Capital Centrality in a Rail Network

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***Abstract*— Vertex centrality in a network is one of many concepts that have been created in social network analysis.**

In this paper, we apply centrality measures on a rail network in Kingdom of Saudi Arabia in the south-west of Asia which includes the city of Riyadh and determines if Riyadh is the city with the highest centrality.

Using degree centrality, closeness centrality, and betweenness centrality, Riyadh is established to be the city with the highest centrality.

Keywords- Network; vertex centrality; degree centrality; closeness centrality; betweenness centrality.

I. INTRODUCTION

Vertex centrality is a real-valued function assigning to each vertex in a network some value. The higher the value, the more central the vertex is for the network [11]. It is a significant concept in the study of any networked system. Measures of centrality try to capture the notion of the importance of vertices in networks. A network consists of a set of nodes (vertices), and ties (edges) representing some relationship between the nodes [7].

Since Riyadh is the capital of the Kingdom of Saudi Arabia and it is known to be a paramount center given it's important financial, business, and manufacturing place [9]. This paper establishes this importance by applying network centrality measures to a rail network in Saudi Arabia which includes Riyadh, Figure 1 [1] is used.

Figure 1. Rail Network Map in KSA

II. METHODOLOGY

Rail network (given above) will be used to study centrality using the following concepts, degree centrality, closeness centrality, and betweenness centrality.

Where degree centrality is defined as the number of edges incident upon a vertex (i.e., the number of edges that a vertex is an incident with) [10].

Closeness centrality of a vertex is the number of other vertices divided by the sum of all distance between the vertex and all others [6].

Lastly, betweenness centrality assesses the degree to which a vertex lies on the shortest paths between pairs of other vertices [4].

Betweenness centrality of the vertex v is defined by $Cb(v) = \sum \sigma_v(u,y)/\sigma(u,y)$, where $\sigma_v(u,y)$ is the number of these shortest paths that pass through vertex v in the network and $\sigma(u,y)$ is the number of shortest paths from vertex u to vertex y [11].

Graph of the rail network map in Figure 1, is given below in Figure 2.

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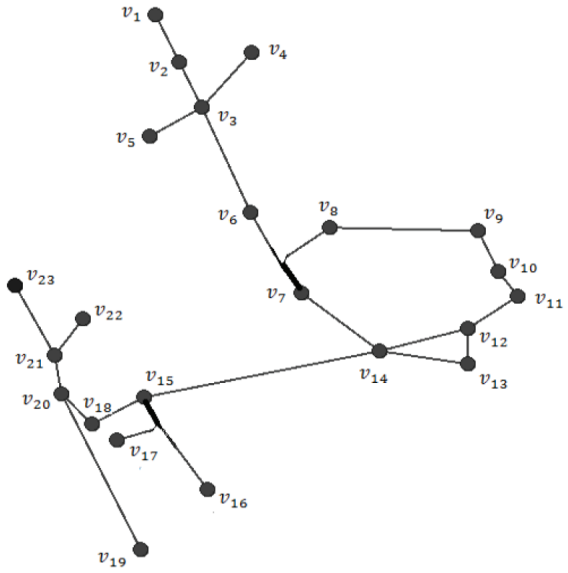

Figure 2. graph of Rail Network Map in KSA

Indexed vertices in Figure 2 are names of cities in the map. Table 1 specifies the city for each vertex.

Table 1

v_1	Al-Hadeetha	v_{13}	Hardh
v_2	Al-Quorayyat	v_{14}	Riyadh
v_3	Al-Jouf	v_{15}	Al-Muwayh
v_4	Hazm	v_{16}	Khamis Mesheet
v_5	Al-Baseeta	v_{17}	Al-Taaf
v_6	Hail	v_{18}	Makkah
v_7	Buraidah	v_{19}	Gizan
v_8	Zubariya	v_{20}	Jeddah
v_9	Ras Al-Zour	v_{21}	Rabigh
v_{10}	Jubail	v_{22}	Madinah
v_{11}	Dammam	v_{23}	Yanbu
v_{12}	Hafouf		

To verify if Riyadh is the city with the highest centrality, degree centrality, closeness centrality, and betweenness centrality are measured.

III. FINDINGS

After calculating degree centrality Cd, closeness centrality Cc and betweenness centrality Cb for each vertex in Figure 2, results are specified in Table 2:

Table 2

v_i	Cd(v_i)	Cc(v_i)	Cb(v_i)
v_1	≈ 0.0455	≈ 0.1679	0
v_2	≈ 0.0909	0.2	≈ 0.0830
v_3	≈ 0.1818	≈ 0.2418	≈ 0.3004
v_4	≈ 0.0455	≈ 0.1964	0
v_5	≈ 0.0455	≈ 0.1964	0
v_6	≈ 0.0909	≈ 0.2821	≈ 0.3360
v_7	≈ 0.1364	≈ 0.3333	≈ 0.4802
v_8	≈ 0.0909	≈ 0.2651	≈ 0.0968
v_9	≈ 0.0909	≈ 0.2651	≈ 0.0395
v_{10}	≈ 0.0909	≈ 0.2115	≈ 0.0138
v_{11}	≈ 0.0909	≈ 0.2341	≈ 0.0534
v_{12}	≈ 0.1364	≈ 0.2821	≈ 0.1166
v_{13}	≈ 0.0909	≈ 0.2716	0
v_{14}	≈ 0.1818	≈ 0.3492	≈ 0.5553
v_{15}	≈ 0.1818	≈ 0.3235	≈ 0.4941
v_{16}	≈ 0.0455	≈ 0.2472	0
v_{17}	≈ 0.0455	≈ 0.2472	0
v_{18}	≈ 0.0909	≈ 0.2785	≈ 0.3360
v_{19}	≈ 0.0455	≈ 0.1947	0
v_{20}	≈ 0.1364	≈ 0.2391	≈ 0.2964
v_{21}	≈ 0.1364	≈ 0.2018	≈ 0.1581
v_{22}	≈ 0.0455	≈ 0.1746	0
v_{23}	≈ 0.0455	≈ 0.1746	0

From Table 2, v_3, v_{14} and v_{15} have the highest degree centrality 0.1818, which means they are quite active in the network. However, they are not necessarily the most powerful cities because they are only directly connected within one degree to vertices in their surroundings, they have to go through each other to get to another surroundings.

For closeness centrality, v_{14} has the highest closeness centrality because it can reach more cities through shorter paths. As such, v_{14} placement allows it to connect to cities in its surrounding, and to cities that span surroundings.

For betweenness centrality, v_{14} has the highest betweenness centrality because it is lying on a high proportion of paths between other vertices in the network.

Therefore, although v_3, v_{15} have the same degree centrality as v_{14}, the fact is v_{14} has additional highest closeness centrality and betweenness centrality. All this results establishes v_{14} to be the vertex with highest centrality.

Our calculations have established v_{14} which is Riyadh to be the capital using centrality concepts.

CONCLUSIONS

The goal is to verify if Riyadh is the capital by using three pertinent centrality measurements.

We found from results of Table 2 that Riyadh acquired the highest values in all accounts of centrality, although I had a belief that v_{15} may be the chosen, these measurements proved their worth and accuracy.

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