Midterm Exam 1, Semester I, 1446
Department of Mathematics, College of Science, KSU
Course: Math 481 — Maximum Marks: 25 — Duration: 1.5 Hours

immediate

November 23, 2024

Question 1 [3+3 points]
Test the following series for convergence:

1. $\sum_{n=1}^{\infty} \frac{2n}{n!n^n}$
2. $\sum_{n=1}^{\infty}(-1)^n \log n$

Question 2 [3+3 points] Determine whether f is integrable over $[0,1]$, and evaluate $\int_0^1 f(x)\,dx$ whenever it exists.

1. $f(x) = \begin{cases} 1, & \text{if } x \in [0,1/2), \\ 0, & \text{if } x \in [1/2,1] \end{cases}$
2. $f(x) = \begin{cases} \frac{1}{2}, & \text{if } x \in \mathbb{Q}, \\ 0, & \text{if } x \notin \mathbb{Q} \end{cases}$

Question 3 [3+3points] Write the following limits as integrals:

1. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n+k}$
2. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{n^2+k^2}$

Question 4 [7 points] Let $f_n : [0,1] \to \mathbb{R}$ be defined as follows:

$$f_n(x) = \begin{cases} n^2 x, & \text{if } x \in [0,1/n], \\ n^2(x-2/n), & \text{if } x \in (1/n,2/n], \\ 0, & \text{if } x \in (2/n,1]. \end{cases}$$

Prove that $\lim_{n\to\infty} f_n(x) = f(x) = 0$. By comparing $\int_0^1 f(x)\,dx$ with $\lim_{n\to\infty}\int_0^1 f_n(x)\,dx$, show that the convergence of f_n is not uniform.