Mean – Variance Approach

Investors are taking their invest. Decisions

Expected Return	Risk TOTAL
$E(R)$	– Variance or – SD
$E(HPR)$	
$E(HPY)$	

positive relationship between Risk and Expected Return

Coefficient of Variation: CV

$$CV = \frac{\sigma(R_i)}{E(R_i)}$$

The Coefficient of Variation provides an idea about the amount of risk per each unit of Expected Return.

Ex: $\begin{cases} \sigma(R_i) = 2{,}5\% \\ E(R_i) = 10\% \end{cases}$, $\Rightarrow CV(R_i) = \frac{2{,}5}{10}$

$= 25\%$

Risk

In Finance, we assume that (σ^2) or (σ) are the best measures of risk.

Computation

Historical Data

$$\boxed{E(R_i) = \frac{\sum_{i=1}^{n} R_i}{n}}$$

n is the number of historical observations.

$$\sigma^2(R_i) = \frac{\sum_{i=1}^{n} \left(R_i - E(R_i)\right)^2}{n}$$

in Statistics

$$\sigma^2(R_i) = \frac{\sum \left(R_i - E(R_i)\right)^2}{n-1}$$

→ Convergent Estimator

Probability distribution

Return	Prob
R_1	p_1
R_2	p_2
⋮	⋮
R_n	p_n

$$\sum_{i=1}^{n} p_i = 1$$

$$\boxed{E(R) = \sum_{i=1}^{n} p_i R_i}$$

Variance:

$$\boxed{\sigma^2(R_i) = \sum_{i=1}^{n} p_i \left(R_i - E(R_i)\right)^2}$$

$$\sigma = \sqrt{\sigma^2}$$

Principle of Dominance:

In Finance, we assume that:

(H1) – Investors are risk-averse

(H2) – Investors are RATIONAL

(H1): Risk Aversion:

If we have 2 Assets A and B.

$$\begin{cases} E(R_A) = E(R_B) \\ \sigma(R_A) < \sigma(R_B) \end{cases} \Rightarrow$$ Select A having the lowest level of risk

(H2) Investors are Rational

Investors use the principle of Dominance when They are taking their investment decisions.

Let's Consider 2 Assets A and B.
With $E(R_A)$, $E(R_B)$, $\sigma(R_A)$, $\sigma(R_B)$ -

Case 1:

$\begin{cases} E(R_A) = E(R_B) \\ \sigma_A < \sigma_B \end{cases} \Rightarrow$ A is better than B

A dominates B

B is dominated by A

$\boxed{A \succ B}$

Case 2:

$\begin{cases} \sigma_A = \sigma_B \\ E(R_A) < E(R_B) \end{cases} \Rightarrow$ B is better than A

B dominates A.

$B \succ A$

Case 3: $\begin{cases} \sigma_A = \sigma_B \\ E_A = E_B \end{cases} \Rightarrow A \simeq B$

Same level of risk

Same level of $E(R_i)$.

Case 4: $\begin{bmatrix} E(R_A) \neq E(R_B) \\ \sigma_A \neq \sigma_B \end{bmatrix} \Rightarrow$ We use the CV

We Select Asset having the lowest Value of the Coefficient of Variation.

Prob

$$E(R_i) = \sum_{i=1}^{n} p_i R_i$$

$$\sum_{i=1}^{n} p_i = 1$$

if

$$p_1 = p_2 = \cdots p_n = \frac{1}{n}.$$

$$E(R_i) = \sum_{i=1}^{n} \frac{1}{n} R_i$$

$$\boxed{E(R_i) = \frac{1}{n} \sum_{i=1}^{n} R_i}$$

H.D

$$E(R_i) = \frac{\sum R_i}{n}.$$

H.D is a probability distribution. when we assume that events have the same prob.

$$p_1 = p_2 \text{ —— } = p_n$$

$$= \frac{1}{n}.$$

Required Rate of Return

- (F1) Time-Value of Money + Inflation
- (F2) ~~Inflation~~
- (F3) Risk

Inf ↑ $\xrightarrow{\text{How?}}$ $E(R_i)$ ↑↑

1)

Inf ↑ → Nominal Interest Rates ↑ → $E(R_1)$ > nominal rate

$E(R_i)$ ↑↑↑

Central Banks have to fix interest Rates higher than Inflation

2)

Inf ↑ → Investors are looking for higher expected Returns to compensate for the time value of Money → Asking for higher expected Returns ↑↑

Inf ↑ → T.V of money ↓↓

– Risk and Required Rate of Return

Risk $\uparrow\uparrow$ $\longrightarrow$ $E(R_i)\uparrow\uparrow\uparrow$

Different Arguments:

– Investors are risk averse $\uparrow\uparrow$ $\xrightarrow{+}$ Ask For Risk Premium $\uparrow\uparrow$

$\downarrow$

One Component of $E(R_i)$

$\downarrow$

using the CAPM

$\uparrow\uparrow\uparrow$

$$E(R_i) = RFR + \underbrace{[E(R_M) - RFR]}_{\uparrow\uparrow\uparrow} \overset{\text{Risk}}{\beta_i}$$

Risk and E(Ri)

Risk ↑↑ ——Why——→ Invt ↓↓ ↓

Cost of Equity ↑↑ ↓

$$\text{WACC} = K_S \times \frac{S}{D+S} + K_D \frac{D}{D+S}(1-T)$$ ↑↑↑↑

Risk ↑ ——→ K_S ↑↑, Risk Premium

Risk ↑ ——→ Debt ↑↑ ——→ K_D ↑↑

WACC ↑↑↑

↓

Shareholders will ask for Higher expected Returns

↑ Required Returns

Nominal and Real Interest Rates:

$$\text{Nominal RFR} = \left[\left(1+\text{Real RFR}\right)\left(1+\text{Inf}\right)\right] - 1.$$

$$\text{Real RFR} = \left[\frac{1+\text{Nominal RFR}}{1+\text{Inf}}\right] - 1$$

Example:

Let's Consider a PF:

- L.T. Bonds : 7,5%, $W_{LTB} = 0,5$
- U.S. T.Bills : 5,5%, $W_{T.B} = 0,25$
- Stocks : 12,5%, $W_S = 0,25$

$$\sum w_i = 1$$

$CPI_{2014} = 174$

$CPI_{2015} = 178$

1°/ Calcule Inflation Rate.

2°/ ——— the real Return for each asset.

Q3) Calculate the Real Return for the whole portfolio.

Answer:

1°/ Inflation:

$$I_{(2014,\,2015)} = \frac{CPI_{2015} - CPI_{2014}}{CPI_{2014}}$$

N.A:

$$\left[\frac{178 - 174}{174}\right] = 2{,}29\% \simeq 2{,}3\%$$

2°/ Real Return For each asset:

$$\text{Real Return} = \left[\frac{1 + \text{Nominal Return}}{1 + \text{Inf}}\right] - 1$$

For L.T. Bonds:

$$\text{Real} = \left[\frac{1 + 7{,}5\%}{1 + 2{,}3\%}\right] - 1 = 0{,}05 = 5\%$$

For T. Bills:

$$\text{Real} = \left[\frac{1 + 5{,}5\%}{1 + 2{,}3\%}\right] - 1 = 3{,}13\%$$

10%

For Stocks:

$$\text{Real} = \left[\frac{1 + 12{,}5\%}{1 + 2{,}3\%}\right] - 1 = \cancel{\;} = \cancel{\;}$$

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Comment :

For all the Selected Assets, the real Return ~~is~~ ~~higher~~ is positive because nominal Returns are higher than inflation.

3°/ Total Real Return For the portfolio :

$$\boxed{\begin{pmatrix} \text{Real} \\ \text{Return} \\ \text{PF} \end{pmatrix} = \sum_{i=1}^{3} w_i \text{ Real Return}_i}$$

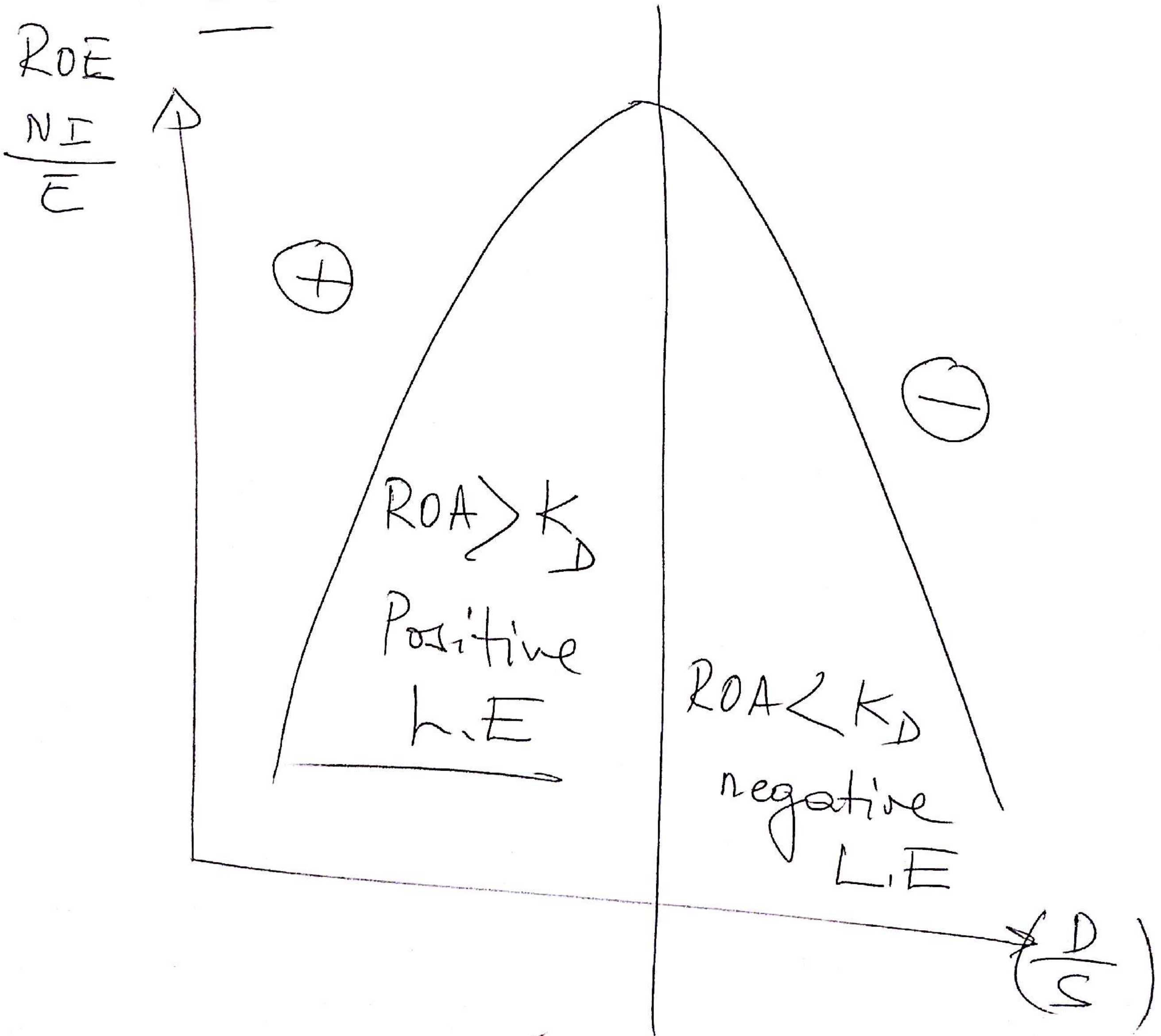
$$= (0{,}5 \times 5\%) + (0{,}25 \times 3.13\%) + (0{,}25 \times 10\%) = \_\_\_\_\_\_$$

Financial Risk

$$\text{Debt}\uparrow\uparrow \longrightarrow \left(\text{Financial Risk}\right)\uparrow\uparrow$$

leverage effect

$$\left(\frac{D}{S}\right) \; \ominus \oplus \; \rightleftarrows \; \text{ROE}$$



Assets	$\cancel{S}$ + D
$ROA = \dfrac{NI}{\Sigma A}$	$ROE = \dfrac{NI}{\Sigma \cancel{S}}$ ↑↑↑

توميتر الفريبي

$ROA > K_D \Longrightarrow D \uparrow\uparrow \longrightarrow ROE \uparrow\uparrow$

$ROA < K_D \Longrightarrow D \uparrow\uparrow \longrightarrow ROE \downarrow\downarrow\downarrow$

Cmaloui@ksu.edu.sa

Chaker aloui