EE-301: Signals and Systems Midterm Exam 1

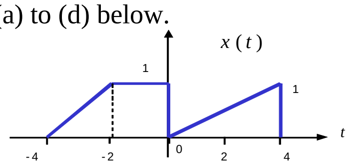
Electrical Engineering Department, College of Engineering, King Saud University (Winters 322)

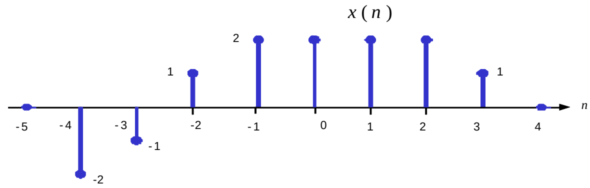
	تسلسل		الشعبة (وقت)		رقم الطالب	SOLUTION	اسم الطالب

Instructions: 1hours 30 mins allowed, answer all questions

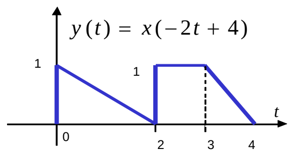
Q1. (30*marks total; 5marks each for parts* (a)-(f))

Given the continuous-time (CT) and discrete-time (DT) signals $x(t)$ and $x(n)$ shown below, sketch and label carefully the signals in (a) to (d) below.



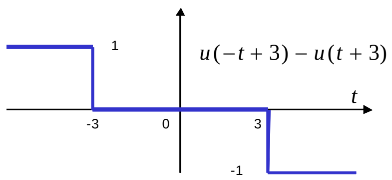


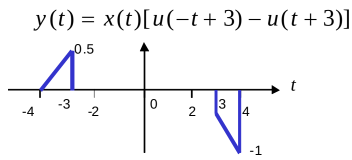
(a) $y(t) = x(-2t + 4)$



Ans: $y(t) = x[-2(t-2)]$

(b) $y(t) = x(t)[u(-t+3) - u(t+3)]$

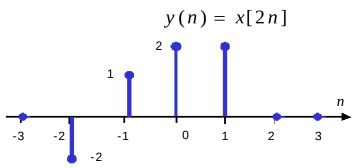


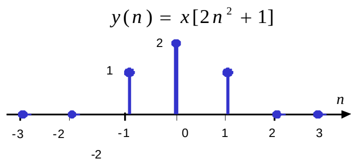


Ans: $y(t) = x(t)\cdot[u\{-(t-3)\} - u(t+3)]$

(c) $y[n] = x[2n]$

(d) $y[n] = x[2n^2 + 1]$





Ans:

(e) The input-output relation for a system is described by the equation $y(n)=x[2n-1]$. State if the system possesses the following properties: (i) Memoryless, (ii) BIBO (Bounded Input, Bounded Output) Stability, (iii) Causality, (iv) Linearity, and (v) Time Invariance.

Ans: (i) Not memoryless, (ii) BIBO stable, (iii) Non-causal, (iv) Linear, (v) Time varying.

(f) Using the properties of the impulse function, evaluate the following integrals:

(i) $\int_{-\infty}^{\infty} \delta(-t+1)\cos(t)\,dt$. Ans: $\int_{-\infty}^{\infty} \delta(-t+1)\cos(t)\,dt = \int_{-\infty}^{\infty} \delta[-(t-1)]\cdot\cos(t)\,dt = \int_{-\infty}^{\infty} \delta[(t-1)]\cdot\cos(t)\,dt = \cos(1)$

(ii) $\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}\,dt$. Ans: $\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}\,dt = e^{-j\omega t}\Big|_{t=0} = 1$

Q2. (30 *marks total; 10marks each for parts* (a)-(c))

(a) Classify the signals in (i) to (v) below as periodic / aperiodic and find the fundamental period, if periodic.

(i) $x(n) = \cos\left(\frac{14}{11}\pi n\right)$. Ans: $\omega_0 = (\frac{14}{11})\pi$, therefore $N = k \cdot \frac{2\pi}{(14/11)\pi} = 11$ for $k = 7$. Periodic with $N = 11$

(ii) $x(n) = \cos\left(\frac{14}{11}n\right)$. Ans: $\omega_0 = (\frac{14}{11})$, therefore $N = k \cdot \frac{2\pi}{(14/11)} \neq$ integer $\forall k$. Aperiodic

(iii) $x(n) = \cos(3\pi n) + \cos(\pi n) + \cos(4\pi n)$. Ans: $\omega_0 = \gcd(3\pi, \pi, 4\pi) = \pi$, thus Periodic with $N = \frac{2\pi}{\pi} = 2$.

(iv) $x(t) = \cos\left(\frac{14}{11}t\right)$. Ans: $\omega_0 = \frac{14}{11}$, thus Periodic with period $T_0 = \frac{2\pi}{\omega_0} = \frac{11}{7}\pi$.

(v) $x(t) = \cos(\pi t) + \sin(2t)$. Ans: $T_1 = \frac{2\pi}{\pi} = 2$, and $T_2 = \frac{2\pi}{2} = \pi$. Thus ratio test $\frac{T_1}{T_2} \neq \text{rational}$. Aperiodic

(b) Consider the CT signal $x(t)$ sketched below,

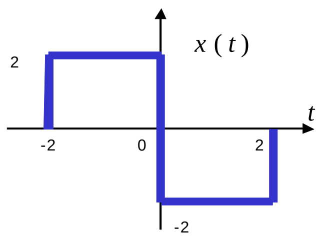
(i) determine the total Energy of the Signal. Ans: $E_{total} = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-2}^{0} (2)^2 \, dt + \int_{0}^{2} (-2)^2 \, dt = 16$ units.

(ii) determine the average Power of the signal. Ans: $P_{ave} = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = 0$.

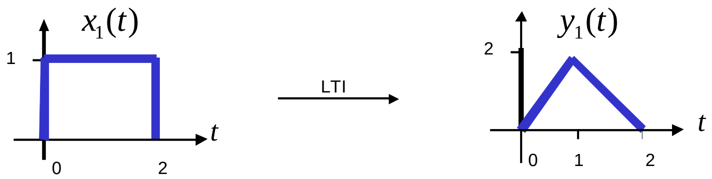
(iii) state if x(t) is a Power or Energy signal. Ans: Energy signal

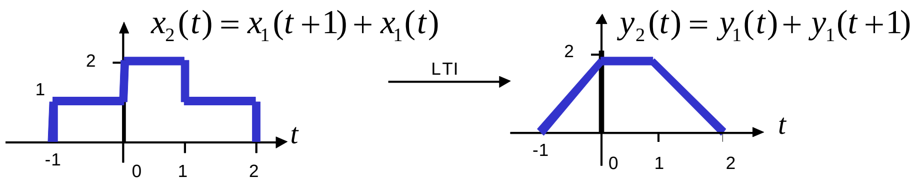
(iv) state if x(t) is an even or odd signal. Ans: Odd signal

(v) state if x(t) is Periodic or Aperiodic signal. Ans: Aperiodic signal



(c) The response of an LTI system to an input signal $x_1(t)$ is $y_1(t)$ as shown below. Determine and sketch the response of the system to the input signal $x_2(t)$.





Q3. (40 *marks total; 10marks each for parts* (a)-(d))

(a) The output of an LTI system with the input signal $x(t)$ can be computed as: $y(t)= x(t) * h(t)$, where $h(t)$ denotes the impulse response of the system, and * denotes the convolution operation. Show that $x(t-t_0) * h(t)= y(t-t_0)$.

(b) Determine and sketch the output of a DT LTI system with input signal $x(n)$ and impulse response $h(n)$ given by:

$$x[n]=u[n-3] \text{ and } h[n]=u[n]-u[n-3] .$$

(c) Determine and sketch the output of a CT LTI system with input signal $x(t)$ and impulse response $h(t)$ given by:

$$x(t)=e^{-4t}u(t) \text{ and } h(t)=u(t-2) .$$

(d) Assess the stability of the LTI systems with the impulse responses given in parts (b) and (c).

Ans:

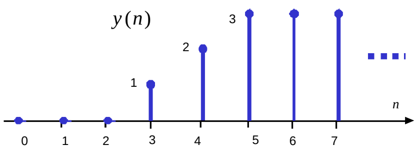
(a) For LTI systems, $y(t) = x(t) * h(t) = \int_0^\infty x(\tau)h(t-\tau)d\tau$ …..(1)

If $x(t)$ is shifted, the resulting output is given by: $x(t-t_0)*h(t)=\int_0^\infty x(\tau-t_0)h(t-\tau)d\tau$.

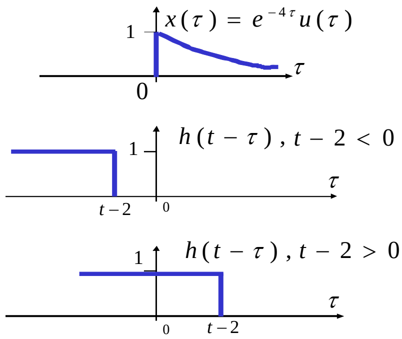
Let $\nu=\tau-t_0$, then we have: $\int_0^\infty x(\nu)h(t-\nu-t_0)d\nu = y(t-t_0)$, since $y(t')=\int_0^\infty x(\nu)h(t'-\nu)d\nu$ equals Eq(1) for $t'=t-t_0$.

(b) $y(n)=h(n)*x(n) =[\delta(n)+\delta(n-1)+\delta(n-2)]*x(n) = x(n)+x(n-1)+x(n-2)$

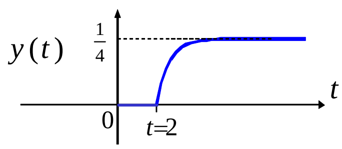
$$= \delta(n-3) + 2\delta(n-4) + \sum_{k=5}^{\infty} 3\delta(n-k)$$



[note: verify that graphical convolution gives same result].



(c) For $t-2<0$, $y(t)=0$



For $t-2>0$, $y(t)=\int_0^{t-2} e^{-4\tau}d\tau = \dfrac{[e^{-4(t-2)}-1]}{-4} = 0.25(1-e^{8}e^{-4t})$

(d) (i) $\sum_{k=0}^{\infty}|h(n)| = \sum_{k=0}^{\infty}|u(n)-u(n-3)| = 3$: stable, (ii) $\int_0^\infty |h(t)|\,dt = \int_0^\infty |u(t-2)|\,dt = \infty$: unstable