\begin{array}{ll}1 & 2 \\ 1 & 3\end{array}\right], A^{-2}$ equals
(a) $\left[\begin{array}{cc}11 & -8 \\ -4 & 3\end{array}\right]$
(b) $\left[\begin{array}{cc}3 & 8 \\ 4 & 11\end{array}\right]$
(c) $\left[\begin{array}{ll}1 & 4 \\ 1 & 9\end{array}\right]$
(d) None of the previous
(4) If $B^{3}=\left[\begin{array}{ccc}-8 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 1\end{array}\right]$, then $\operatorname{tr}(B)$ equals
(a) $\left[\begin{array}{ccc}-2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1\end{array}\right]$
(b) 20
(c) 2
(d) None of the previous
(5) For $\left[\begin{array}{ccc}1 & 3 & 4 \\ 2 & 1 & 0 \\ 5 & 0 & -1\end{array}\right]$, the minor $M_{12}$ equals
(a) 2
(b) -2
(c) -3
(d) None of the previous
(6) $\left|\begin{array}{llll}1 & 4 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 3 & 1 & 5 \\ 1 & 2 & 3 & 5\end{array}\right|$ equals
(a) 5
(b) -10
(c) 10
(d) None of the previous
[III] Let $A=\left[\begin{array}{ccc}1 & 0 & 1 \\ 1 & 1 & 1 \\ -1 & 1 & 0\end{array}\right], B=\left[\begin{array}{ccc}2 & -1 & 3 \\ 1 & 2 & 4 \\ 5 & 0 & 1\end{array}\right]$. Find the following [6 Points]
(a) $A+B$
(b) $(3 A)^{-1}$
(c) The matrix $X$ for which $A X=B$
(a) Solve the following system

$$
\begin{aligned}
x+y+3 z & =0 \\
2 x+y+4 z & =1 \\
3 x+y+5 z & =2
\end{aligned}
$$

(b) Is the coefficient matrix of the previous system invertible? Justify your answer.

