الاعداد المركبة

تذكير:

$\mathbb{N} = \{1, 2, 3, \dots\}$ الأعداد الطبيعية

$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$ الأعداد الصحيحة

$\mathbb{D} = \left\{ \frac{a}{10^n} : a \in \mathbb{Z}, n \in \mathbb{N} \right\}$ الأعداد العشرية

$\mathbb{Q} = \left\{ \frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z} \setminus \{0\} \right\}$

$= \left\{ \frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{N} \right\}$ الأعداد الكسرية

$\mathbb{R} = (-\infty, +\infty)$: الأعداد الحقيقية

ملاحظات: (1)

$$1.\underbrace{32501}_{\text{الجزء العشري}} = \frac{132501}{10}$$

الجزء الصحيح

(2)

$$\frac{-a}{-b} = \frac{a}{b}$$

$$\frac{a}{-b} = \frac{-a}{b}$$

(3)

$$\frac{2}{3} = 0.666\ldots\ldots$$

$$= 0.\underline{6} \in \mathbb{Q}$$

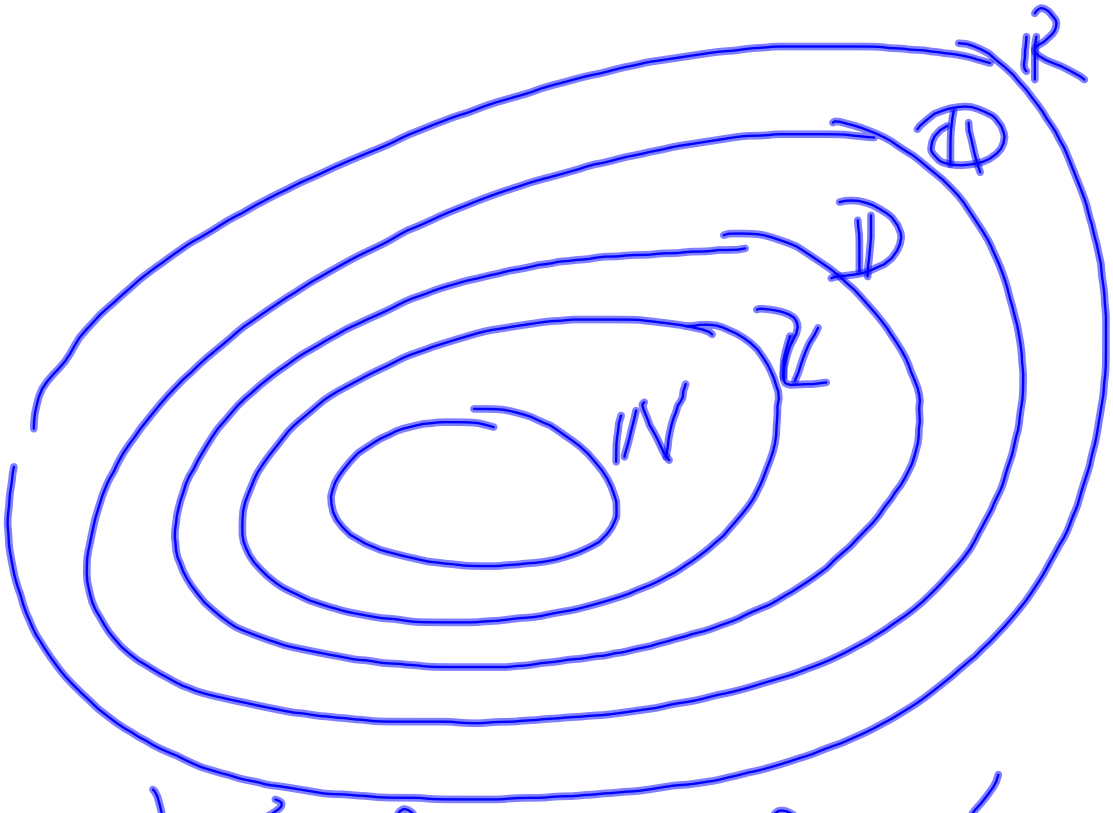
$$\frac{2}{3} \notin \mathbb{N}$$

(4)

$$-12.231014\underline{312} \in \mathbb{Q}$$

(5)

$$\sqrt{2} \in \mathbb{R}, \quad \sqrt{2} \notin \mathbb{Q}$$



( ) $x^2 \in \mathbb{R}_+$ لأن $x \in \mathbb{R}$ لأن

تعريف: * $\mathbb{C}$: مجموعة الأعداد المركبة

* x عدد مركب إذا كان يوجد $a, b \in \mathbb{R}$

حيث $x = a + ib$ و $i^2 = -1$

a : الجزء الحقيقي للعدد x

b : الجزء المركب للعدد x.

$a+ib$: الكتابة الجبرية للعدد x

a : الجزء الحقيقي للعدد x

b : الجزء الخيالي للعدد x

تذكير:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$(a-b)(a+b) = a^2 - b^2$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$(a+b)(a^2 - ab + b^2) = a^3 + b^3$$

$$(a-b)(a^2 + ab + b^2) = a^3 - b^3$$

$$(ab)^n = a^n b^n$$

(٢)

$$a^n \cdot a^p = a^{n+p}$$

$$(a^n)^p = a^{n \cdot p}$$

$$\frac{1}{a^p} = a^{-p} \quad ; \quad \frac{a^n}{p} = a^n$$

مثال (1):

$i^2 = -1 \; ; \; i^3 = i^2 \cdot i = -i$

$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$

$$i^n = \begin{cases} 1 & ; \; n = 4p \\ i & ; \; n = 4p+1 \\ -1 & ; \; n = 4p+2 \\ -i & ; \; n = 4p+3 \end{cases}$$

$i^{4p} = (i^4)^p = 1^p = 1$

$i^{4p+1} = i^{4p} \cdot i = 1 \cdot i = i$

$i^{4p+2} = i^{4p} \cdot i^2 = 1 \cdot (-1) = -1$

$i^{4p+3} = i^{4p} \cdot i^3 = 1 \cdot (-i) = -i$

$$(1+i)^2 = 1^2 + 2(1)i + i^2 \qquad (\ )$$

$$= 1 + 2i + (-1) = 2i$$

$$(1+i)^3 = (1+i)^2(1+i) = 2i(1+i)$$

$$= 2i + 2i \cdot i = 2i + 2i^2 = 2i - 2$$

$$= -2 + 2i$$

$$(1+i)^4 = (1+i)^2(1+i)^2 = (2i)(2i) = 4i^2 = -4$$

$(1-i)^2, (1-i)^3, (1-i)^4$ (٣)

$(2+i)^2 ; (3-2i)^3$ (٤)

الحل: (٣)

$(1-i)^2 = 1^2 - 2(i) + i^2 = 1 - 2i - 1$

$(1-i)^2 = -2i$

ط ١

$(1-i)^3 = 1^3 - 3(i) + 3i^2 - i^3$

$= 1 - 3i - 3 + i = -2 - 2i$

ط ٢

$(1-i)^3 = (1-i)^2(1-i)$

$= -2i(1-i) = -2i + 2i^2 = -2 - 2i$

$(1-i)^4 = (1-i)^2(1-i)^2 = (-2i)(-2i) = -4$

(٤)

$(2+i)^2 = 2^2 + 2(2)i + i^2 = 4 + 4i - 1$

$= 3 + 4i$

$(3-2i)^3 = 3^3 - 3(3^2)(2i) + 3(3)(2i)^2 - (2i)^3$

$(3-2i)^3 = 27 - 54i - 36 + 8i$

$(3-2i)^3 = -9 - 48i$

المرافق: ليكن z عدد مركب حيث $z = a + ib$

و $a, b \in \mathbb{R}$.

المرافق للعدد z ونرمز له $\bar{z}$ هو العدد المركب

$$\bar{z} = a - ib$$

ملاحظة:

$$z \cdot \bar{z} = (a+ib)(a-ib)$$

$$= a^2 - (ib)^2 = a^2 + b^2$$

$$\boxed{z\bar{z} = (a+ib)(a-ib) = a^2 + b^2}$$

لكل $z = a + ib \in \mathbb{C}$ فإن $a^2 + b^2 = z\bar{z} \geq 0$

مثال: أوجد الكتابة الجبرية للاعداد التالية

(١) $\frac{1+i}{2-i}$ (٢) $\frac{(1+i)^2}{(1+2i)^2}$ (٣) $\frac{(2+i)^2}{(1+3i)^3}$

الحل: (١)

$$\frac{1+i}{2-i} = \frac{(1+i)(2+i)}{(2-i)(2+i)} = \frac{2+i+2i+i^2}{2^2+1^2}$$

$$= \frac{1+3i}{5} = \frac{1}{5} + i\frac{3}{5}$$

(٢)

$$\frac{(1+i)^2}{(1+2i)^2} = \left(\frac{1+i}{1+2i}\right)^2 = \left(\frac{(1+i)(1-2i)}{(1+2i)(1-2i)}\right)^2$$

$$= \left(\frac{1-2i+i-2i^2}{1^2+2^2}\right)^2 = \left(\frac{3-i}{5}\right)^2 = \frac{(3-i)^2}{5^2}$$

$$= \frac{3^2-2(3)i+i^2}{25} = \frac{8}{25} - \frac{6}{25}i$$

$$\frac{(2+i)^2}{(1+3i)^3} = \frac{2^2+2(2)i+i^2}{1^3+3(3i)+3(3i)^2+(3i)^3}$$ (٣)

$$= \frac{4+4i-1}{1+9i-27-27i} = \frac{3+4i}{-26-18i}$$

$$= -\frac{1}{2}\,\frac{(3+4i)(13-9i)}{(13+9i)(13-9i)}$$

$$= -\frac{1}{2}\,\frac{39-27i+52i-36i^2}{13^2+9^2}$$

$$= -\frac{1}{2}\,\frac{75+25i}{250} = -\frac{1}{2}\,\frac{25(3+i)}{25(10)}$$

$$= -\frac{3}{20} - \frac{1}{20}i$$

خصائص المرافق: ليكن z_1, z_2

كل منهما عدد مركب لدينا:

$$\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$$

$$\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$$

$$\overline{z_1^n} = \overline{z_1}^n \qquad n \in \mathbb{N}$$

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$$

$$\overline{\left(\frac{z_1}{z_2}\right)^n} = \frac{\overline{z_1}^n}{\overline{z_2}^n} \qquad n \in \mathbb{N}$$

مثال: أوجد المرافق للأعداد التالية بطريقتين

(١) $(1+i)(2+3i)$ (٢) $\frac{1+2i}{1-i}$ (٣) $\frac{(1+2i)^2}{(2+i)}$

الحل: (١) ط ١

$(1+i)(2+3i) = 2+3i+2i+3i^2$

$= -1+5i = -1-5i$

(١) ط ٢

$(1+i)(2+3i)$

$= (1+i)\cdot(2+3i)$

$= (1-i)(2-3i)$

$= 2-3i-2i+3i^2$

$= -1-5i$

(٢) ط ١

$\left(\frac{1+2i}{1-i}\right) = \left(\frac{(1+2i)(1+i)}{(1-i)(1+i)}\right)$

$= \left(\frac{1+i+2i+2i^2}{1^2+1^2}\right)$

$= \left(\frac{-1+3i}{2}\right) = -\frac{1}{2}-\frac{3}{2}i$

(٢) ط ٢

$\left(\frac{1+2i}{1-i}\right)$

$= \frac{1+2i}{1-i} = \frac{1-2i}{1+i}$

$= \frac{(1-2i)(1-i)}{(1+i)(1-i)}$

$= \frac{1-i-2i+2i^2}{1^2+1^2}$

$= -\frac{1}{2}-\frac{3}{2}i$

1 a (5)

$$\overline{\left(\frac{(1+2i)^2}{(2+i)^2}\right)} = \overline{\left(\frac{1+2i}{2+i}\right)^2}$$

$$= \overline{\left(\frac{(1+2i)(2-i)}{(2+i)(2-i)}\right)^2} = \overline{\left(\frac{2-i+4i-2i^2}{2^2+1^2}\right)^2}$$

$$= \overline{\left(\frac{4+3i}{5}\right)^2} = \overline{\left(\frac{4^2+2(4)3i+(3i)^2}{25}\right)}$$

$$= \overline{\left(\frac{7+24i}{25}\right)} = \frac{7}{25} - \frac{24}{25}i$$

2 b

$$\overline{\left(\frac{(1+2i)^2}{(2+i)^2}\right)} = \frac{\overline{(1+2i)}^2}{\overline{(2+i)}^2}$$

$$= \frac{\overline{(1+2i)}^2}{\overline{(2+i)}^2} = \frac{(1-2i)^2}{(2-i)^2} = \frac{1-4i+(2i)^2}{2^2-4i+i^2}$$

$$= \frac{-3-4i}{3-4i} = \frac{(-3-4i)(3+4i)}{(3-4i)(3+4i)}$$

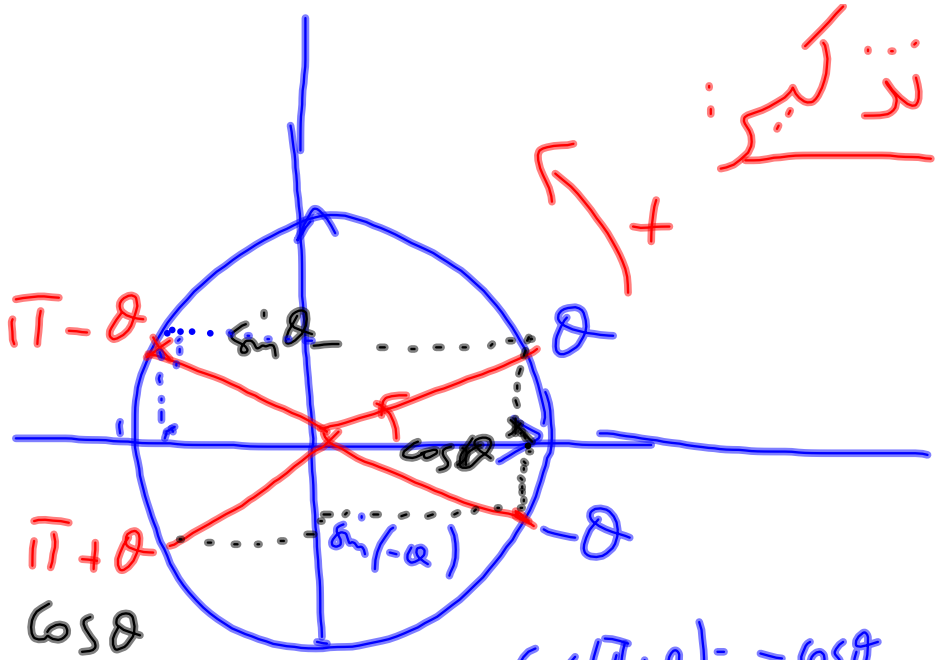
$$= \frac{-9-12i-12i-(4i)^2}{3^2+4^2} = \frac{7-24i}{25}$$

$$= \frac{7}{25} - \frac{24}{25}i$$

مثال: أوجد مرافق العدد في الحالات الثلاثة

(١) $(1+3i)(-2+i)$ (٢) $\frac{1+2i}{-3-i}$ (٣) $\frac{(1+2i)^2}{(1-i)^3}$

تذكير:



$$\begin{cases} \cos(-\theta) = \cos\theta \\ \sin(-\theta) = -\sin\theta \end{cases}$$

$$\begin{cases} \cos(\pi-\theta) = -\cos\theta \\ \sin(\pi-\theta) = \sin\theta \end{cases}$$

$$\cos(\pi+\theta) = -\cos\theta$$
$$\sin(\pi+\theta) = -\sin\theta$$

$$\cos(2\pi+\theta) = \cos\theta$$
$$\sin(2\pi+\theta) = \sin\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$
$$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$$

(( )

θ	0	$\frac{\pi}{6} \sim 30^\circ$	$\frac{\pi}{4} \sim 45^\circ$	$\frac{\pi}{3} \sim 60^\circ$	$\frac{\pi}{2} = 90^\circ$
$\sin\theta$	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \; ; \; \cos\theta \neq 0$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} \; ; \; \sin\theta \neq 0$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

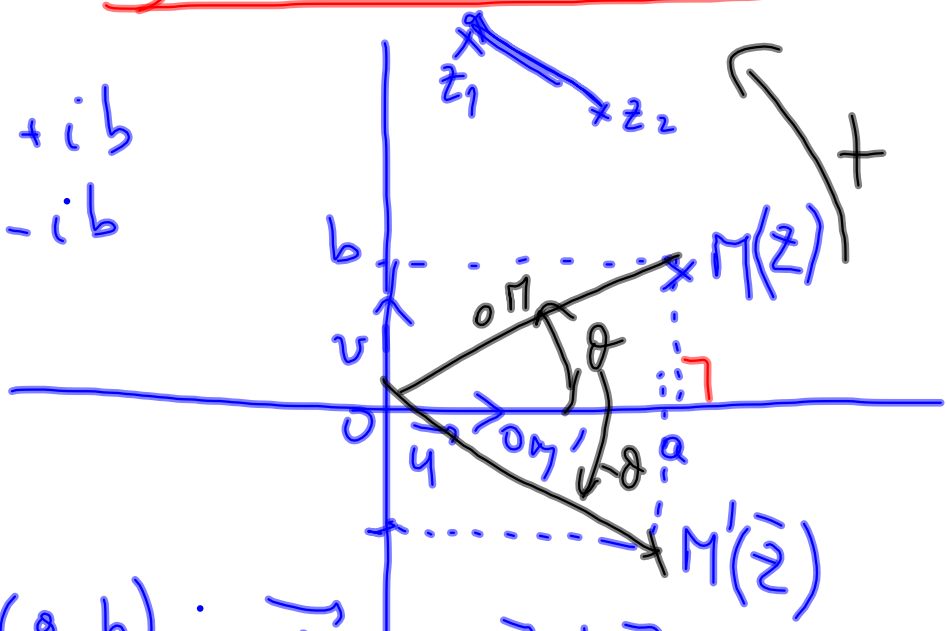
$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \; ; \; \frac{\frac{a}{b}}{c} = \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{bc}$$

$$\frac{a}{\frac{b}{c}} = a \cdot \frac{c}{b} = \frac{ac}{b}$$

الكتابة القطبية للعدد المركب

$z = a + ib$

$\bar{z} = a - ib$



$M(a,b)$; $\vec{OM} = a.\vec{u} + b\vec{v}$

$z = a + ib$

$|z| = OM = \sqrt{a^2 + b^2}$

$Arg(z) = \theta = (\widehat{\vec{u}, \vec{OM}})$

الكتابة القطبية للعدد z

$$\boxed{z = |z|(\cos\theta + i\sin\theta)}$$

$$\begin{cases} \cos\theta = \dfrac{a}{|z|} \\ \sin\theta = \dfrac{b}{|z|} \end{cases}$$

البعد بين نقطتين:

$M_2(z_2)$, $M_1(z_1)$

$d(M_1, M_2) = |z_2 - z_1|$

مثال: اكتب على الصيغة القطبية الاعداد التالية

$z = i$ (١) $z = 1+i$ (٢) $z = 1-i$ (٣)

الحل: (١)

$z = i = 0 + i \cdot 1$

$|z| = \sqrt{0^2+1^2} = \sqrt{1} = 1$

$\theta = Arg(z) = ??$

$\cos\theta = \frac{a}{|z|} = \frac{0}{1} = 0$; $\theta = \frac{\pi}{2}$

$\sin\theta = \frac{b}{|z|} = \frac{1}{1} = 1$

$z = 1\left(\cos\left(\frac{\pi}{2}\right) + i\sin\left(\frac{\pi}{2}\right)\right)$

(٢)

$z = 1+i$

$|z| = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta = Arg(z) = ?$

$\cos\theta = \frac{a}{|z|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin\theta = \frac{b}{|z|} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}$

$z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(٣)

$z = 1-i = 1 + i(-1)$

$z = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) - i\sin\frac{\pi}{4}\right)$

$= \sqrt{2}\left(\cos\frac{\pi}{4} + i\left(-\sin\left(\frac{\pi}{4}\right)\right)\right)$

$z = \sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

مثال: أوجد أقرب نقطة من النقاط $M_1(z_1)$ و $M_2(z_2)$ و $M_3(z_3)$ إلى النقطة $M(z)$

حيث $z_1 = 1+2i$ و $z_2 = 2-i$ و $z_3 = 3+i$

و $z = i$

الحل:

$d(M_1, M) = |1+2i-i|$

$= |1+i| = \sqrt{1^2+1^2} = \sqrt{2}$

$d(M_2, M) = |2-i-i| = |2-2i|$

$= \sqrt{2^2+(-2)^2} = \sqrt{8} = 2\sqrt{2}$

$d(M_3, M) = |3+i-i|$

$= |3| = 3$

لدينا $\sqrt{2} < 2\sqrt{2} < 3$

وبالتالي M_1 هي أقرب نقطة إلى M.

خصائص الكتابة القطبية

لتكن z عدد مركب: $z = |z|(\cos\theta + i\sin\theta)$

ولتكن $n \in \mathbb{N}$ فإن

(١) $\bar{z} = |z|\left(\cos(-\theta) + i\sin(-\theta)\right)$

(٢) $z \neq 0$; $\frac{1}{z} = \frac{1}{|z|}\left(\cos(-\theta) + i\sin(-\theta)\right)$

(٣) $z^n = |z|^n\left(\cos(n\theta) + i\sin(n\theta)\right)$

(٤) لتكن $z_1 = |z_1|\left(\cos(\theta_1) + i\sin(\theta_1)\right)$,

$z_2 = |z_2|\left(\cos(\theta_2) + i\sin(\theta_2)\right)$

$z_1 z_2 = |z_1|\cdot|z_2|\left(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)\right)$

$\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}\left(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)\right)$

مثال: أرجد الكتابة الجبرية والقطبية للأعداد التالية

(١) $z_1=(1+i)^3(1-i)^2$ (٢) $z_2=\frac{(1+i)^2}{(1-i)^3}$ (٣) $z_3=(1-i)^{10}$

الحل: (١)

$1+i=?$

$|1+i|=\sqrt{1^2+1^2}=\sqrt{2}$

$\theta=?$

$\cos\theta=\frac{a}{|z|}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$\sin\theta=\frac{b}{|z|}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$ $\Rightarrow \theta=\frac{\pi}{4}$

$(1+i)=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$(1+i)^3=\sqrt{2}^3\left(\cos\left(3\frac{\pi}{4}\right)+i\sin\left(3\frac{\pi}{4}\right)\right)$ إذن

ولدينا

$(1-i)=\overline{(1+i)}$

$=\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$

$(1-i)^2=\sqrt{2}^2\left(\cos\left(-\frac{2\pi}{4}\right)+i\sin\left(-\frac{2\pi}{4}\right)\right)$

إذن

$z_1=\sqrt{2}^3\sqrt{2}^2\left(\cos\left(\frac{3\pi}{4}-\frac{2\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}-\frac{2\pi}{4}\right)\right)$

$z_1=4\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(\frac{\pi}{4}\right)\right)$ الكتابة القطبية

$z_1=4\sqrt{2}\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)=4+4i$ الكتابة الجبرية

(٣) $z_3=(1+i)^{10}$

لدينا

$1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

إذن

$z_3=(1+i)^{10}$

$=\sqrt{2}^{10}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^{10}$

$z_3=\sqrt{2}^{10}\left(\cos\frac{10\pi}{4}+i\sin\frac{10\pi}{4}\right)$

$=32\left(\cos\frac{5\pi}{2}+i\sin\frac{5\pi}{2}\right)$

$\boxed{z_3=32\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)}$

والكتابة الجبرية للعدد z_3 هي:

$z_3=32(0+i1)=32i$

(ج) لدينا $1+i=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$, $1-i=\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)$

إذن

$$z_2=\frac{(1+i)^2}{(1-i)^3}=\frac{\sqrt{2}^2\left(\cos\left(\frac{2\pi}{4}\right)+i\sin\left(\frac{2\pi}{4}\right)\right)}{\sqrt{2}^3\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)}$$

$$=\frac{1}{\sqrt{2}}\left(\cos\left(\frac{2\pi}{4}-\left(-\frac{3\pi}{4}\right)\right)+i\sin\left(\frac{2\pi}{4}-\left(-\frac{3\pi}{4}\right)\right)\right)$$

$$\boxed{z_2=\frac{\sqrt{2}}{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)}$$

الكتابة الجبرية للعدد z_2 هي

$$z_2=\frac{\sqrt{2}}{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)=\frac{\sqrt{2}}{2}\left(\cos\left(\pi+\frac{\pi}{4}\right)+i\sin\left(\pi+\frac{\pi}{4}\right)\right)$$

$$=\frac{\sqrt{2}}{2}\left(-\cos\frac{\pi}{4}-i\sin\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}\left(-\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}\right)$$

$$\boxed{z_2=-\frac{1}{2}-i\frac{1}{2}}$$

مثال: (١) أوجد الكتابة القطبية للعددين $z_1 = 1+i$ و $z_2 = 1-i$

(٢) أوجد الكتابة القطبية والجبرية $z = \frac{z_1}{z_2}$

(٣) استنتج $\cos\frac{\pi}{12}$, $\sin\frac{\pi}{12}$

الحل: (١)

$z_1 = 1+i\sqrt{3}$

$|z_1| = \sqrt{a^2+b^2} = \sqrt{1^2+\sqrt{3}^2} = 2$

$\theta_1 = ?$

$\left.\begin{array}{l}\cos\theta_1 = \frac{a}{|z_1|} = \frac{1}{2} \\ \sin\theta_1 = \frac{b}{|z_1|} = \frac{\sqrt{3}}{2}\end{array}\right\} \theta_1 = \frac{\pi}{3}$

إذن $z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$z_2 = 1-i$

$|z_2| = \sqrt{a^2+b^2} = \sqrt{1^2+1^2} = \sqrt{2}$

$\theta_2 = ?$

$\left.\begin{array}{l}\cos\theta_2 = \frac{a}{|z_2|} = \frac{1}{\sqrt{2}} \\ \sin\theta_2 = \frac{b}{|z_2|} = \frac{1}{\sqrt{2}}\end{array}\right\} \Rightarrow \theta_2 = \frac{\pi}{4}$

إذن $z_2 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(2) الكتابة القطبية للعدد $z = \frac{z_1}{z_2}$

لدينا

$$z = \frac{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)}{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)} = \sqrt{2}\left(\cos\left(\frac{\pi}{3}-\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)\right)$$

$$\boxed{z = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)}$$

الكتابة الجبرية للعدد z هي:

لدينا:

$$z = \frac{z_1}{z_2} = \frac{1+i\sqrt{3}}{1+i} = \frac{(1+i\sqrt{3})(1-i)}{(1+i)(1-i)}$$

$$= \frac{(1+\sqrt{3}) + i(\sqrt{3}-1)}{2}$$

$$\boxed{z = \frac{1+\sqrt{3}}{2} + i\,\frac{\sqrt{3}-1}{2}}$$

(3) بما أن:

$$z = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right) = \frac{1+\sqrt{3}}{2} + i\,\frac{\sqrt{3}-1}{2}$$

فإن

$$\begin{cases} \sqrt{2}\cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2} \\ \sqrt{2}\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2} \end{cases}$$

فإن

$$\cos\frac{\pi}{12} = \frac{1+\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{4}$$

$$\sin\frac{\pi}{12} = \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$$