EE 320 Communications Principles

Midterm# # 1 (2nd Semester 1431/1432H G2011) *Name:*____

Wednesday, March 30, 2011 *Student-ID:*_____________________________________

Total Marks=60 Total Time = 90 Minutes

Question#1 (20 *marks*)**:**

(a) [6] Determine the autocorrelation function for: g(t)=exp(at)u(-t)

(b) [6] A sinc pulse x(t) = 8 sinc(4t) is passed through an ideal low-pass filter of bandwidth B and magnitude spectrum $|H(f)| = 3$, $-B \le f \le B$, and zero otherwise. Calculate the output energy for $B = 2$.

(c) [6] Consider a Low pass filter h(t) whose frequency response is given by $H(f) = \frac{4}{2+j2\pi f}$
Determine the 3 dB bandwidth of the filter h(t).

(d) [2] Write down two reasons for the need of modulation in communication systems.

Question#2(20 *marks*)**:**

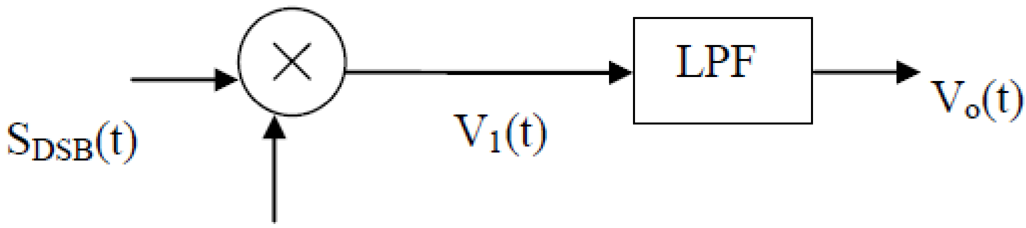
Consider an AM signal with carrier $c(t) = A_c \cos(2\pi f_c t)$ and the message signal $m(t) = A_m \cos(2\pi f_m t)$. Then the AM signal is $s(t) = A_c[1 + k_a A_m \cos(2\pi f_m t)] \cos(2\pi f_c t)$. Suppose the total transmitted power of this AM wave is 400 watts with the percentage modulation, μ=50%. For this AM wave determine:

a) [4] The carrier amplitude, A_c
b) [4] Power in the Carrier.
c) [4] Power in Upper and Lower Side Bands.
d) [4] Percentage of total power wasted in carrier i.e. wasted power.
e) [4] Percentage of total power in side bands, i.e. efficiency

Question#3(20 *marks*)**:**

Consider the DSB-SC signal with carrier $c(t) = 10\cos(2\pi 2000t)$ and the message signal $m(t) = 10\cos(2\pi 500t) + 20\sin(2\pi 1000t)$. For these signals,

a) [5] Write the time-domain equation for DSB-SC signals, and then write the time-domain equations for upper-sideband and lower-sideband.
b) [5] Sketch the frequency spectrum of this DSB-SC signal, clearly labeling the frequency and amplitude axis.
c) [10(4+4+2)] Now consider the demodulation of this DSB-SC signal using coherent demodulation as shown in the figure below. Assume the carrier used here is $c(t) = 20\cos(2\pi 2000t)$. For this coherent demodulation determine
    a. Time-domain equation for $V_1(t)$ and also sketch $V_1(f)$.
    b. What should be the specification (amplitude and cut-off frequency) of LPF (low-pass filter) for exact recovery of message signal in terms of both amplitude and frequency (there should not be any amplitude scaling of message signal) ?
    c. With LPF specifications in previous part: Sketch $V_o(f)$ and write down $V_o(t)$.

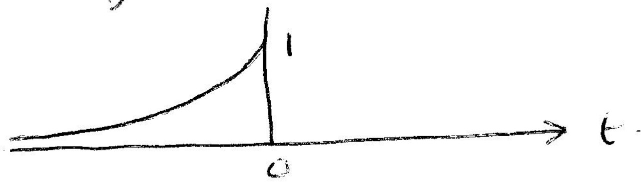


LPF
$S_{DSB}(t)$
$V_1(t)$
$V_o(t)$

EE: 320, Mid #1 Solution

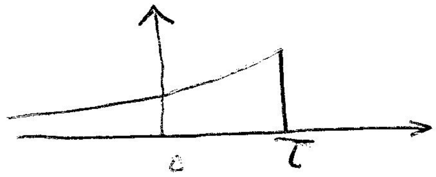
Q #1

(a) $g(t) = \exp(at)u(-t)$



$$R_x(\tau) = \int_{-\infty}^{+\infty} g(t)\, g(t-\tau)\, dt$$

for $\boxed{\tau > 0}$ $g(t-\tau)$



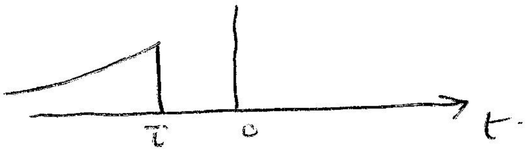
$$R_x(\tau) = \int_{-\infty}^{0} \exp(at) \exp(a(t-\tau))\, dt$$

$$= \exp(-a\tau) \int_{-\infty}^{0} \exp(2at)\, dt$$

$$= \frac{\exp(-a\tau)}{2a} \exp(2at) \Big|_{-\infty}^{0}$$

$$R_x(\tau) = \frac{1}{2a} \exp(-a\tau) \qquad \tau > 0$$

$\boxed{\tau < 0}$ $g(t-\tau)$



$$R_x(\tau) = \int_{-\infty}^{\tau} \exp(at) \exp(a(t-\tau))\, dt$$

$$= \exp(-a\tau) \int_{-\infty}^{\tau} \exp(2at)\, dt$$

$$= \frac{\exp(-a\tau)}{2a} \cdot \exp(2at) \Big|_{-\infty}^{\tau}$$

$$= \frac{1}{2a} \exp(-a\tau) \left[\exp(2a\tau) - 0 \right]$$

$$R_x(\tau) = \frac{\exp(a\tau)}{2a} \qquad \tau < 0$$

Q # 1

(b)

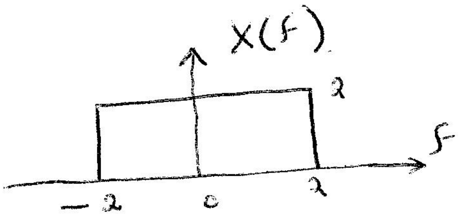
$$x(t) = 8\,\text{Sinc}(4t) = A\,\text{Sin}(2Wt)$$

$$X(f) = \frac{A}{2W}\,\text{rect}\left(\frac{f}{2W}\right)$$

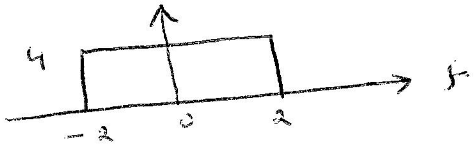
$\boxed{2W = 4}$ $\boxed{A = 8}$

$$X(f) = \frac{8}{4}\,\text{rect}\left(\frac{f}{4}\right)$$

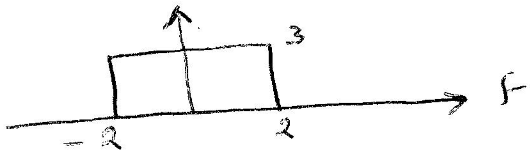
$$X(f) = 2\,\text{rect}\left(\frac{f}{4}\right)$$



$$\Psi_x(f) = |X(f)|^2 \longrightarrow$$

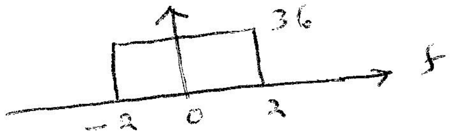


LPF :



$$\Psi_Y(f) = |H(f)|^2 |X(f)|^2$$

=



0 otherwise

$$E_y = 2\int_0^2 36\,df$$

$$\boxed{E_y = 2 \times 36 \times 2 = 144 \text{ units}}$$

Q # 1 (c)

$$H(f) = \frac{4}{2 + j2\pi f}$$

$$|H(f)| = \frac{4}{\sqrt{4 + (2\pi f)^2}}$$

$$\text{Max of } |H(f)|_{f=0} = \frac{4}{\sqrt{4}} = \frac{4}{2} = 2.$$

$$f_{3db}: \quad \frac{4}{\sqrt{4 + (2\pi f_{3d})^2}} = \frac{1}{\sqrt{2}} \cdot 2.$$

$$\sqrt{4 + (2\pi f_{3db})^2} = 2\sqrt{2}$$

$$4 + (2\pi f_{3db})^2 = 8$$

$$2\pi f_{3db} = \sqrt{8 - 4}$$

$$f_{3db} = \frac{2}{2\pi} = \frac{1}{\pi}$$

$$\boxed{f_{3db} = \frac{1}{\pi}}$$

Q # 1 (d)

* Reduce Antenna Size
* Multiplexing
* Combat against noise.

Q # 2: $s(t) = A_c[1 + k_a A_m \cos(2\pi f_m t)] \cos 2\pi f_c t$

$= A_c \cos 2\pi f_c t + \mu A_c \cos 2\pi f_c t \cos 2\pi f_m t$

$\boxed{\mu = k_a A_m}$

$s(t) = A_c \cos(2\pi f_c t) + \frac{\mu A_c}{2}\left[\cos 2\pi (f_c + f_m)t + \cos(2\pi(f_c - f_m)t)\right]$

(a) Power in carrier $= \frac{1}{2} A_c^2$

" " USB $= \frac{1}{8}\mu^2 A_c^2$

" " LSB $= \frac{1}{8}\mu^2 A_c^2$

Total Power $= \frac{1}{2} A_c^2 + \frac{1}{8}\mu^2 A_c^2 + \frac{1}{8}\mu^2 A_c^2$

$P_T = \frac{1}{2} A_c^2 + \frac{1}{4}\mu^2 A_c^2$

$\mu = 0.5$ $\quad P_T = 400$ Watts.

$400 = A_c^2 [0.5 + 0.0625]$

$400 = A_c^2 \times 0.5625$ $\quad \boxed{A_c = 26.67}$

(b) $P_c = 355.64$ W

(c) $P_{USB} = P_{LSB} = 22.23$ W

(d) Wasted Power $= \frac{P_c}{P_T} = \frac{355.64}{400} = 88.9\%$ ← or use: $\frac{2}{2+\mu^2}$

(e) effeciency $= \frac{P_{USB} + P_{LSB}}{P_T} = \frac{2 \times 22.23}{400} = 11.1\%$

↳ or use: $\frac{\mu^2}{2+\mu^2}$

Q #3

$c(t) = 10 \cos(2\pi 2000 t)$

$m(t) = 10 \cos(2\pi 500 t) + 20 \sin(2\pi 1000 t)$

(a) DSB _ SC $\quad s(t) = c(t)\, m(t)$

$= 100 \cos(2\pi 2000t) \cos(2\pi 500 t) + 200 \cos(2\pi 2000 t) \sin(2\pi 1000 t)$

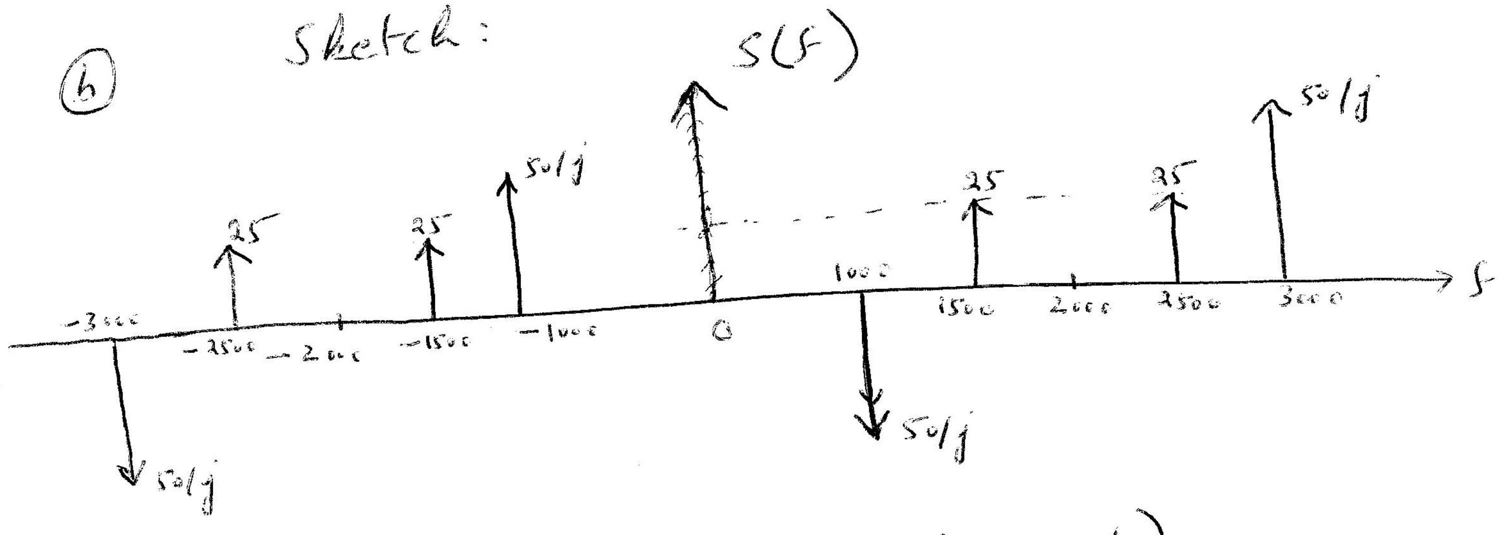
$= 50 [\cos(2\pi 2500 t) + \cos(2\pi 1500 t)] + 100 [\sin(-2\pi 1000 t) + \sin(2\pi 3000 t)]$

$s(t) = \underbrace{50 \cos(2\pi 2500 t) + 100 \sin(2\pi 3000 t)}_{USB} + \underbrace{50 \cos(2\pi 1500 t) - 100 \sin(2\pi 1000 t)}_{LSB}$

USB : $50 \cos(2\pi 2500 t) + 100 \sin(2\pi 3000 t)$

LSB : $50 \cos(2\pi 1500 t) - 100 \sin(2\pi 1000 t)$

(b) Sketch:



Demodulation $\quad c(t) = 20 \cos(2\pi 2000 t)$

$v_1(t) = s(t)\, 20 \cos(2\pi 2000 t)$

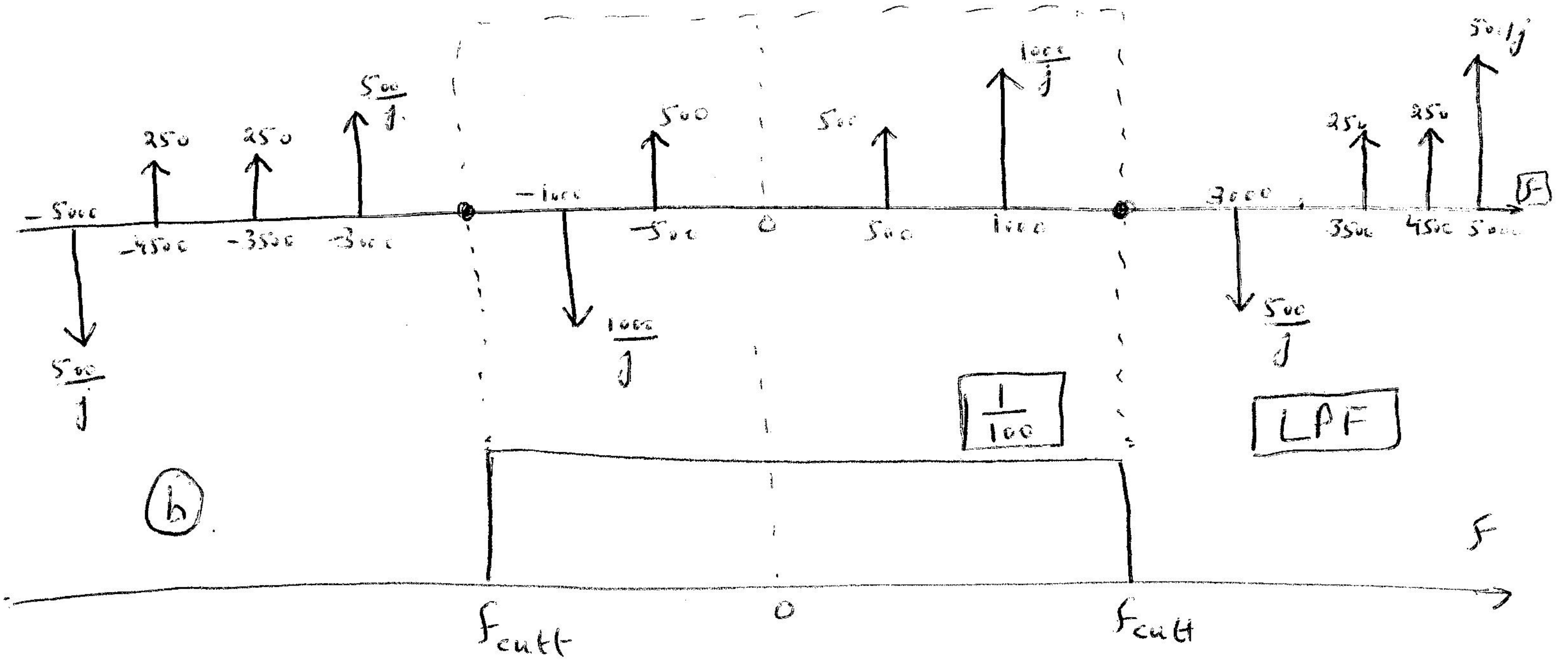
$v_1(t) = 2000 \cos^2(2\pi 2000 t) \cos(2\pi 500 t) + 4000 \cos^2(2\pi 2000 t) \sin(2\pi 1000 t)$

$= 1000 [1 + \cos(2\pi 4000 t)] \cos(2\pi 500 t) + 2000 [1 + \cos(2\pi 4000 t)] \sin(2\pi 1000 t)$

(c)

$$v_1(t) = 1000\cos(2\pi 500t) + 1000\cos(2\pi 4000t)\cos(2\pi 500t) + 2000\sin(2\pi 1000t) + 2000\cos(2\pi 4000t)\sin(2\pi 1000t)$$

$$v_1(t) = 1000\cos(2\pi 500t) + 500\left[\cos(2\pi 4500t) + \cos(2\pi 3500t)\right] + 2000\sin(2\pi 1000t) + 1000\left[\sin(2\pi 5000t) - \sin(2\pi 3000t)\right]$$



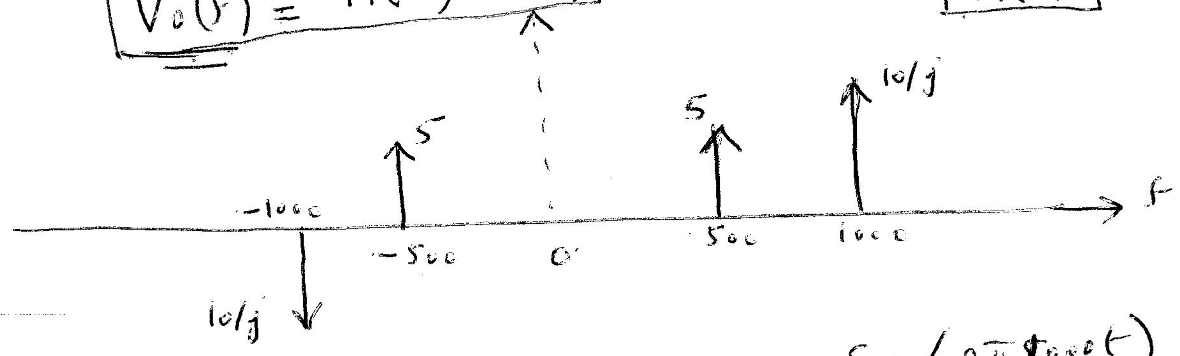
(b)

$1000\ Hz < f_{cutt} < 3000\ Hz$

LPF amplitude in Passband $= \frac{1}{100}$

(c) $V_o(f) = H(f)V_i(f)$

$V_i(f)$



$$v_o(t) = 10\cos(2\pi 500t) + 20\sin(2\pi 1000t)$$