King Saud University Department of Mathematics 2nd Semester 1432-1433 H	MATH 244 (Linear Algebra) 1st Midterm Exam Duration: 90 Minutes

Student's Name	**Student's ID**	**Group No.**	**Lecturer's Name**

Question No.	**I**	**II**	**III**	**IV**	**Total**
Mark					

[**I**] Determine whether the following is **True** or **False**.

(1) A homogeneous system of linear equations must have a unique solution. (   )

(2) For an $n \times n$ matrix A, if the system $A\mathbf{x} = 4\mathbf{x}$ has a unique solution then the matrix $A - 4I_n$ is invertible. (   )

(3) The matrix $\begin{bmatrix} 1 & 3 \\ 2 & 0 \end{bmatrix}$ is invertible. (   )

(4) The matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 5 \\ 3 & -5 & -1 \end{bmatrix}$ is symmetric. (   )

(5) If A and B are $n \times n$ matrices for which $AB = I_n$ then $BA = I_n$. (   )

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[II] Choose the correct answer.

(1) $\begin{vmatrix} 5 & 2 & 2 \\ -1 & 1 & 2 \\ 3 & 0 & 0 \end{vmatrix}$ equals

(a) -18 **(b)** 6 **(c)** -6 **(d)** None of the previous

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(2) For $a = 4$, the system

$$\begin{aligned} x_1 + x_2 + x_3 &= 1 \\ (a^2 - 4)x_3 &= a \end{aligned}$$

has

(a) No solution **(b)** Exactly one solution **(c)** Infinitely many solutions

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(3) The values of k for which $\det(A) = 0$, where $A = \begin{bmatrix} k & -k & 3 \\ 0 & k+1 & 1 \\ k & -8 & k-1 \end{bmatrix}$ are

(a) $k = 0, 1$ **(b)** $k = 1, 2$ **(c)** $k = 0, 2$ **(d)** None of the previous

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(4) If $B^3 = \begin{bmatrix} -8 & 0 & 0 \\ 0 & -64 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then

(a) $B^2 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **(b)** $B^2 = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **(c)** $B^2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **(d)** None of the previous

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(5) For $C = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & -9 \end{bmatrix}$, $tr(4C^T)$ equals

(a) -4 **(b)** -1 **(c)** 4 **(d)** None of the previous

OVER

[III]

(a) Solve the following matrix equation for X, where $A = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 0 \\ 1 & -2 \end{bmatrix}$.

$$X - (2AB + C^T)^{-1} = 0_{2\times 2}$$

(b) Find a matrix Y for which $DY = \begin{bmatrix} 1 & 2 \\ 0 & 3 \\ 0 & 0 \end{bmatrix}$, where $D = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$.

OVER

[IV] Solve the following system if possible

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\begin{aligned}
x_1 - 2x_2 + x_3 - 4x_4 &= 1 \\
2x_1 + 6x_2 + 14x_3 + 4x_4 &= 4 \\
x_1 - 12x_2 - 11x_3 - 16x_4 &= -3
\end{aligned}
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Good Luck