Controller Design by Pole placement

1. Introduction to control
2. Design of two position controller
3. Control design by pole placement
4. Control design by PID control

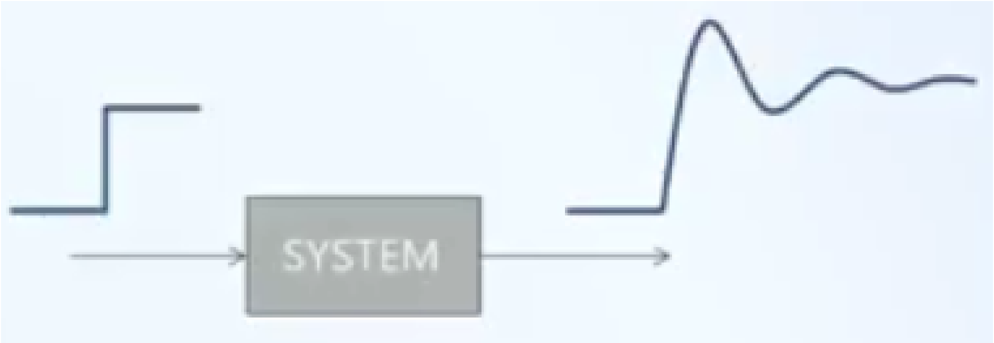
1 Introduction to Control

• So far we have modeled systems (mechanical, electromechanical and electric) and analyzed their time-response behavior.

• Because systems that stores energy cannot respond instantaneously, they exhibit a transient response when they are subjected to inputs or disturbances.

• Specifications may exists based on response to simple inputs.

•If a system does not have desired response (peak time too large, time response large, much oscillation, large overshoot), then it can be modified with control.



- t_d **(Delay Time)**: is the time needed for the response to reach half of its final value the very first time.
- t_r **(Rise Time)**: is the time required for the response to rise from 10% to 90%.
- t_p **(Peak Time):** is the time required for the response to reach the first peak of the overshoot.
- M_p **(Maximum percent Overshoot):** is the maximum peak value of the response curve.
- t_s **(Settling Time):** is the time required for the response curve to reach and stay within 2% of the final value.

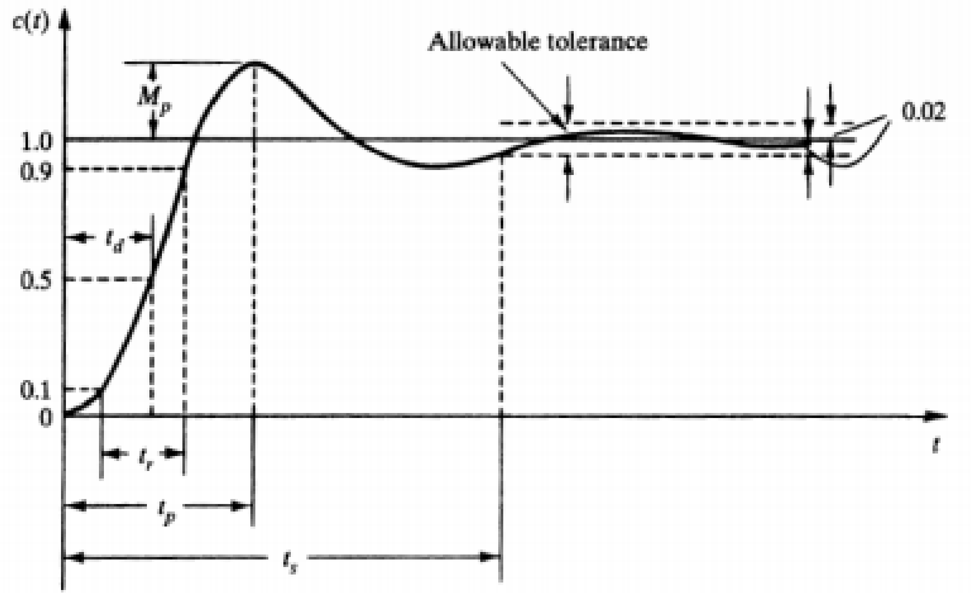


Figure 10–21 Transient-response specifications.

1.1 Control architecture (feed-forward control)

- **Open-loop control (feed-forward control).**

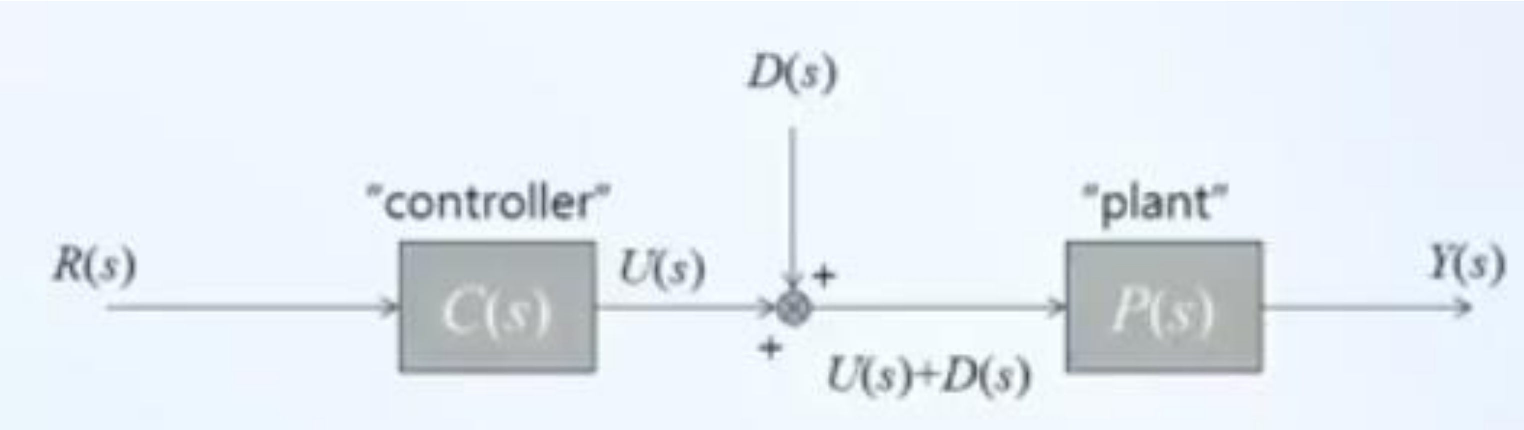
- **Simple to design (plant inversion).**

U(s) = C(s) R(s).

If we ignore D(s)

Y(s) = P(s) C(s) R(s) .

To have Y(s) = R(s) we need to put P(s) C(s) =1



Therefore, C(s) = 1/P(s) (Design a controller = inverse of the plant)

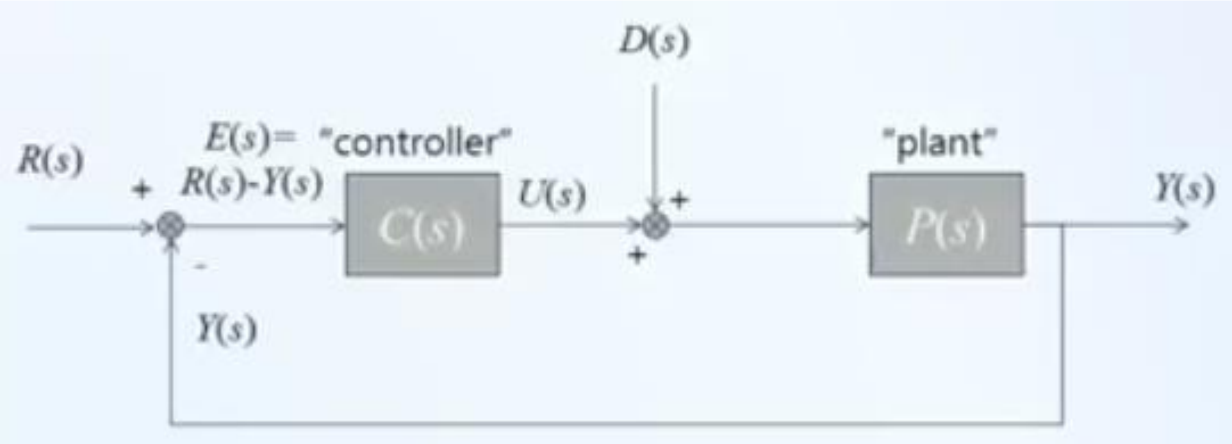
-**This is very simple and cheap** (no feedback so no sensor needed, no software to interact with sensor, no signal processing to use with the sensor).

-**Not robust**. (model error -> bad controller C(s)= 1/P(s) is not robust, disturbances change the plant , U(s) is not the only input)

- one way to address this limitations is to add feed-back.

1.3 Control architecture (feed-back control)

- **Closed-loop control (feed-back control).**



-More robust:
by measuring the output we can see the fact of no convergence of the output to the input (due to bad modeling or disturbances), and the controller changes the control signal to improve the system behavior.

-More complicated and expensive:
Add sensors and hardware or software to process the sensors' signals

- Can cause a stable system to become unstable.
For a stable system (poles in LHP) and stable controller (poles in LHP) but when we close the loop we obtain an unstable system .

-The response can be slower.
With feedback control we have to wait for the error to develop (dynamic of the plant) before establishing the control signal.

1.4 Control requirements

Fig.1 controlled process

- Experiences and knowledge in physics, mathematics, and control theory are required to design a stable controller with good performance.

- **Approach to Control design**: Translate engineering specifications into control requirements, then design a controller to meet those specifications.
    1. Determine what the system should do and how to do it (design specification),
    2. Determine the controller or compensator configuration relative to how it is connected to the controlled process.
    3. Determine the parameter values of the controller to achieve the design goals.

- **Example:** example in cars, increases the comfort and safety of driving a car or reduces the fuel consumption and exhaust gas emissions.
- **Examples transient specifications (system step response):** Time constantτ (speed of response of a first order system), over shoot M_p , peak time t_p , settle time t_s and rise time t_r. (feasible analytically only for second-order systems, or systems that can be approximated by a second-order system).

- **Example steady-state specifications:** Typically it is required that steady-state error be less than some amount, for example e_{ss} <0.02. We can use the final value theorem for different types of reference inputs $e_{ss} = lim_{s \to 0} sE(s)$

2 On-Off (Two-position) Controller

- This is a nonlinear controller which is very simple and it does not need any design. The On-Off Controller is defined as:

$$u(t) = \begin{cases} U_{max} & if\ e(t) > 0 \\ U_{min} & if\ e(t) < 0 \end{cases}$$

Where : $e(t) = r(t) - y(t)$ is the tracking error and u(t) is the applied control system.

- Control signal u(t) can have only two possible values. (fully-on, High u(t) =$U_{\max}$ or fully-off, low u(t) =$U_{\min}$) depending if error is positive or negative. The main idea in this way of control, which only two control levels achieve desired value of the controlled variable in shortest time possible.

- The control signal will oscillate between two levels (high frequency, damage the actuators) and good tracking accuracy is achieved.

- The control signal never reach the zero value which means the process consumes energy (cost is high).

- To avoid this high frequency oscillation an hysteresis ε is introduced.

$$u(t) = \begin{cases} U_{max} & if\ e(t) > +\varepsilon \\ U_{min} & if\ e(t) < -\varepsilon \end{cases}$$

2.1 Advantages and Drawbacks of ON-OFF controller

- The typical behavior of ON-OFF controller system is shown in Fig.1.
- **Advantages**: very simple, robust, and inexpensive.
- **Drawbacks**: large overshoot and undershoot.
- An on-off controller is the simplest form of temperature control device.
- On-off control is usually used where a precise control is not necessary

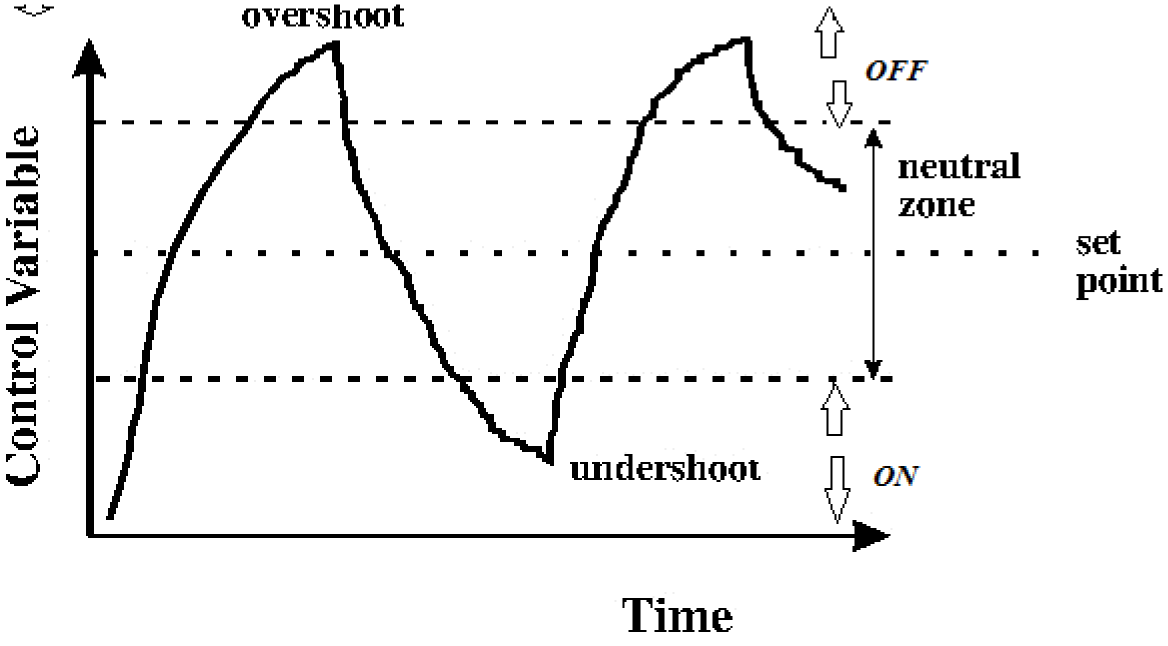


Fig.1 ON-OFF controller behavior

2.2 Example 1

Control the water level system by two-position controller. The transfer function of the water level system is: $\dot{h} = u(t)$where h(t) is the water level system.

SOLUTION

- Simulation using: SIMULINK : With eps=0.1 and U_{max}=5 and U_{min}=-5,
- We got the following simulation results:

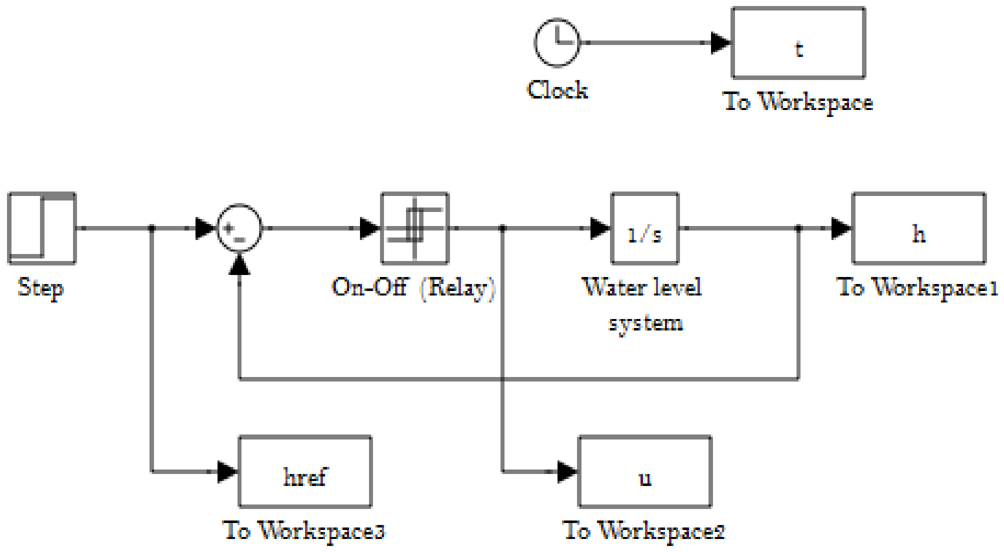


Fig.1 Simulink Program

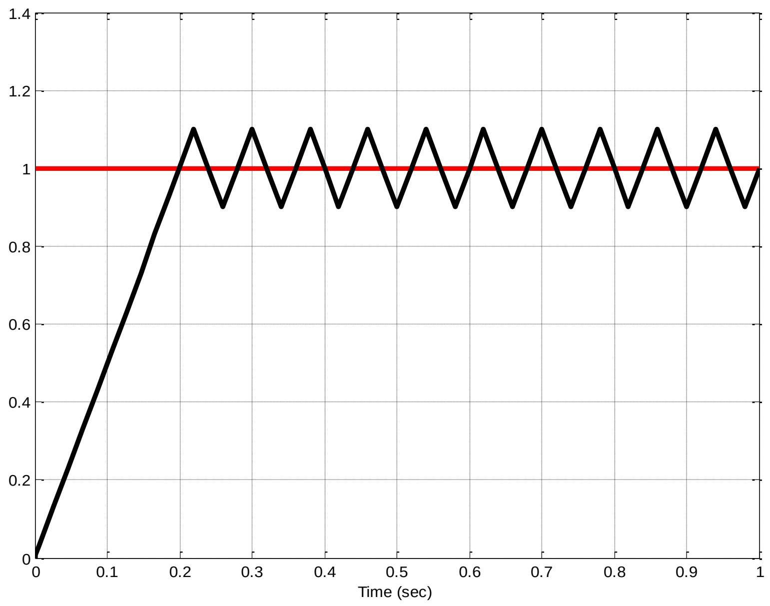


Fig.2 Output signal

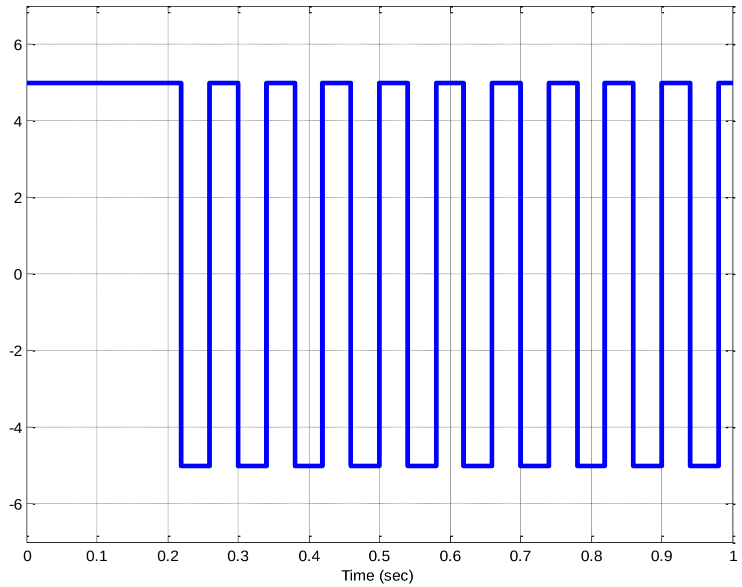


Fig.3 control signal

2.3 Example 2

- For another case (ref=8v; ε=±1; On=5v and Off=-5v).

We have: $u = 5v$ when $e(t) \succ -1; y_{ref} - y(t) \succ -1 \; or \; y(t) < yref + 1 = 9$

Or $u = -5v$ when $e(t) < \; 1; y_{ref} - y(t) < 1 \; \; or \; y(t) > yref - 1 = 7$

The problem is when $7 < y(t) < 9$, the control signal keeps the previous value.

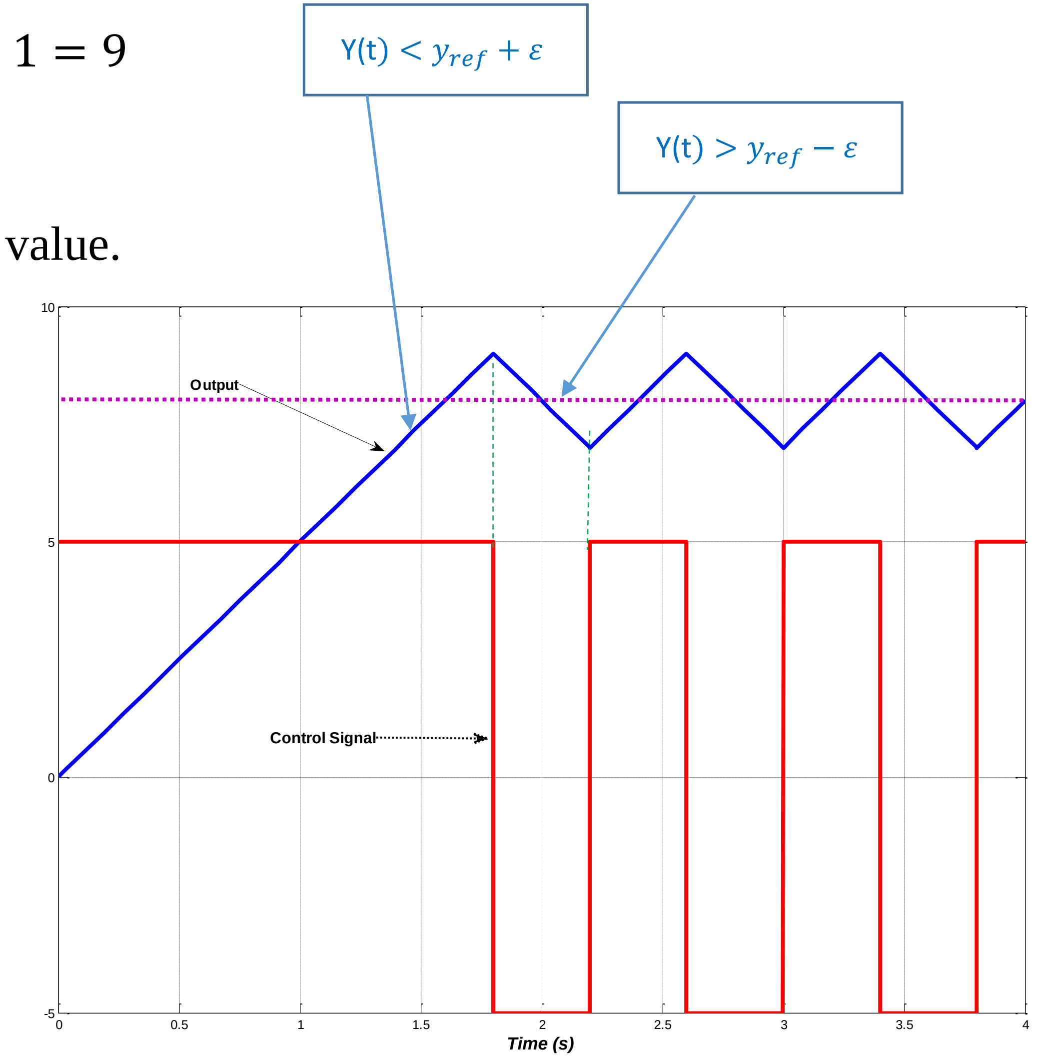


Fig.1 output signal and control signal

- We need to design a controller with smooth control signal, less energy consuming, and good tracking performances.

3 Design by pole placement

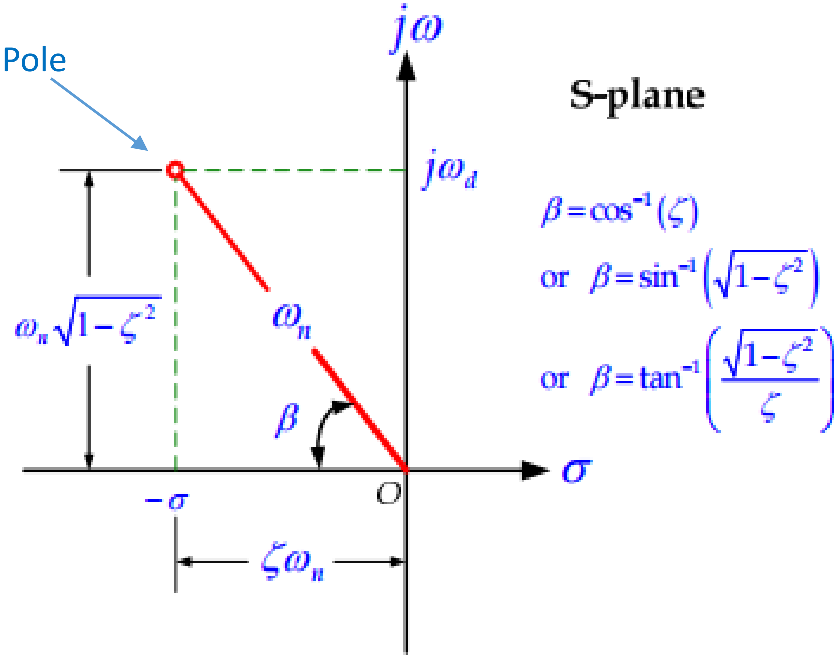
- Poles of a system affect the time response.

2nd order system transfer function: $G(s) = \frac{\omega_n^2}{s^2+2\xi\,\omega_n+\omega_n^2}$

The poles of the system: $P_{1,2} = \sigma \pm j\omega_d = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$

ξ is Damping Ratio

ω_n is the Natural frequency



Rise Time: $T_r = \frac{\pi-\beta}{\omega_d}$

Settling Time: $T_s = \frac{4}{\xi\,\omega_n}$

Delay Time: $T_d = \frac{1+0.7\,\xi}{\omega_n}$

Peak Time: $T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

Maximum Overshoot: $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100\%$

Controller Design

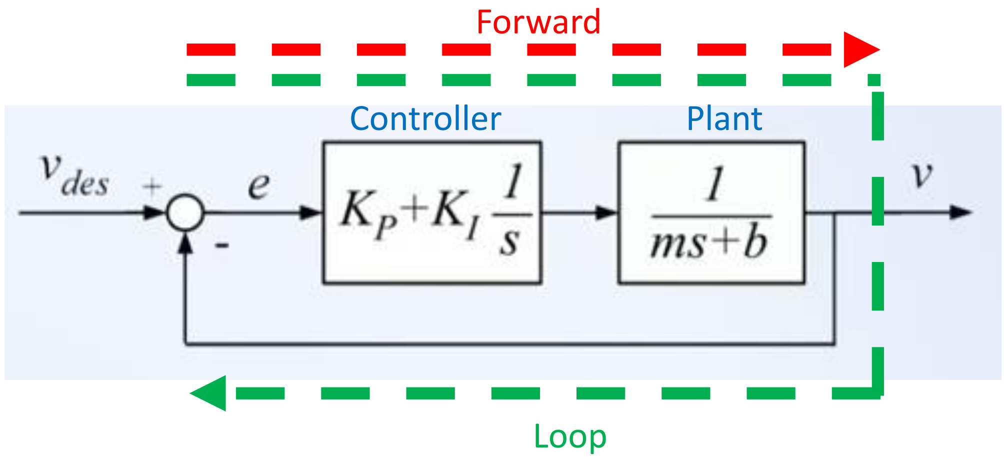
- Choose controller gains (for ex: K_p, K_I, and K_D) so that poles of the close-loop system meet given requirements.

- If the system is simple (canonical first order system or canonical second order system) can employ algebra (precise relationships between poles and the shape of the step response).

- If the system is non-canonical (higher order system) the relationships is not exact. One idea is to place one or two dominant poles to meet our requirements and place the rest of poles to be significantly faster, so that we can one sense neglect the effect of the dynamics for the other transient response.

3.1 Example

- Find the close-loop transfer function for a cruise control system with PI controller (How to design a PI controller, in other words how to choose the gains proportional gain K_p and integral gain K_I for a first order system).



- First step in the design is to find the close-loop transfer function to see the effect of the controller gains on the close-loop poles.

$$\frac{V(s)}{V_{des}(s)} = \frac{forward}{1 + loop} = \frac{(K_p + K_I \frac{1}{s})(\frac{1}{ms + b})}{1 + (K_p + K_I \frac{1}{s})(\frac{1}{ms + b})} = \frac{K_p\, s + K_I}{s\,(ms + b) + K_p\, s + K_I} = \frac{K_p\, s + K_I}{m\, s^2 + \left(b + K_p\right)s + K_I}$$

- we can see that the two gains affect the coefficients of the denominator.

3.1 Example (Continued)

$$\frac{V(s)}{V_{des}(s)} = \frac{K_p\, s + K_I}{m\, s^2 + (b + K_p)s + K_I}$$

- with two parameters (K_p and K_I) can place poles of this second order system anywhere we like (2 d.o.f. control)

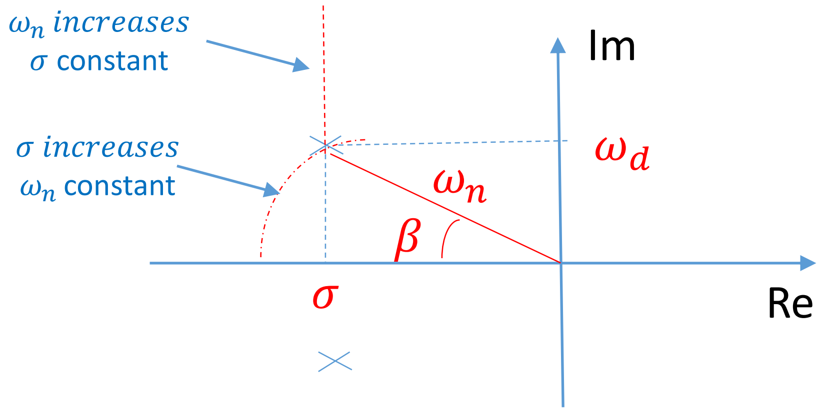
- To see the effect of changing the control gains we match this denominator to the canonical second order system denominator :

$$s^2 + 2\,\xi\,\omega_n\, s + {\omega_n}^2 = s^2 + \frac{(b + K_p)}{m} s + \frac{K_I}{m}$$

$$\Rightarrow \begin{cases} \xi\,\omega_n = \sigma = \dfrac{(b + K_p)}{2\,m} \\ {\omega_n}^2 = \dfrac{K_I}{m} \Rightarrow \omega_n = \sqrt{\dfrac{K_I}{m}} \end{cases}$$

K_p will directly affect σ which is the real part of our poles.

K_I directly affects ω_n which is the naturel frequency.



$$t_s = \frac{4}{\sigma}, \qquad t_p = \frac{\pi}{\omega_d}, \qquad M_p = e^{-\xi\pi/\sqrt{1-\xi^2}}$$

- Increasing K_p (holding K_I constant) ->σ increases (t_s smaller), ω_n = constant (the poles move on a circle of radius ω_n), ω_d decreases ($t_p$increases), β decreases → ξ larger less oscillations (towards real poles, M_p decreases).
- Increasing K_I (holding K_p constant) -> σ constant (t_s *inchanged*) ->ω_d increases (t_p decreases) ->ξ smaller (M_p increases)

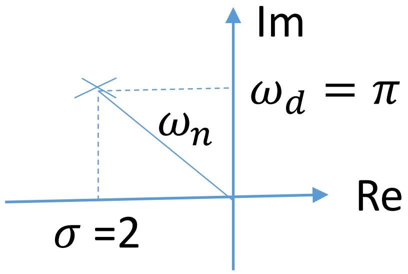
Example (Continued)

- Let m = 5 and b = 1. Choose K_P and K_I to achieve setting time of 2 seconds and a peak time of 1 second.
- the close-loop transfer function is:

$$\frac{V(s)}{V_{des}(s)} = \frac{1}{m}\frac{K_p\, s + K_I}{s^2 + \frac{(b+K_p)}{m}s + \frac{K_I}{m}} \Longrightarrow CLTF = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{1}{5}\frac{K_P s + K_I}{s^2 + \frac{1+K_P}{5}s + \frac{K_I}{5}}$$

$$t_s = \frac{4}{\sigma} \Longrightarrow \sigma = \frac{4}{2} = 2$$ Matching the coefficients $$\sigma = \xi\omega_n = \frac{1+K_P}{2\,(5)} = 2 \Longrightarrow K_P = 2(10) - 1 = 19$$

$$t_p = \frac{\pi}{\omega_d} = 1 \Longrightarrow \omega_d = \pi$$ Matching the coefficients $$\omega_n = \sqrt{\frac{K_I}{5}}$$



Recall that: $\omega_d = \omega_n\sqrt{1-\xi^2}$ Or we can use geometry $\omega_n = \sqrt{2^2 + \pi^2} \approx 3.72$

$$\Longrightarrow \omega_n = \sqrt{\frac{K_I}{5}} \approx 3.72 \Longrightarrow K_I \approx (3.72)^2\, 5 \approx 69.3$$

- The close-loop system with this controller will not have settling time of 2 seconds and peak time of 1 second, because we don't have a canonical system (presence of a zero in the numerator). The calculated gains will be a good starting point.

- the presence of a zero increase the overshoot and decrease the peak time (makes the system faster).

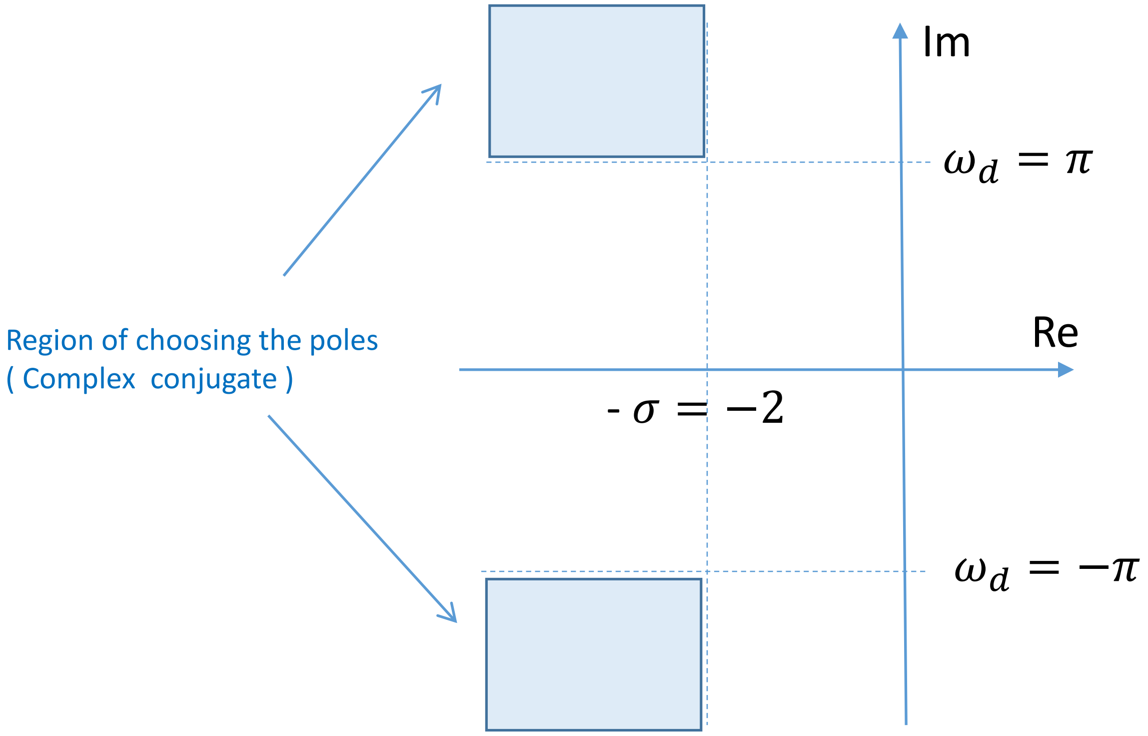
Example (Continued)

- Often requirements are given as inequalities.

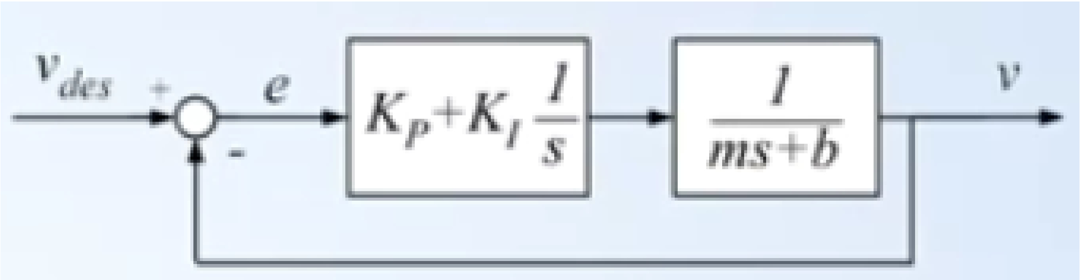
- Plot region in complex plane where poles of a canonical 2nd order system must be located in order to achieve a settling time **less** than 2 seconds and a peak time **less** than 1 second.

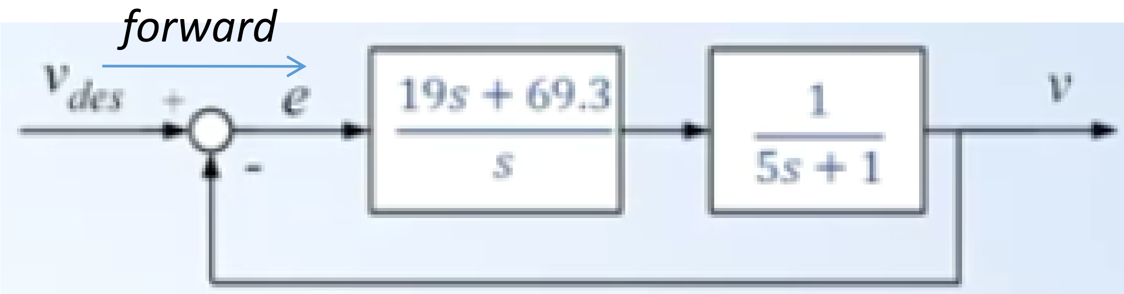
$$t_s = \frac{4}{\sigma} < 2 \qquad \Longrightarrow \qquad \sigma > 2$$

$$t_p = \frac{\pi}{\omega_d} < 1 \qquad \Longrightarrow \qquad \omega_d > \pi$$



Example





- What is the resulting steady-state error of this system for a unit ramp reference?

- We can calculate the error from $E(s) = V_{des}(s) - V(s)$

- One way to have the error using the transfer function of the input $V_{des}(s)$ and the output the error $E(s)$:

$$G(s) = \frac{E(s)}{V_{des}(s)} = \frac{forward}{1 + loop} = \frac{1}{1 + \frac{19s + 69.3}{s(5s+1)}} = \frac{s(5s+1)}{s(5s+1) + 19s + 69.3}$$

- We can then find the steady-state error by applying the final value theorem

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \left[\frac{s(5s+1)}{s(5s+1) + 19s + 69.3}\right]\frac{1}{s^2} = \frac{1}{69.3} = \frac{1}{K_I} \approx 0.0144 \qquad (input\ rampe\colon V_{des}(s) = \frac{1}{s^2})$$

- By increasing the integral gain $K_I$we reduce the steady-state error e_{ss}.

- The proportional gain K_P does not appear to affect the steady state error.

- If we have had a condition on the steady-state error that conflicted with the condition on the peak time or the settling time we may have to iterate in the design of our controller.

PID Controller (three-term Controller)

- The transfer function of the PID controller is given by:

$$C(s) = \frac{U(s)}{E(s)} = k_p + \frac{K_I}{s} + K_D\, s$$

Proportional To the error

Integral of the error

derivative Of the error

Where K_p is the proportional term, K_I is the integral term, and K_D is the derivative term.

- PID controller is used to increase or to achieve the desired performances in transient and steady state response.

- The controller input is the error (difference between the reference and the actual output) and the controller output is the control signal input to the plant).

- PID controllers are ubiquitous (very widely used in the industry).

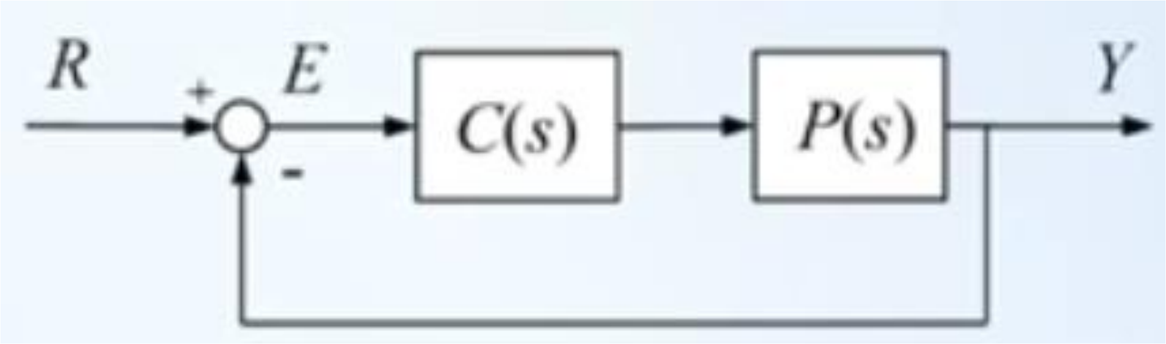
- Very intuitive, easy to implement.

- The three-term or PID control method has an older pedigree (origin) that was developed through long experience and by trial and error.

- Provide sophistication in that the control is based not only on the current error, but also the history of the error (integral of error) and the anticipated future error (derivative of the error).

PID Controller

• We will examine effect of PID control (effect of proportional term, derivative term and integral term) on a canonical 2nd order system to gain insight



• The transfer function of the plant (process) that we wish to control

$$P(s) = \frac{b}{s^2 + as + b}$$

• The closed-loop transfer function (negative feedback system)

$$\frac{Y(s)}{R(s)} = \frac{C(s)P(s)}{1 + C(s)P(s)} = \frac{bC(s)}{s^2 + as + b(1 + C(s))}$$

Proportional Control (P)

$$\frac{Y(s)}{R(s)} = \frac{bC(s)}{s^2 + as + b(1 + C(s))}$$

- ***We will see the effect of P control on transient performance***
- The close-loop transfer function
- Control effort is proportional to the amount of error .

$$C(s) = K_P$$

control signal = constant times the error , so bigger is the error bigger is the control signal

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{bK_P}{s^2 + as + b(1 + K_P)}$$

the P controller is used in our closed-loop TF. It has a form similar to a canonical 2nd order system (no zeros, only two poles).

- Matching with the parameters of a 2nd order canonical system (standard form).

$$\omega_n^2 = b(1 + K_P) \Rightarrow \omega_n = \sqrt{b(1 + K_P)}$$

Increasing K_p makes ω_n larger (make the system stiffer (rigid))

$$2\zeta\omega_n = a \Rightarrow \zeta = \frac{a}{2\omega_n} = \frac{a}{2\sqrt{b(1 + K_P)}}$$

Increasing K_p makes ξ smaller (decreasing the damping)

- Cannot set ω_n and ξ independently (one d.o.f only one parameter to tune).

Proportional Control (P)

- ***Effect of P control on steady-state performance***

  Steady -state value for a unit step reference. We apply the final value theorem to the output Y(s) of our close-loop system

$$y_{ss} = \lim_{s\to 0} sY(s) = \lim_{s\to 0} s \frac{bK_p}{s^2 + as + b(1+K_p)} \frac{1}{s} = \frac{bK_p}{b(1+K_p)} = \frac{K_p}{1+K_p} \neq 1$$

Transfer function

Unit step input

- The poles are all in the left side of the s-plane because all the coefficients of the characteristic equation have the same sign

- The final value of the system does not go to one (with P control we will have Some amount of error for unit step reference).

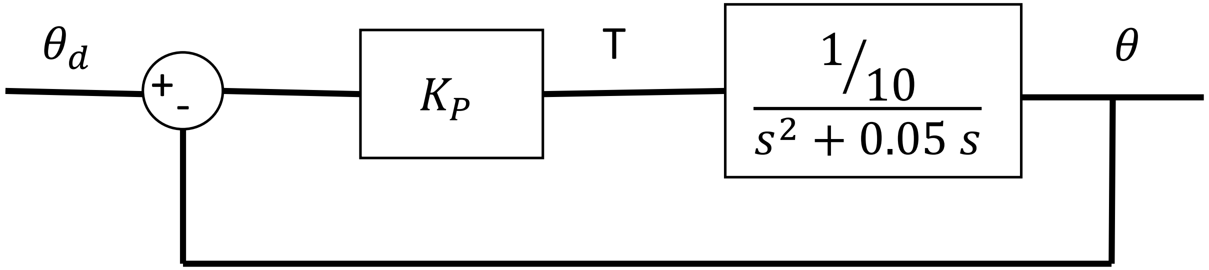
- In order to reduce the steady-state error we must increase K_p.

larger K_p makes $y_{ss} \to 1$ ... error goes to zero

- if we increase K_p to reduce the steady-state error we introduce other adverse effects such as increasing the overshoot with the oscillation in the performance of he system.

Example

The plant is a rotating mass with inertia J=10, the viscous friction b=0.5. To command the position θ the torque T is used, Find K_P for a proportional control to have $\xi = 0.7$.



For proportional control we have: $T = K_P\,(\theta_d - \theta)$

We have the close-loop transfer function:

$$CLTF = \frac{\frac{K_P/10}{s^2 + 0.05\,s}}{1 + \frac{K_P/10}{s^2 + 0.05\,s}} = \frac{K_P/10}{s^2 + 0.05\,s + K_P/10}$$

$$CLCE = s^2 + 0.05\,s + K_P/10$$

$$CLCE\ desired = s^2 + 2\xi\omega_n s + \omega_n^2$$

Matching

For $\xi = 0.7$

$$0.05 = 2\xi\omega_n = 2(0.7)\omega_n \Rightarrow \omega_n = \frac{0.05}{2(0.7)} = 0.0357$$

$$K_P/10 = \omega_n^2$$

$$\Rightarrow K_P = 10\,(0.0357)^2 = 0.0127$$

Proportional-Derivative Controller (PD)

- ***We will see the effect of adding a derivative action to our P controller***

- The close-loop transfer function

$$\frac{Y(s)}{R(s)} = \frac{bC(s)}{s^2 + as + b(1 + C(s))}$$

- With derivative control, do not have to wait for error to get large before control action becomes large, control anticipates the error (by using the slope of the variation of the error and even error is relatively small the control signal increase before the error gets large, so large control signal, and then improve damping).

- Never use D control by itself, amplifies noise (for D control we use PD or PID).

- The PD controller has a constant proportional to the error plus the proportional (constant K_D) to the differentiation of the error (the term s).

$$C(s) = K_P + K_D s$$

- Substituting in the close-loop controller

$$\Rightarrow \frac{Y(s)}{R(s)} = \frac{b(K_P + K_D s)}{s^2 + (a + bK_D)s + b(1 + K_P)}$$

K_D affects sigma term ξ with ω_n term

K_p affects ω_n

- Not canonical form (because of the zero in the numerator), but trends are meaningful (we can use this transfer function to build some insight and see the effect of the controlling gains).

- Can place two poles anywhere (by tuning two parameters K_p and K_D gaind, 2 d.o.f)

- If we apply the final value theorem to see the effect on the steady-state error we obtain the same affect as the proportional control by itself, so the addition of the derivative action didn't improve the steady-state error.

- Often, K_p is used first to increase the speed of the system and reduce the steady-state error through its effect on ω_n, and then K_D is used to bring down the overshoot through its effect on ξ.

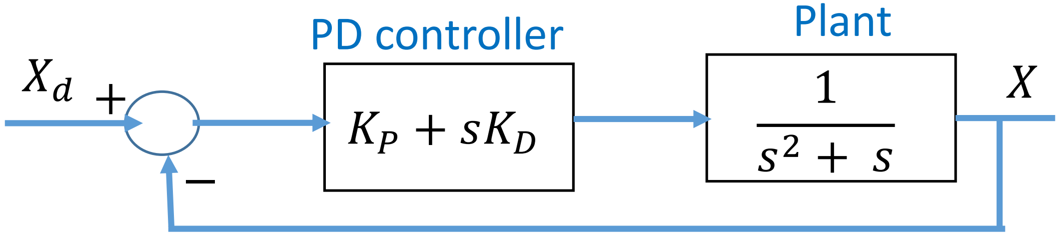
Example

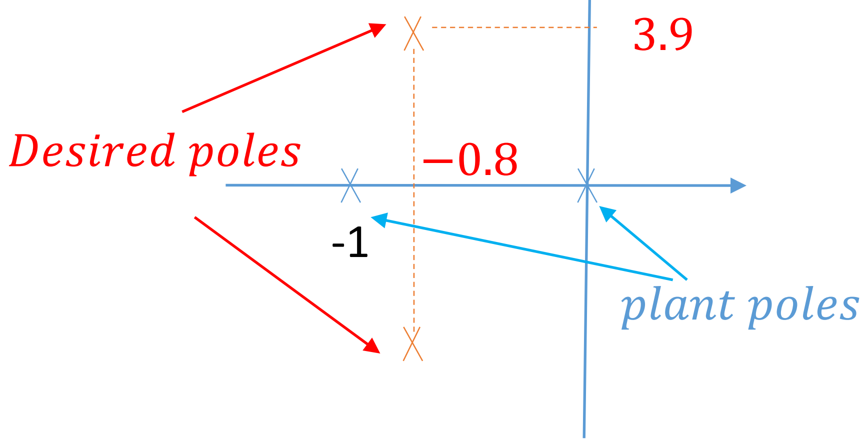
The plant G(s) is given in figure, use a PD controller to have the desired system response $\omega_n = 4\frac{rad}{s}$ *and* $\xi = 0.2$

$$G(s) = \frac{1}{s^2 + s}$$

For $\omega_n = 4$, $\xi = 0.2$ Poles at $-8 \pm 3.9\,i$ (using $s^2 + 2\xi\,\omega_n\,s + \omega_n{}^2 = 0$) ⇒ $(s + 0.8 + 3.9i)(s + 0.8 - 3.9i) = 0$

- The open-loop system poles are 0 and -1, it did not meet the desired specifications, we need to add a controller to have a new system.





$$CLTF = \frac{(K_P + sK_D)(\frac{1}{s^2 + s})}{1 + (K_P + sK_D)(\frac{1}{s^2 + s})} = \frac{(K_P + sK_D)}{s^2 + s + (K_P + sK_D)}$$

⇒ $CLCE = s^2 + s + (K_P + sK_D) = 0$

Example

- For the design we need to find the values of K_P and K_D such that the new system satisfies the design criteria. For second order system we have the characteristic equation (not canonical because of the zero in the numerator but a good start)

$$s^2 + 2\xi\, \omega_n\, \mathrm{s} + {\omega_n}^2 = 0$$

desired $s^2 + 2(0.2)\, 4\, \mathrm{s} + 4^2 = 0 \Rightarrow s^2 + 1.6s + 16 = 0$

$$CLCE = s^2 + (1 + K_D)s + K_P = 0$$

$$Desired\ CLCE = s^2 + 1.6s + 16 = 0$$

Matching coefficients $\Rightarrow$

$$\begin{cases} 16 = K_P \\ 1.6 = 1 + K_D \end{cases} \Rightarrow \begin{cases} K_P = 16 \\ K_D = 0.6 \end{cases}$$

Proportional-Integral Controller (PI)

- ***We will see the effect of adding integral action to our P controller***
- The close-loop transfer function

$$\frac{Y(s)}{R(s)} = \frac{bC(s)}{s^2 + as + b(1 + C(s))}$$

- The integral Control effort gets larger as error is accumulated (integrated)

$$C(s) = K_P + \frac{K_I}{s}$$

- The form of PI controller (term proportional to the error K_p with term proportional K_I to the integral (1/s) of the error)
- substituting in the close-loop controller $\Rightarrow \frac{Y(s)}{R(s)} = \frac{b\left(K_P + \frac{K_I}{s}\right)}{s^2 + as + b\left(1 + K_P + \frac{K_I}{s}\right)} = \frac{b(K_P s + K_I)}{s^3 + as^2 + b(1 + K_P)s + bK_I}$
- Cannot use canonical relations because third order with a zero, but tends to make system slower (sum of error)and more oscillatory.
- The transfer function of the controller can be written as: $C(s) = \frac{K_P s + K_I}{s} = K_P\left(\frac{s + b}{s}\right)$ Where $b = \frac{K_I}{K_p}$

Proportional-Integral Controller (PI)

- ***Effect of PI control on steady-state performance***

  Steady -state value for a unit step reference. We apply the final value theorem to the output Y(s) of our close-loop system

$$y_{ss} = \lim_{s\to 0} sY(s)$$

$$= \lim_{s\to 0} s \frac{b(K_P s + K_I)}{s^3 + as^2 + b(1+K_P)s + bK_I} \frac{1}{s} = \frac{bK_I}{bK_I} = 1$$

Transfer function

Unit stp input

For a reference one (unit step) the output is one

- Therefore, the steady-state error is zero for a step reference, even for small K_I (just takes longer to reach steady-state).

•The benefit of using the integral term is to eliminate the steady-state error.

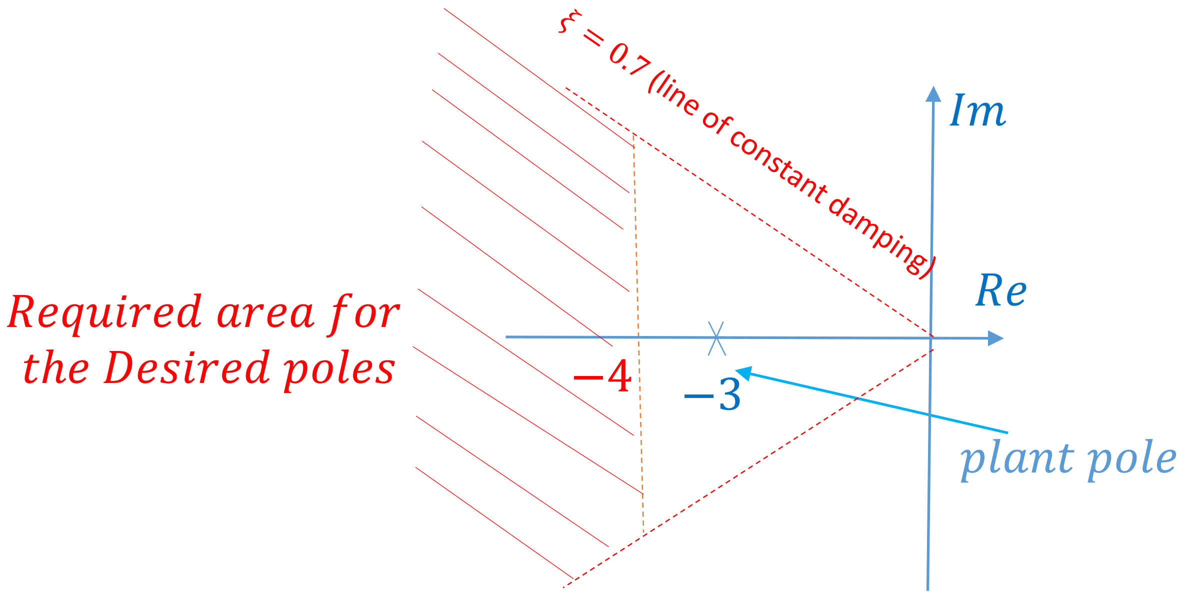
Example

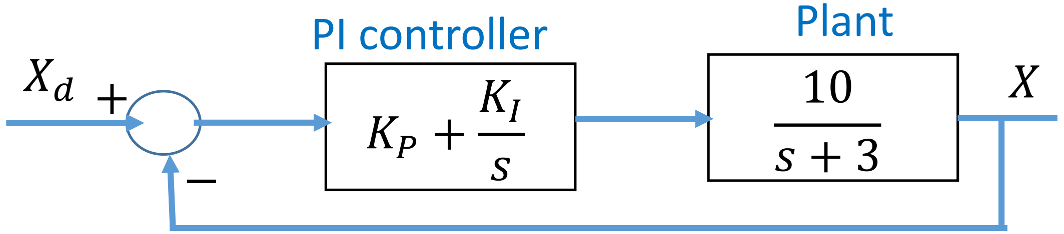
The plant G(s) is given, use a PI controller to have the desired system response:

$$G(s) = \frac{10}{s+3}$$

$$\xi = 0.7, \qquad T_s < 1s, e_{ss} = 0 \; for\ step\ response$$

- The plant poles: -3
- The desired system poles: $T_s = \frac{4}{\sigma} < 1 \rightarrow \sigma > 4$
- The open-loop system did not meet the desired specifications, we need to add a controller to have a new system.





$$CLTF = \frac{\left(K_P + \frac{K_I}{s}\right)\left(\frac{10}{s+3}\right)}{1 + \left(K_P + \frac{K_I}{s}\right)\left(\frac{10}{s+3}\right)} = \frac{10(K_P s + K_I)}{s^2 + 3s + 10(K_P s + K_I)} \Rightarrow CLCE = s^2 + (10K_P + 3)s + 10K_I = 0$$

$$CLCE = s^2 + (10K_P + 3)s + 10K_I = 0$$
$$s^2 + 2\xi\, \omega_n\, s + \omega_n^2 = 0$$

Matching coefficients

$$\begin{cases} \omega_n^2 = 10\, K_I \\ 2\xi\, \omega_n = 10K_P + 3 \end{cases}$$

We have

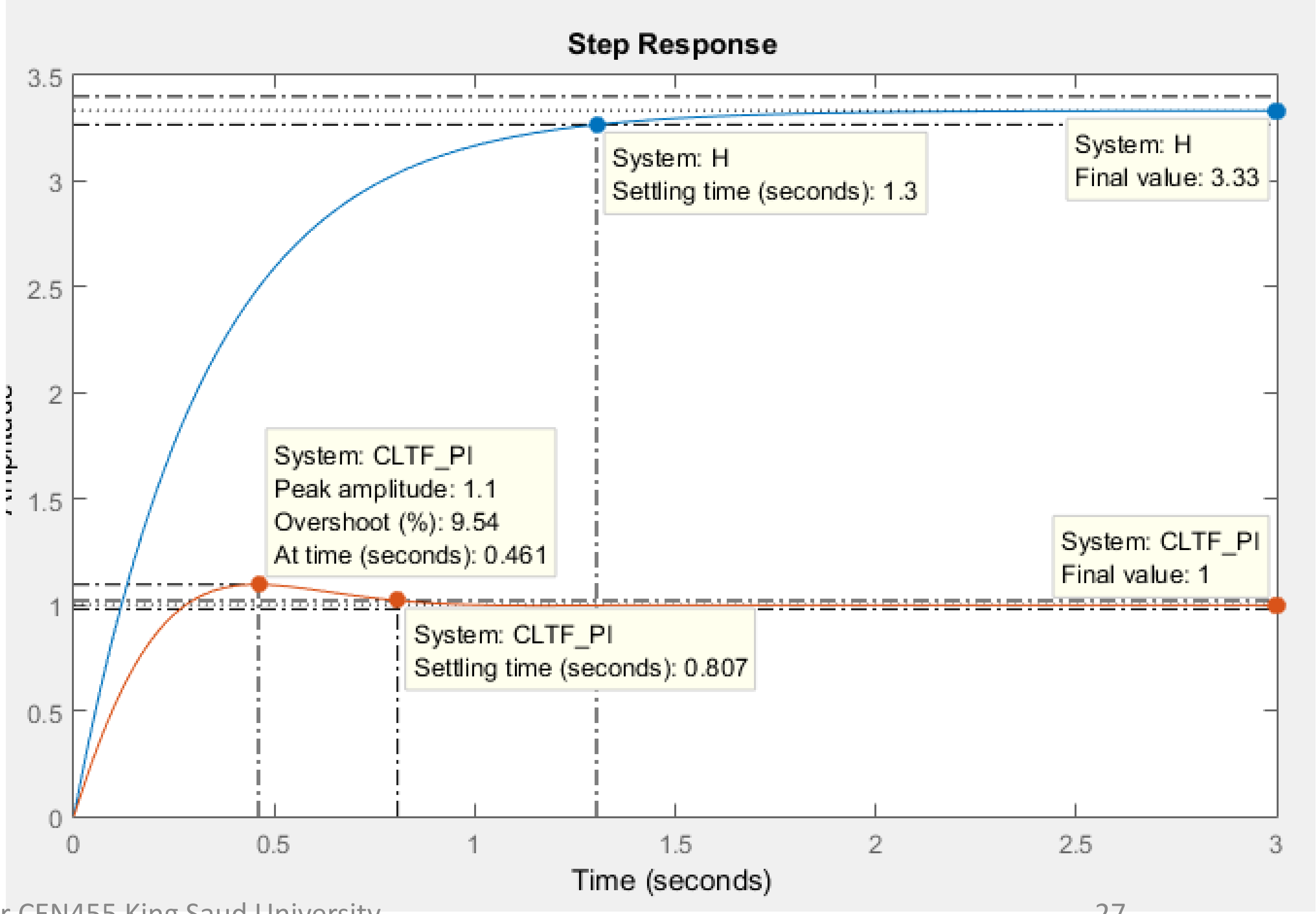
$$\begin{cases} T_s = \dfrac{4}{\xi\, \omega_n} < 1 \\ \xi = 0.7 \end{cases} \Rightarrow \begin{cases} \omega_n > \dfrac{4}{0.7} = 5.71 \\ K_I = \dfrac{\omega_n^2}{10} > 3.27 \\ K_P > \dfrac{2\xi\, \omega_n - 3}{10} \approx 0.5 \end{cases}$$

For

$$\begin{cases} K_P = 0.6 \\ K_I = 4 \end{cases}$$

- using Matlab we can find the values of K_P and K_I to put the poles in the desired area to meet the design specifications.

```
%-- mASS SYSTEM -------
clc;
%- Mechanical system --
H=tf(10,[1 3])
step(H)
%------ PI Control ---
figure(1)
rlocus(H)
kp=0.6; % proportional
ki=4; % integral
GPI =tf([kp ki],[1 0]);
CLTF_PI=feedback(GPI*H,1)
figure(2)
step(H,CLTF_PI)
```



PID Control

1. Some intuition about the effect of the PID terms.

• **Increasing** K_p : Same amount of error generates a proportionally larger amount of control, makes system faster, but overshoot more (less stable).

•**Increasing** K_D : Allows controller to anticipate an increase in error, adds damping to the system (reduce overshoot), can amplify noise.

• **Increasing** K_I : Control effort builds as error is integrated over time, helps reduce steady-state error, but can be slow to respond.

2. For systems that are not canonical first or second order, need to use trial and error. Can look for reduced-order approximation (try to identify if there are any poles that dominate the response, try to identify if there are any of zeros cancel with any of the poles, and if we can take a higher order system and approximates it as first or second order system.

3. Following table is helpful, though not always true

	$K_P \uparrow$	$K_D \uparrow$	$K_I \uparrow$
ss error	$\downarrow$	---	$\downarrow$
rise time	$\downarrow$	---	?
settling time	---	$\downarrow$	? ($\uparrow$)
overshoot	$\uparrow$	$\downarrow$	? ($\uparrow$)