Example: Determine the value of P_{ave} & E for each of the Following:

a) $x(t) = e^{-2t} u(t)$

ans $E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt = \lim_{T\to\infty} \int_{-T}^{T} |e^{-2t} u(t)|^2 \, dt = \lim_{T\to\infty} \int_{-T}^{T} e^{-4t} u(t) \, dt$

$$= \lim_{T\to\infty} \int_{0}^{T} e^{-4t} \, dt = \lim_{T\to\infty} \left(\frac{e^{-4t}}{-4} \right)_0^T = \lim_{T\to\infty} \left(\frac{e^{-4T}}{-4} - \frac{1}{-4} \right) = \frac{1}{4}$$

$\Rightarrow E = 1/4 \longrightarrow P_{ave} = 0$

b) $x(t) = e^{j(2t + \pi/4)}$

ans $E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt = \lim_{T\to\infty} \int_{-T}^{T} |e^{j(2t+\pi/4)}|^2 \, dt = \lim_{T\to\infty} \int_{-T}^{T} 1 \, dt = \lim_{T\to\infty} (2T)$

$E = \infty$; $P_{ave} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 = \lim_{T\to\infty} \frac{1}{2T}(2T) = 1$

c) $x[n] = (\frac{1}{2})^n u[n]$

ans

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N\to\infty} \sum_{n=-N}^{N} |(\tfrac{1}{2})^n u(n)|^2 = \lim_{N\to\infty} \sum_{n=-N}^{N} |(\tfrac{1}{2})^{2n} u(n)|$$

$$= \lim_{N\to\infty} \sum_{n=0}^{N} (\tfrac{1}{4})^n = 4/3$$

$$\sum_{n=0}^{N-1} a^n = \begin{cases} N & a=1 \\ \frac{1-a^N}{1-a} & a\neq 1 \end{cases}$$

$$\sum_{n=0}^{\infty} a^n = \begin{cases} \infty & |a|>1 \\ \frac{1}{1-a} & |a|<1 \end{cases}$$

$\Rightarrow E = 4/3 \longrightarrow P_{ave} = 0$

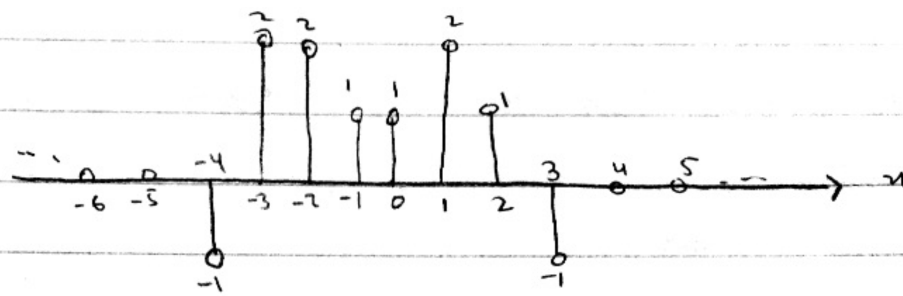
d) $x(n) = e^{j(\frac{\pi}{2}n + \pi/8)}$

$$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N\to\infty} \sum_{n=-N}^{N} 1 = \lim_{N\to\infty} (2N+1) = \infty$$

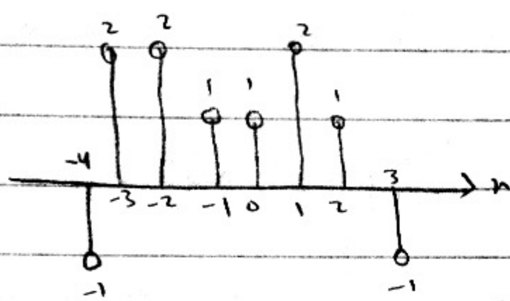
$$\Rightarrow P_{ave} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 = 1$$

Sketch the even & Odd parts of $x[n]$

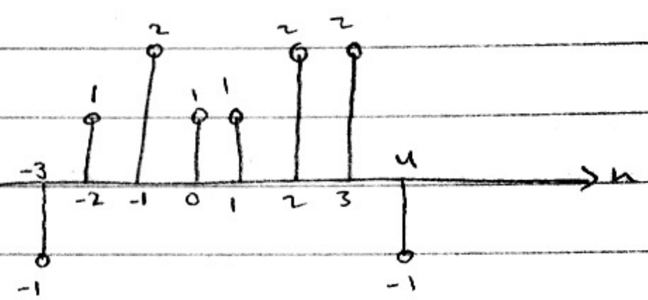
$x(n)$



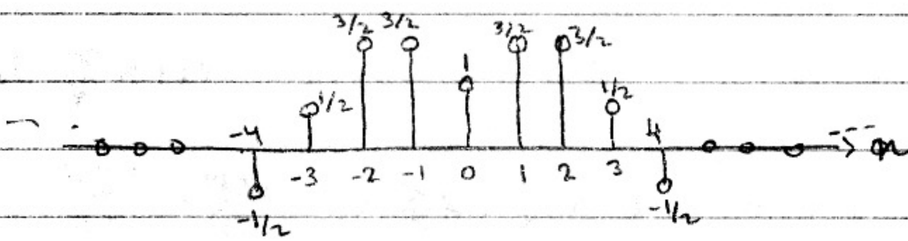
$x[n]$



$x[-n]$



$\mathcal{E}v\{x[n]\}$



$\mathcal{O}d\{x[n]\}$