

$$a_n = 2n + 1$$

$$a_0 = 1$$

$$a_1 = 3$$

⋮

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Methods of proof

1. proof by contradiction
2. sequential reasoning

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$n^2 - 2$ is not divisible by 3

$$n^2 - 2 \not\equiv 0 \pmod{3}$$

$$n = 3k$$

$$n = 3k + 1$$

$$n = 3k + 2$$

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Case 1

$$n = 3k$$

$$n^2 - 2 = (3k)^2 - 2 = 9k^2 - 2$$

$$= 3(3k^2) - 2$$

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Case 2: $n = 3k + 1$

$$(3k + 1)^2 - 2$$

$$9k^2 + 6k + 1 - 2$$

$$9k^2 + 6k - 1$$

$$3[3k^2 + 2k] - 1$$

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$$n = 3k + 2$$

$$(3k + 2)^2 - 2$$

$$9k^2 + 12k + 4 - 2$$

$$9k^2 + 12k + 2$$

$$3[3k^2 + 4k] + 2$$

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$$H_2$$

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$$\{a_n\} \quad n \leq N$$

$$a_n = 3n + 5 \quad n \geq 0$$

$$- \sum_{n=0}^k a_n = n$$

$$\sum_{n=0}^k a_n = 1 + 2 + \dots + k$$

$$\left. \begin{array}{l} a_0 = 0 \\ a_1 = 1 \\ a_2 = 2 \\ \dots \end{array} \right\}$$

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$$1 + 2 + 3 + \dots + (N-1) + N$$

$$N + (N-1) + (N-2) + \dots + 2 + 1$$

$$(N+1) + (N+1) + (N+1) + \dots$$

$$= N(N+1)$$

$$a + a = B \Rightarrow a = \frac{B}{2}$$

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$$1 + 2 + \dots + N = \frac{N(N+1)}{2}$$

$$a_{n+1} = a_n + r \quad \text{Badr}$$

$$a_{n+1} = a_n \times r$$

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$$\sum_{j=0}^n a^j = \frac{a^{n+1} - 1}{a - 1}$$

$$\sum_{j=l}^n a^j = \frac{(a^{n-l+1} - a^l)}{a - a^l}$$

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$$a_n = (-1)^n \quad n \geq 0$$

$$\sum_{n=0}^N a_n = 1 - 1 + 1 - 1 + 1$$

$$\sum_{n=0}^N a_n = \begin{cases} 1 & \text{if } N \text{ is even} \\ 0 & \text{if } N \text{ is odd} \end{cases}$$

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Logic,

$\neg(x < 4) \text{ then } y > 5$

TRUE

$\neg(x > 4) \text{ then } y < 5$

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$x > 0 \wedge y > 0 \rightarrow x + y > 0$

$p \rightarrow q$

contraposition, $\neg q \rightarrow \neg p$

$x + y \leq 0 \rightarrow x \leq 0 \text{ OR } y \leq 0$

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prove that:

$x + y > 73 \rightarrow x > 36 \vee y > 36$

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

$x \leq 36 \wedge y \leq 36 \rightarrow x + y \leq 73$

$\neg x > 73$

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$\neg\neg p \equiv p$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

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$p \wedge p \equiv p$

$p \vee p \equiv p$

$p \vee 0 \equiv p$ | $p \vee 1 \equiv 1$

$p \wedge 1 \equiv p$ | $p \wedge 0 \equiv 0$

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$p \rightarrow q \equiv \neg p \vee q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

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$$\neg p \vee q \equiv \neg \neg (p \vee q)$$

$$\neg (p \wedge \neg q)$$

Domain: {students at the university}

$P(x)$: student x is intelligent

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All students are intelligent

$\forall x \in D: P(x)$

$\exists x$

$\forall x \in \mathbb{N}: P(x)$

$P(x): x > 0$

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$P(x): x > 0$ [false] $-3 > 0$

$\forall x \in \mathbb{R}: P(x)$ [FALSE]

$\exists x \in \mathbb{R}: P(x)$ [TRUE]

↳ counterexample

$-3 \in \mathbb{R} \wedge P(-3) \text{ false}$

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$\forall x P(x)$

$\neg (\forall x P(x)) \equiv \exists x \neg P(x)$

$\neg (\exists x P(x)) \equiv \forall x \neg P(x)$

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Give the converse of

- $q \rightarrow r$
- if I am smart then I am rich
- if $x^2 = x$ then $x=0 \vee x=1$
- if $2+2=4$ then $2+4=8$

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$p \rightarrow r$

- $\neg r \rightarrow \neg p$
- if I am not rich then I am not smart
- $x \neq 1 \wedge x \neq 0$ then $x^2 \neq x$
- if $2+4 \neq 8$ then $2+2 \neq 4$

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Example

$$[\forall n \in \mathbb{N} : n^3 < 3^n]$$

$$P(n) : n^3 < 3^n$$

$n=0$	$0 < 1$	T
$n=1$	$1 < 3$	T
$n=2$	$8 < 9$	T
$n=3$	$3^3 < 3^3$	F

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$$\forall x \in \mathbb{R} (x+1)^2 \geq x^2$$

$x=-1$	0	≥ 1
$x=-2$	1	$\geq (-2)^2$

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