Design of Experiments

(Lecture III)

Dr. Adham Ragab

Outline

- Review of normal distribution basics.

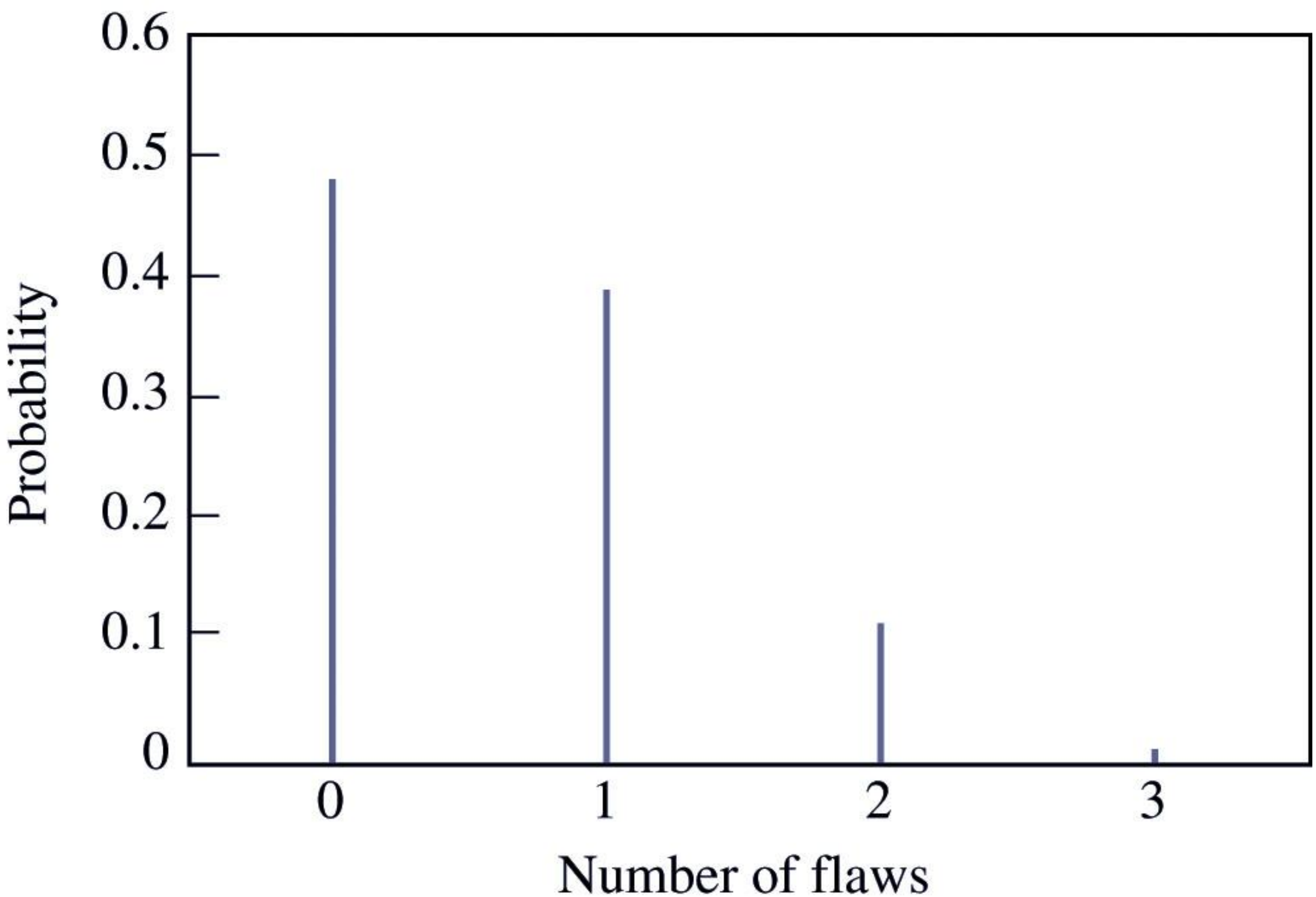
Objectives

- By the end of this lecture the student should be able to:
  - Recognize both discrete and continuous probability distribution.
  - Define the normal distribution
  - Calculate different requirements from the normal distribution

Discrete and Continuous Random Variables

- Discrete random variable is one whose possible values form a discrete set
- A continuous random variable is one whose possible values form a continuous set.

Probability Mass Function



Definition

- The description of the possible values of a random value *X* and the probabilities of each has a name: the probability distribution.

Section 4.5: The Normal Distribution

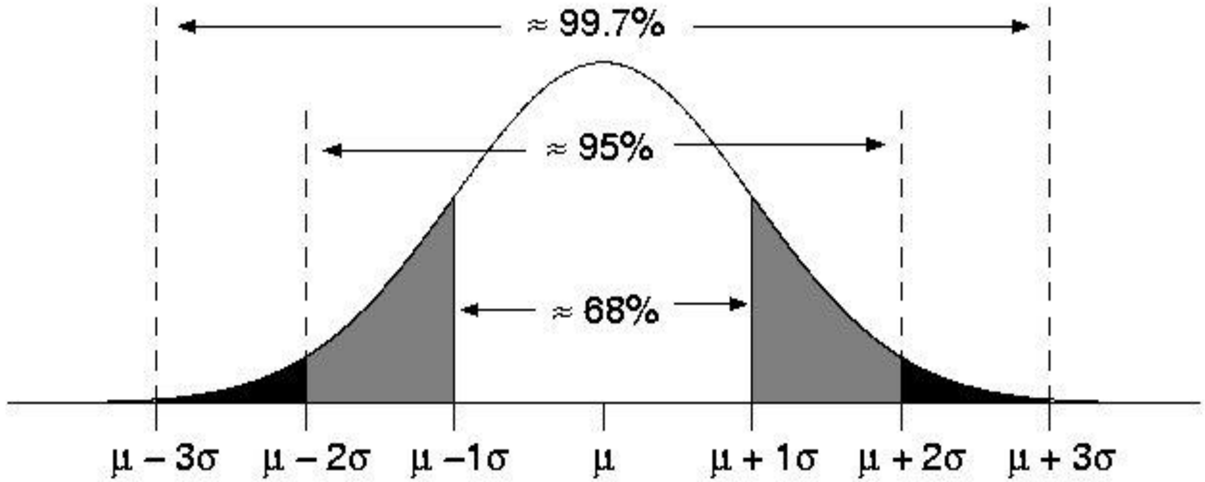
- The **normal distribution** (also called the Gaussian distribution) is by far the most commonly used distribution in statistics. This distribution provides a good model for many, although not all, continuous populations.
- The normal distribution is continuous rather than discrete. The mean of a normal population may have any value, and the variance may have any positive value.

Normal R.V.: pdf, mean, and variance

The probability density function of a normal population with mean μ and variance σ^2 is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \quad -\infty < x < \infty$$

68-95-99.7% Rule



This figure represents a plot of the normal probability density function with mean μ and standard deviation σ. Note that the curve is symmetric about μ, so that μ is the median as well as the mean. It is also the case for the normal population.

- About 68% of the population is in the interval $\mu \pm \sigma$.
- About 95% of the population is in the interval $\mu \pm 2\sigma$.
- About 99.7% of the population is in the interval $\mu \pm 3\sigma$.

Standard Normal Distribution

In general, we convert to standard units by subtracting the mean and dividing by the standard deviation. Thus, if *x* is an item sampled from a normal population with mean μ and variance σ^2, the standard unit equivalent of *x* is the number *z*, where

$$z = (x - \mu)/\sigma.$$

The number *z* is sometimes called the "z-score" of *x*. The *z*-score is an item sampled from a normal population with mean 0 and standard deviation of 1. This normal distribution is called the **standard normal distribution**.

Example

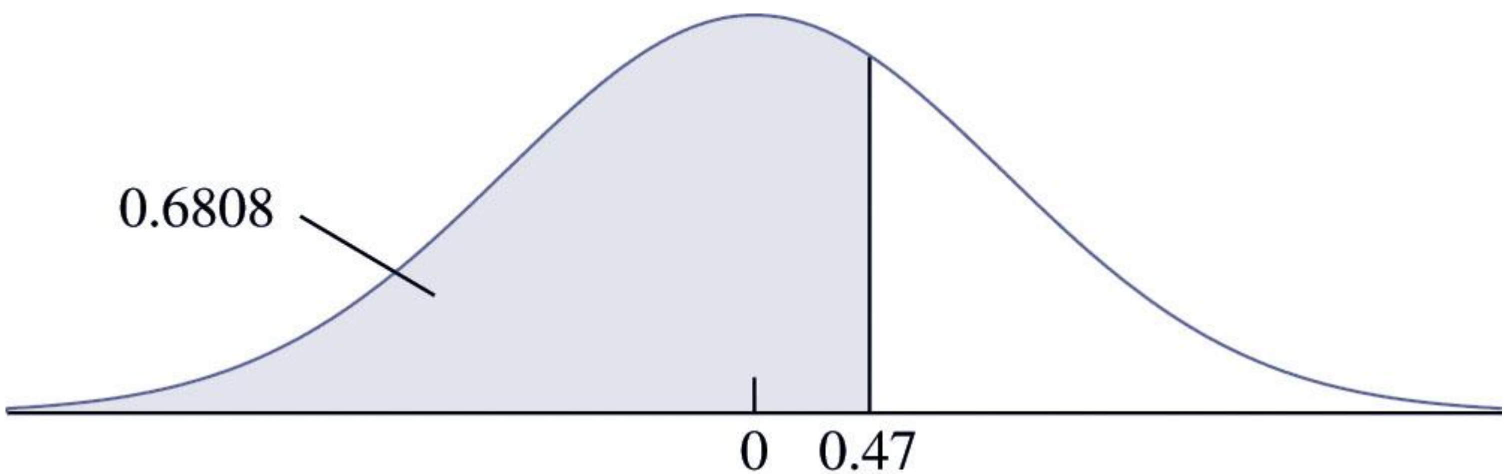
Aluminum sheets used to make beverage cans have thicknesses that are normally distributed with mean 10 and standard deviation 1.3. A particular sheet is 10.8 thousandths of an inch thick. Find the z-score.

Example

The thickness of a certain sheet has a z-score of -1.7. Find the thickness of the sheet in the original units of thousandths of inches.

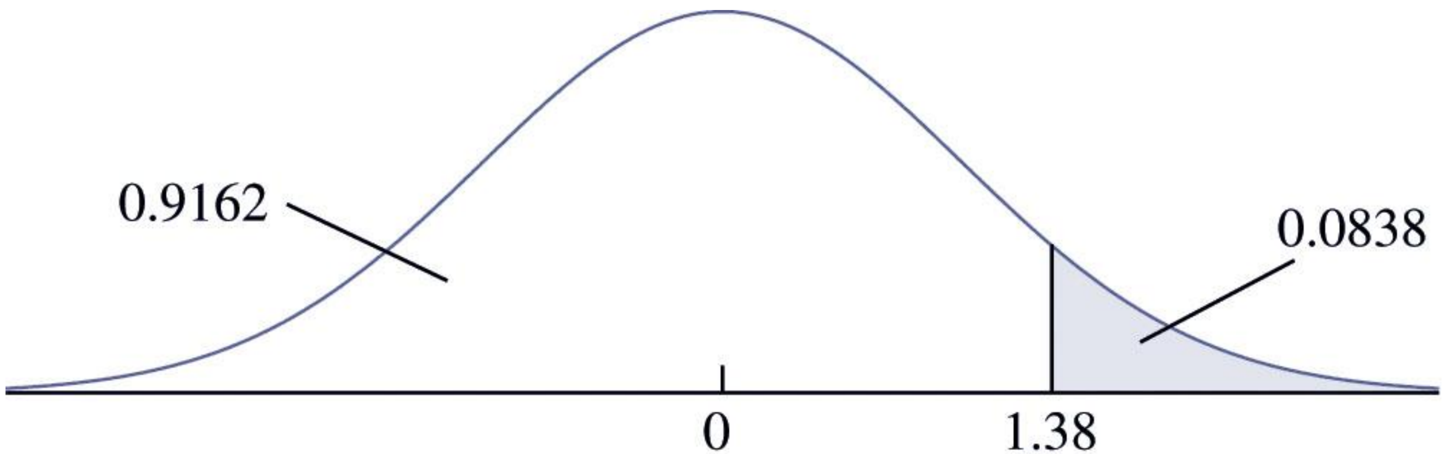
Examples

Find the area under normal curve to the left of $z = 0.47$.



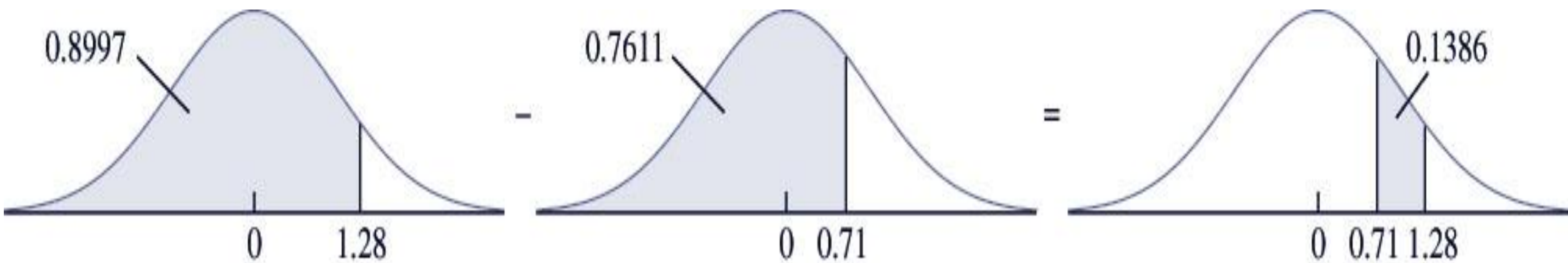
Examples

- Find the area under the curve to the right of $z = 1.38$.



Example 4.43

Find the area under the normal curve between $z = 0.71$ and $z = 1.28$.



Summary

- Normal distribution is the most common probability distribution used in statistics.
- Any normal distribution could be transformed to a standard normal distribution for the convenience of using the tables.