RACSAM
DOI 10.1007/s13398-017-0422-7

ORIGINAL PAPER

Compactness theorems for sequences of pseudo-holomorphic coverings between domains in almost complex manifolds

Nabil Ourimi[1]

Received: 31 May 2016 / Accepted: 25 July 2017

Abstract Our aim in this paper is to characterize smooth domains (D, J) and (D', J') in almost complex manifolds of real dimension $2n + 2$ with a covering orbit $\{f_k(p)\}$, accumulating at a strongly pseudoconvex boundary point, for some (J, J')-holomorphic coverings $f_k : (D, J) \to (D', J')$ and $p \in D$. It was shown that such domains are both biholomorphic to a model domain, if the source domain (D, J) admits a bounded strongly J-plurisubharmonic exhaustion function. Furthermore, if the target domain (D', J') is strongly pseudoconvex, then both (D, J) and (D', J') are biholomorphic to the unit ball in $\mathbb{C}^{n+1}$ with the standard complex structure. Our results can be considered as compactness theorems for sequences of pseudo-holomorphic coverings. Lin and Wong (Rocky Mt J Math 20(1):179–197, 1990) and Ourimi (Proc AMS 128(3):831–836, 2000) generalize for relatively compact domains in almost complex manifolds.

Keywords Almost complex manifolds · Coverings · Orbits · Strongly pseudoconvex domains

Mathematics Subject Classification 32H02 · 32H40 · 32H35 · 53C15

1 Introduction and results

The classical local version of Wong–Rosay theorem [9,27,28,33] states that the unit ball $\mathbb{B}^{n+1}$ in $\mathbb{C}^{n+1}$ is a model for the class of $\mathcal{C}^2$-strongly pseudoconvex domains in $\mathbb{C}^{n+1}$ (or more generally, complex manifolds of dimension $n + 1$) at an accumulation point of an automorphism orbit. This local version is valid only in almost complex manifolds of real

Supported by King Saud University, Deanship of Scientific Research, College of Science Research Center.

✉ Nabil Ourimi
ourimi@ksu.edu.sa

1 Department of mathematic, King Saud University, P. O. Box 2455, Riyadh 11451, Kingdom of Saudi Arabia

Published online: 16 August 2017

dimension four (see [10]) and fails in general for higher dimensions; the Siegel half-plane (see Theorem 1) admits an automorphism orbit accumulating at a strongly pseudoconvex boundary point and whose almost complex structure is non-integrable. Our purpose in this paper is to extend this theorem to unbranching proper holomorphic mappings (pseudo-holomorphic coverings) in almost complex manifolds.

If D and D' are domains in some almost complex manifolds (M, J) and (M', J') of the same dimension, respectively, we let $\mathcal{P}_{(J,J')}(D, D')$ be the set of all (J, J')-holomorphic coverings from D to D' (see preliminaries for definitions). Our main theorem is the following

Theorem 1 *Let D and D' be relatively compact domains in some almost complex manifolds (M, J) and (M', J') of real dimension $2n + 2$, respectively. Suppose that D admits a bounded strongly J-plurisubharmonic exhaustion function. Assume that there exist a sequence $\{f_k\}_k \subseteq \mathcal{P}_{(J,J')}(D, D')$ and a point $p \in D$ such that $\{f_k(p)\}$ converges to some J'-strongly pseudoconvex boundary point $q \in \partial D'$. Then both (D, J) and (D', J') are biholomorphic to $(\mathbb{H}, J_B)$, where $\mathbb{H} = \{(z_0, {}'z) \in \mathbb{C} \times \mathbb{C}^n : 2Re(z_0) + |{}'z|^2 < 0\}$ is the Siegel half-plane and J_B is a simple model structure.*

The Siegel half-plane $\mathbb{H}$ with the standard complex structure J_{st} is biholomorphic by the Cayley transformation to the unit ball $(\mathbb{B}, J_{st})$. Thus, $(\mathbb{H}, J_B)$ may be viewed as a deformation of $(\mathbb{B}, J_{st})$. Recall that a real valued negative continuous function $u : \Omega \to [a, 0[$ is called a bounded exhaustion function for a domain $\Omega \subset \mathbb{C}^{n+1}$ if for each constant $a \leq b < 0$, $\{z \in \Omega : a \leq u(z) \leq b\}$ is compact in Ω. Note that such a function u extends continuously to the boundary and that $u(z) = 0$ for $z \in \partial\Omega$. In [6], it was proved that if an almost complex manifold (M, J) admits a strongly plurisubharmonic function, then any relatively compact pseudoconvex domain with $\mathcal{C}^3$-smooth boundary in (M, J) admits a bounded strongly J-plurisubharmonic exhaustion function. The existence of such a function seems to be important in the proof of Theorem 1. Indeed, when such a function exists, the domain (D, J) is hyperbolic (see for example [10,15,16,29]) and the Kobayashi distance induces the usual topology on D (see for example, [16]). This leads us to show that the family of correspondences obtained by scaling is locally equicontinuous and then a normal family by the classical Ascoli's theorem (since (D, J) is relatively compact). The existence of a bounded plurisubharmonic exhaustion function is also important to prove that the limit map obtained by scaling is proper. Theorem 1 recovers the classical Wong–Rosay theorem for relatively compact domains in almost complex manifolds. Related results were proved in [10,21–25].

For relatively compact domains D and D' in almost complex manifolds, the noncompactness of the set $\mathcal{P}_{(J,J')}(D, D')$ is equivalent to the existence of a covering orbit $\{f_k(p)\}$ for some $\{f_k\} \subseteq \mathcal{P}_{(J,J')}(D, D')$ and $p \in D$ which accumulating at a boundary point. As an application of Theorem 1, one has the following

Theorem 2 *Let D and D' be relatively compact domains in some almost complex manifolds (M, J) and (M', J') of real dimension $2n+2$, respectively. Suppose that (D', J') is strongly pseudoconvex. Then $\mathcal{P}_{(J,J')}(D, D')$ is noncompact if and only if both (D, J) and (D', J') are biholomorphic to the unit ball $(\mathbb{B}, J_{st})$ in $\mathbb{C}^{n+1}$.*

In the integrable case, Theorem 2 was proved by Lin and Wong [22]. The main techniques of their proofs are basically differential geometry (curvature, estimates of some canonical Khäler metrics and intrinsic measures). Theorem 2 generalizes Theorem 1.1 in [9] for pseudo-holomorphic coverings, since $\mathcal{P}_{(J,J')}(D, D') = Aut(D, J)$ when $(D, J) = (D', J')$. Another application of Theorem 1 can be stated as follows.

Corollary 1 *Let D and D' be relatively compact domains in some almost complex manifolds (M, J) and (M', J') of real dimension $2n+2$, respectively. Suppose that the domain (D, J) is homogeneous and admits a bounded strongly J-plurisubharmonic exhaustion function and the boundary of D' possesses J'-strongly pseudoconvex points. If either (D, J) or (D', J') is not biholomorphically equivalent to a model domain, then any proper pseudo-holomorphic map $f : (D, J) \to (D', J')$ is branched (its critical set is nonempty).*

According to [27], a proper holomorphic map between strongly pseudoconvex domains in $\mathbb{C}^{n+1}$ (or complex manifolds of complex dimension $n+1$) is a covering. This result is unknown in the non-integrable case, except in some special cases. For example, it is easy to prove that any proper holomorphic self-mapping $F : (\mathbb{H}, J_B) \to (\mathbb{H}, J_B)$ of a model domain in $\mathbb{C}^{n+1}$ is a covering (and therefore an automorphism, since $\mathbb{H}$ is simply connected). Indeed, according to [17], there exists a real constant c such that for all $(z_0, {}'z) \in \mathbb{H}$, $F(z_0, {}'z) = (cz_0 + f({}'z), {}'F({}'z))$, where $f : \mathbb{C}^n \to \mathbb{C}$ is antiholomorphic and ${}'F : \mathbb{C}^n \to \mathbb{C}^n$ is holomorphic (with respect to the standard complex structure). In particular, F extends smoothly to the boundary and maps $\partial\mathbb{H}$ to $\partial\mathbb{H}$. Since $(\mathbb{H}, J_B)$ is strongly pseudoconvex, according to [2,5], F is locally a smooth diffeomorphism on $\partial\mathbb{H}$. So, for any arbitrary point $a \in \partial\mathbb{H}$, there exists a neighborhood U_a of a such that the restriction $\varphi : F_{/U_a} : U_a \cap \overline{\mathbb{H}} \to \mathbb{C}^{n+1}$ is a smooth diffeomorphism on $U_a \cap \partial\mathbb{H}$, pseudo-holomorphic on $U_a \cap \mathbb{H}$ and $\varphi(U_a \cap \partial\mathbb{H}) \subset \partial\mathbb{H}$. According to [26] (Theorem of Poincaré–Alexander), φ extends as an automorphism of $(\mathbb{H}, J_B)$ (this extension is still denoted by $\varphi = (\varphi_0, {}'\varphi)$). As ${}'\varphi$ is holomorphic in $\mathbb{C}^n$ and ${}'F = {}'\varphi$ on some open set of $\mathbb{C}^n$, then by the classical uniqueness theorem, ${}'F = {}'\varphi$ on $\mathbb{C}^n$. According to [17], the function φ_0 may be written as $\varphi_0(z_0, {}'z) = dz_0 + h({}'z)$ for some constant $d \in \mathbb{C}$ and some antiholomorphic function $h : \mathbb{C}^n \to \mathbb{C}$. By differentiation with respect to z_0, it follows that $c = d$ and again by the uniqueness theorem, we deduce that $\varphi_0 = F_0$. Hence, $F = \varphi$ on $\mathbb{H}$.

On the other hand, it was proved in [2] that the set of critical points (branch locus) of a proper holomorphic map between strongly pseudoconvex domains in almost complex manifolds of the same dimension is relatively compact in the source domain (the existence of a strongly plurisubharmonic function on the source domain suffices to deduce this result). It seems to be difficult to assert that this set is empty as in the standard case. One of the main difficulty is the missing of the notion of analytic sets.

Our approach of proof of Theorem 1 uses the scaling method introduced by Pinchuk [27] and successfully applied in different problems for holomorphic and CR mappings. This technique was adapted to almost complex manifolds by Gaussier and Sukhov (see [10,11]). The scaling process in complex manifolds deals with deformations of domains under holomorphic transformations called dilations. When the manifolds are almost complex, the transformations operating on the domains are not pseudo-holomorphic and so we simultaneously dilate the almost complex structures. This provides, as limits, a quadratic domain and a linear deformation of the standard structure in $\mathbb{R}^{2n+2}$, called model structure. It is worth to remark that here we adapt the scaling technique for unbranched proper holomorphic mappings between domains in almost complex manifolds. I'm not able to adapt this notion in the case of proper mappings. The crucial point is how to define the convergence of the sequence inverse?

Note that if $n = 2$, then one can normalize the initial structures to obtain the standard structure as a limit. In the general case, the limit of almost complex structures are not necessarily integrable.

2 Preliminaries

2.1 Almost complex manifolds and almost complex structures

Recall that if M is a smooth manifold of dimension $2n+2$, an almost complex structure on M is an endomorphism J on the tangent bundle TM of M, satisfying $J^2 = -I$. If J is an almost complex structure on M then the 2-tuple (M, J) is called an almost complex manifold. An almost complex structure J on M is said to be integrable if J is induced from the standard complex structure J_{st} of $\mathbb{C}^{n+1}$ in a local coordinate system about z for each point $z \in M$. In this paper we will restrict to $\mathcal{C}^\infty$ almost complex structures on smooth $\mathcal{C}^\infty$ manifolds. Every almost complex structure admits a Hermitian metric and also provides an orientation on the manifold.

For any 1-form ω on (M, J), $J^*\omega$ is defined by $J^*\omega(v) = \omega(Jv)$. A $\mathcal{C}^2$ real-valued function u on M is strongly J-plurisubharmonic on M if $\mathcal{L}^J(u)(p)(v) := -d(J^*du)(v, Jv)$ is positive for every $p \in M$, $v \in T_pM\backslash\{0\}$. A relatively compact domain D in M with boundary of class $\mathcal{C}^2$ is strongly J-pseudoconvex at p if there is a neighborhood $U \subset M$ of p and a smooth $\mathcal{C}^2$ function ρ, strongly J-plurisubharmonic on U, such that $d\rho \neq 0$ on U and $D \cap U = \{\rho < 0\}$. We will say that (D, J) is a strongly pseudoconvex domain if D is defined by $\{\rho < 0\}$, where ρ is a $\mathcal{C}^2$-regular defining function that is strongly J-plurisubharmonic on $\bar{D}$.

Throughout this paper, we denote by $(x_0, y_0, \ldots, x_n, y_n)$ the coordinates in $\mathbb{R}^{2n+2}$ and by $z = (z_0, \ldots, z_n) = (z_0, {}'z) \in \mathbb{C} \times \mathbb{C}^n$ the associated complex coordinates. An almost complex structure J on $\mathbb{R}^{2n+2}$ is called a model structure if it is defined by $\begin{pmatrix} J_{st}^{(1)} & B^J({}'z) \\ 0 & J_{st}^{(n)} \end{pmatrix}$, where $B^J({}'z) \in \mathcal{M}_{2,2n}(\mathbb{R})$ is $\mathbb{R}$-linear in $x_1, \ldots, x_n, y_1, \ldots, y_n$, $J_{st}^{(1)}$ is the standard structure on $\mathbb{R}^2$ and $J_{st}^{(n)}$ is the standard structure on $\mathbb{R}^{2n}$. A pair (D, J) is called a model domain if $D = \{z \in \mathbb{C}^{n+1} : Re z_0 + P({}'z, {}'\bar{z}) < 0\}$, where P is some real homogeneous polynomial of degree 2 and J is a model structure such that D is strongly J-pseudoconvex at 0. Model structures and model domains were introduced in [10]. The complexification of the matrix B^J is $B^J_{\mathbb{C}}({}'z) = \left(\sum_{k=1}^n (a^J_{1,k}\, z_k + b^J_{1,k}\, \bar{z}_k) \cdots \sum_{k=1}^n (a^J_{n,k}\, z_k + b^J_{n,k}\, \bar{z}_k)\right)$, where $a_{j,k}$ and $b_{j,k}$ are complex constants. The model structure J is called simple if $a_{j,k} = 0$ for all j, k.

Given two almost complex manifolds (M, J) and (M', J') and a map f from M to M' of class $\mathcal{C}^1$, we say that f is (J, J')-holomorphic (or pseudo-holomorphic) if its differential $df : TM \to TM'$ satisfies $df \circ J = J' \circ df$ on TM. Pseudo-holomorphic maps may be viewed as solutions of non-linear elliptic operators. The set of (J, J')-holomorphic maps from M to M' is generically empty. However, model domains are homogeneous domains. Hence, there are examples of non integrable almost complex manifolds with a large group of automorphisms.

For $p \in D$ and $v \in T^c_pM$, recall that the Kobayashi–Royden infinitesimal pseudometric $K_{(D,J)}(p, v)$ is the infimum of the set of positive real number α such that there exists a J-holomorphic disc $f : \Delta \to D$ satisfying $f(0) = p$ and $df(0)(\partial/\partial x) = v/\alpha$. For $p, q \in D$, we define the Kobayashi pseudodistance as:

$$d^K_{(D,J)}(p, q) = \inf_{\gamma \in \Gamma_{p,q}} \int_0^1 K_{(D,J)}(\gamma(t), \gamma'(t))dt,$$

where $\Gamma_{p,q}$ is the set of all $\mathcal{C}^1$-paths $\gamma : [0, 1] \to D$ satisfying $\gamma(0) = p$, $\gamma(1) = q$. As in the complex case, the Kobayashi pseudodistance is decreasing under the action of (J, J')-

holomorphic maps; if $f : (M, J) \to (M', J')$ is a pseudo-holomorphic mapping, then for any points p, q in M and a tangent vector $v \in T_p^c M$, we have

$$K_{(M',J')}(f(p), df(p)(v)) \leq K_{(M,J)}(p, v)$$

and

$$d^K_{(M',J')}(f(p), f(q)) \leq d^K_{(M,J)}(p, q).$$

If $d^K_{(M,J)}$ is a distance, it induces the standard topology on M. In this case, we say that (M, J) is (Kobayashi) hyperbolic. When the Kobayashi ball

$$B^K_{(M,J)}(p, r) = \left\{q \in M : d^K_{(M,J)}(p, q) < r\right\}$$

is relatively compact in M for any $p \in M$ and any $r > 0$, we say that (M, J) is complete hyperbolic.

Remark 1 As it was mentioned before, the existence of a bounded strongly J-plurisubharmonic exhaustion function on the domains D ensures the hyperbolicity of the domain (D, J). Furthermore, under hypotheses of Theorem 1, the domain (D, J) is complete hyperbolic. Indeed, according to the first part of Lemma 3.5 in [17]; for any $R > 0$, there exists a neighborhood V_R of q such that $B^K_{(D',J')}(z, R)$ is relatively compact in D' for any $z \in V_R \cap D'$. Choose any point $z_0 \in D$ and any positive real number R. For $R' = d^K_{(D,J)}(p, z_0)$, there exists an integer k_0 such that $f_{k_0}(p) \in V_{R+R'}$. By using the distance-decreasing property of the Kobayashi distance, we obtain that

$$f_{k_0}(B^K_{(D,J)}(z_0, R)) \subset B^K_{(D',J')}(f_{k_0}(p), R + R') \Subset D'.$$

Since f_{k_0} is a covering (then proper), it follows that $B^K_{(D,J)}(z_0, R)$ is relatively compact in D.

2.2 Coverings and proper holomorphic mappings between almost complex manifolds

Recall that a covering map between topological spaces Y and X is a continuous surjective map $f : Y \to X$ satisfying for every $x \in X$, there exists an open neighborhood U of x such that $f^{-1}(U)$ is a union of disjoint open sets in Y, each of which is mapped homeomorphically onto U by f. The space X is often called the base space of the covering and the space Y is called the total space of the covering. For any point x in the base the inverse image of x in Y is necessarily a discrete space called the fiber over x. Let (D, J) and (D', J') be relatively compact domains in almost complex manifolds respectively M and M' of dimension equal to $2n + 2$. It is well known that if $f : D \to D'$ is a covering, then $f_* : \pi_1(D) \to \pi_1(D')$ is injective. Furthermore, as D' is connected and D is relatively compact, then all fibers $f^{-1}(b), b \in D'$ are finite and have the same cardinal (called multiplicity or degree of f). Moreover, the order of $\pi_1(D', b)/f_*(\pi_1(D, x)), b \in D'$ and $x \in f^{-1}(b)$ is equal to the multiplicity of f. If in addition, D is simply connected, the multiplicity of f is equal to the order of $\pi_1(D')$ (see e.g., [12]). It follows that any covering map between relatively compact domains in almost complex manifolds of the same dimension is proper.

Let $f : (D, J) \to (D', J')$ be a (J, J')-holomorphic mapping between relatively compact domains in almost complex manifolds of dimension $2n + 2$. The set V_f denotes the set of critical points; the set of points $p \in D$ where the Jacobian of f vanishes at p. A critical value is the image by f of some critical point and a regular value is any point which is not a critical value. The Hausdorff dimension of the set $f(V_f)$ of all critical values is $\leq 2n$. This follows from the fact that for any critical point p, $Ker\ df(p)$ contains a subspace of dimension 2 (since it is preserved by $J(p)$, see [7]). If in addition, f is proper ($f^{-1}(K)$ is compact in D whenever K is a compact in D') and D admits a strongly J-plurisubharmonic function, then f is surjective and all regular values of f have the same (finite) number of antecedents, say m, ($m = \text{degree} f = \sum_{p \in f^{-1}(q)} \text{sgn}(\det df(p))$ for any regular value q, see [32, Theorem 12 page 275]) and they form a path-connected open set that is dense in D (see [2]). It follows that f^{-1} splits locally into distinct (J', J)-holomorphic maps $h^1, \ldots, h^m$ in $D' \backslash f(V_f)$. The multivalued map $h = f^{-1}$ is called proper pseudo-holomorphic correspondence associated to f. Moreover, any unbranched proper holomorphic mapping between relatively compact domains in almost complex manifolds of the same dimension is a finite covering if the source domain admits a strongly plurisubharmonic function (the existence of a strongly plurisubharmonic function on the source domain ensures the existence of at least one point $p \in D$ such that $f(p)$ is a regular value, see [2] for details).

3 Proof of results

3.1 Proof of Theorem 1

By the attraction property proved in [11, Lemma 6.2], the sequence $\{f_k\}$ converges uniformly to q on compact subsets of D.

Claim 1 *D is simply connected.*

Proof Assume that D is not simply connected. Then there exists a nontrivial closed loop γ in $\pi_1(D)$. The boundary of D' is smooth near q; then there exists a neighborhood V of q such that $D' \cap V$ is simply connected. For large k's, $f_k(\gamma)$ are closed loops in $D' \cap V$. Nevertheless, $f_k : D \to D'$ is a covering, and $(f_k)_* : \pi_1(D) \to \pi_1(D')$ is one to one. This contradicts the fact that $f_k(\gamma)$ must be a nontrivial element in $(f_k)_*(\pi_1(D))$. □

It follows from Claim 1 that for all k, the multiplicity of f_k is equal to the order of $\pi_1(D')$ (say $m \geq 1$). According to [31, Corollary 3.1.2], there exist a neighborhood U' of q in M' and complex coordinates $\phi : U' \to \mathbb{R}^{2n+2}$, $\phi(q) = 0$, so that $\phi_*(J')(0) = J_{st}$. We may choose U' such that $D' \cap U' = \{r < 0\}$ where r is a function of class $\mathcal{C}^2$, strongly J'-plurisubharmonic on U', satisfying $dr \neq 0$ on U'. Moreover, we may assume that the domain $G := \phi(D' \cap U')$ is defined by $G = \{w \in \phi(U') : (r \circ \phi^{-1})(w) < 0\}$, that $T_0(\partial G) = \{w \in \mathbb{C}^{n+1} : Re(w_0) = 0\}$, and that the defining function $\rho := r \circ \phi^{-1}$ defined on $\phi(U')$, is given by

$$\rho(w) = Re(w_0) + Re\left(w_0 \sum_{j \geq 1} \rho_{\bar{j}} \bar{w}_j + \rho_j w_j\right) + P('w, '\bar{w}) + \rho_\epsilon(w)$$

with P a real homogeneous polynomial of degree 2 and $\rho_\epsilon(w) = o(|w|^2)$. Set $q^k = f_k(p)$. For sufficiently large k, let w^k be the unique point on ∂G such that $|q^k - w^k| = dist(q^k, \partial G)$. For large k's, we consider the change of variables α_k defined by:

$$\begin{cases} \xi_j^k = \dfrac{\partial \rho}{\partial \bar{z}_0}(w^k)(z_j - w_j^k) - \dfrac{\partial \rho}{\partial \bar{z}_j}(w^k)(z_0 - w_0^k), \quad 1 \leq j \leq n \\ \xi_0^k = \displaystyle\sum_{0 \leq j \leq n} \dfrac{\partial \rho}{\partial z_j}(w^k)(z_j - w_j^k) \end{cases}$$

Set $G_k = \alpha_k(G)$. The mapping α_k maps w^k to 0 and q^k to $(-\delta_k, '0)$, where $\delta_k = dist(\alpha_k(q^k), \partial G_k)$. The tangent space to ∂G_k at 0 is $\{Rew_0 = 0\}$. Since α_k converges to the identity mapping on any compact subset of $\mathbb{R}^{2n+2}$ with respect to the $\mathcal{C}^2$-topology, it follows that the sequence of almost complex structures $J'^k := (\alpha_k)_* J'$ converges to J' on any compact subset of $\mathbb{R}^{2n+2}$ with respect to the $\mathcal{C}^1$-topology, and is expressed by $\begin{pmatrix} J'^k_{(1,1)}(0) & 0_{2,2n} \\ J'^k_{(2,1)}(0) & J'^k_{(2,2)}(0) \end{pmatrix}$. For every integer k, we define the inhomogeneous dilatation $\Lambda_k(w_0, 'w) = (\sqrt{\delta_k} w_0, \delta_k 'w)$. Set $\hat{G}_k = \Lambda_k(G_k)$, $\hat{\rho}_k = \frac{1}{\delta_k}\rho \circ \alpha_k^{-1} \circ \Lambda_k^{-1}$ and $\hat{J}'^k = (\Lambda_k)_* J'^k$. For all k, $\hat{f}_k = \Lambda_k \circ \alpha_k \circ \phi \circ f_k : (f_k)^{-1}(D' \cap U') \to \hat{G}_k$ is a $(J, \hat{J}'^k)$-holomorphic covering with multiplicity equal to m and $\hat{f}_k(p) = (-1, '0)$. Set $\hat{G} = \{w \in \mathbb{R}^{2n+2} : \hat{\rho}(w) < 0\}$, where $\hat{\rho}(w) = Rew_0 + P('w, '\bar{w})$. The sequence $(\hat{\rho}_k)$ converges to $\hat{\rho}$ at second order with respect to the compact-open topology. Moreover, the sequence of domains $\hat{G}_k$ converges for the local Hausdorff set convergence on $\mathbb{R}^{2n+2}$ to $\hat{G}$. Finally, the sequence of almost complex structures $(\hat{J}'^k)_k$ converges on any compact subset of $\mathbb{R}^{2n+2}$ to a model structure $\hat{J}'$ in the $\mathcal{C}^1$-sense (for details, see [17]). Now, according to [11], $(\hat{G}, \hat{J}')$ is a model domain. By repeating the arguments developed in [2,10], we obtain that the sequence $\{\hat{f}_k\}$ is a normal family. Passing to a subsequence, we may assume that $\{\hat{f}_k\}$ converges uniformly on compact subsets of D to a $(J, \hat{J}')$-holomorphic mapping $\hat{f} : D \to \overline{\hat{G}}$ with $\hat{f}(p) = s = (-1, '0)$. According to [18, Proposition 6.4 and Corollary 6.11], there exists a simple model structure J_B and $(\hat{J}', J_B)$-biholomorphism $\psi : \hat{G} \to \mathbb{H}$, fixing s, continuous and one-to-one to the boundary. Consider $\hat{g} = \psi \circ \hat{f} : D \to \overline{\mathbb{H}}$.

Next, we will prove that $\hat{g}$ maps D to $\mathbb{H}$. By contradiction, assume that there exists $z_0 \in D$ such that $\hat{g}(z_0) \in \partial\mathbb{H}$ and let γ be a $\mathcal{C}^1$-path in D such that $\gamma(0) = p$, $\gamma(1) = z_0$. Set $t_0 \le 1$ such that $\hat{g}(\gamma([0, t_0[) \subset \mathbb{H}$ and $\hat{g}(\gamma(t_0)) \in \partial\mathbb{H}$. The domain $(\mathbb{H}, J_B)$ is homogeneous ($\mathrm{Aut}(\mathbb{H}, J_B)$ acts transitively on $\mathbb{H}$), see [19], then according to Lemma 3.5 in [17], $(\mathbb{H}, J_B)$ is complete hyperbolic. It follows that

$$\lim_{t \to t_0} d^K_{(\mathbb{H}, J_B)}(s, \hat{g}(\gamma(t))) = \infty.$$

But, from the compactness of $\gamma([0, 1])$ in D, we have for every $t < 1$,

$$d^K_{(\mathbb{H}, J_B)}(s, \hat{g}(\gamma(t))) \le \sup_{s \in [0,1]} d^K_{(D,J)}(p, \gamma(s)) < \infty$$

This contradiction proves that $\hat{g}$ maps D to $\mathbb{H}$.

Convergence of the sequence $\{\hat{f}_k^{-1}\}$. For simplicity, we denote by $\{\hat{h}_k\}$ the sequence of correspondences $\{\hat{f}_k^{-1}\}$. Let $L \ni s$ be a simply connected compact in $\hat{G}$ (with nonempty interior). Starting from some integer k_0, $L \subset \hat{G}_k$ and $\hat{h}_k$ is defined on L. For all k, the correspondence $\hat{h}_k$ splits globally into m distinct $(\hat{J}'^k, J)$ holomorphic mappings $(\hat{h}_k^1, \dots, \hat{h}_k^m)$ in L (the global splitting is due to the simply connectedness of L). For all k and for all $j = 1, \dots, m$, the $(\hat{J}'^k, J)$-holomorphic mappings $\hat{h}_k^j$ maps L to D. As it was mentioned in page 2, from the existence of a bounded strongly plurisubharmonic function on (D, J), it follows that the domain (D, J) is hyperbolic and the Kobayashi distance induces the usual topology on D. Let q_0 be an arbitrary point in $\overset{\circ}{L}$ and $V \subset \overset{\circ}{L}$ be a relatively compact neighborhood of q_0 ($\overset{\circ}{L}$ denotes the interior of L). Let $\hbar$ be a Hermitian metric on V that is smooth up to $\overline{V}$. We denote by $d_\hbar$ the distance function on V. Let us show that for $j = 1, \dots, m$, the family $\{\hat{h}_k^j\}$ is equicontinuous on V. By Lemma 2.4 in [8], there exists a positive constant c

such that

$$K_{(\overset{\circ}{L},\hat{J}'^k)}(z',v') \leq c||v'||_{\hbar}$$

for any $z' \in V$ and any $v' \in T^c_{z'}M'$ and for sufficiently large k. By integration, we get that

$$d^K_{(\overset{\circ}{L},\hat{J}'^k)}(p',q') \leq c d_{\hbar}(p',q')$$

for any p' and q' in V. Hence, for any $p' \in V$ and $\epsilon > 0$, $d_{\hbar}(p',q') < \frac{\epsilon}{c}$ implies that $d^K_{(D,J)}(\hat{h}^j_k(p'),\hat{h}^j_k(q')) < \epsilon$. Since the Kobayashi distance induces the usual topology on D, then for all $j = 1,\ldots,m$, the family $\{\hat{h}^j_k\}$ is equicontinuous on V. As the choice of q_0 was arbitrary, $\{\hat{h}^j_k\}$ is equicontinuous on L for all $j = 1,\ldots,m$. Since D is relatively compact, by the Arzela–Ascoli theorem there is a convergent subsequence in the compact-open topology. Then, after taking a subsequence, we may assume that for all j, $\{\hat{h}^j_k\}_k$ converges uniformly on L to a $(\hat{J},J)$-holomorphic map $\hat{h}^j : L \to \bar{D}$. Since $\hat{G}$ may be exhausted by an increasing sequence of simply connected compacts containing s, by passing to some diagonal subsequence, we obtain that for all j, $\hat{h}^j : \hat{G} \to \bar{D}$ globally defines a $(\hat{J},J)$-holomorphic mapping on $\hat{G}$ and the sequence of correspondences $\{\hat{h}_k\}$ converges uniformly to $\hat{h} = (\hat{h}^1,\ldots,\hat{h}^m)$ on any compact L of $\hat{G}$, in the sense that for all $j = 1,\ldots,m$, $\hat{h}^j_k \to \hat{h}^j$ uniformly on any compact L of $\hat{G}$.

Claim 2 *For any $z \in D$ and $w \in \hat{G}$, we have*:

$$z \in \hat{h}_\infty(w) \Longleftrightarrow w = \hat{f}(z)$$

Proof First, assume that $z \in \hat{h}_\infty(w)$. Then there exists a sequence $\{z_k\} \in D$, $z_k \to z$ and $\hat{f}_k(z_k) = w$. But, $|\hat{f}(z) - w| \leq |\hat{f}(z) - \hat{f}(z_k)| + |\hat{f}(z_k) - \hat{f}_k(z_k)|$. Passing to the limit, we get $w = \hat{f}(z)$.

Conversely, if $w = \hat{f}(z)$, there exists a sequence $\{w_k\}$, $w_k = \hat{f}_k(z)$, that converges to w. Therefore, $z \in \hat{h}_k(w_k)$ and in particular, $z = \hat{h}^j_k(w_k)$ for some $j \in \{1,\ldots,m\}$ and for an infinite number of k. Passing to the limit, it follows that $z = \hat{h}^j(w)$ and so $z \in \hat{h}_\infty(w)$. □

Claim 3 *The mapping $\hat{g} : D \to \mathbb{H}$ is a (J, J_B) biholomorphism.*

Proof In view of the simply connectedness of $\mathbb{H}$, it suffices to prove that $\hat{g}$ is an unbranched proper (J, J_B)-holomorphic mapping. First, we prove that $\hat{g}$ is proper. Since $\hat{g} = \Psi \circ \hat{f}$ and Ψ is a biholomorphism, it suffices to show that $\hat{f} : D \to \hat{G}$ is proper. We follow the ideas of Bell in [1]. Let ρ be the bounded strongly J-plurisubharmonic exhaustion function for D (bounded J-plurisubharmonic exhaustion function suffices for this part of the proof). We define

$$R^k_N(w) = \sup\left\{\sum_{j\in I_N} \rho(\hat{h}^j_k(w)),\ I_N \in S_N\right\},$$

where S_N, $1 \leq N \leq m$, is the set of all subsets of $\{1,\ldots,m\}$ of cardinal N. These functions are continuous and $\hat{J}^k$-plurisubharmonic on $\hat{G}^k$. The sequence $\{R^k_N\}$ converges uniformly on compact subsets of $\hat{G}$ to a continuous $\hat{J}$-plurisubharmonic function R_N on $\hat{G}$. Since $\hat{J}$ is continuous, the classical maximum principle holds for the $\hat{J}$'s plurisubharmonic functions (see [14, Remark A.1, page 3854]). It follows (by the maximum principle) that either $R_N < 0$

on $\hat{G}$ or $R_N \equiv 0$ on $\hat{G}$. Since, $R_m(s) \leq \rho(p) < 0$, then $R_m < 0$ on $\hat{G}$. Let N_∞ be the smallest integer such that R_{N_∞} is not identically equal to zero on $\hat{G}$. Then, exactly $N_\infty - 1$ of the component of $\hat{h}(w)$, $w \in \hat{G}$, lie in the boundary of D and exactly $m_\infty = m - N_\infty + 1$ lie in D. Let $\hat{h}_\infty(w)$ denote the vector in M^{m_∞} whose components are equal to the components of $\hat{h}(w)$ (counted with multiplicity) which lie in D.

It is clear that $R_{N_\infty}(w) = \sup\{\rho(z),\ z \in \hat{h}_\infty(w)\}$. Let L be a compact in $\hat{G}$. Since R_{N_∞} is continuous and negative on $\hat{G}$, there exists a constant $c < 0$ such that $R_{N_\infty} < c$ on L. By Claim 2 and the definition of R_{N_∞}, it follows that $\hat{f}^{-1}(L)$ is a subset of $\{z \in D : \rho(z) < c\} \Subset D$. Since ρ is a bounded exhaustion function, we conclude that $\hat{f}^{-1}(L)$ is a compact. Consequently, $\hat{f}$ is proper.

Now, we prove that $\hat{f}$ is locally one to one. Let $z_0 \in D$, $w_0 = \hat{f}(z_0) \in \hat{G}$ and $L \subset \hat{G}$ be a simply connected compact, neighborhood of w_0. Since $\hat{f}$ is continuous, there exists a compact neighborhood K of z_0 such that $\hat{f}(K) \subset L$. But, $\hat{f}_k \to \hat{f}$ uniformly on K. Then starting from some integer k_0, $\hat{f}(K) \subset \hat{L}$, where $\hat{L}$ is a simply connected compact neighborhood of w_0 in $\hat{G}$, containing L. For large k's and after taking a subsequence, we may assume that the correspondence $\{\hat{h}_k\}$ splits globally on $\hat{L}$ to $(\hat{J}^k, J)$ holomorphic mappings $(\hat{h}_k^1, \ldots, \hat{h}_k^m)$ and converges uniformly to $\hat{h} = (\hat{h}^1, \ldots, \hat{h}^m)$. Since $z \in \hat{h}_k \circ \hat{f}_k(z)$ for all $z \in K$. Passing to the limit, it follows that $z \in \hat{h} \circ \hat{f}(z)$ for all $z \in K$. Then for some $j = 1, \ldots, m$, $z = \hat{h}^j \circ \hat{f}(z)$ for all $z \in K$. This shows that $\hat{f}$ is one to one on $K \ni z_0$. As z_0 was an arbitrary point in D, then $\hat{f}$ is unbranched. This implies that $\hat{g} : D \to \mathbb{H}$ is an unbranched proper (J, J_B)-holomorphic mapping and completes the proof of Claim 3. $\square$

Claim 4 *The domain (D', J') is biholomorphic to $(\mathbb{H}, J_B)$.*

To prove Claim 4, we need to recall some properties of $(\mathbb{H}, J_B)$. These properties are essentially studied in [19]. First recall that $(\mathbb{H}, J_B)$ is hyperbolic and strongly pseudoconvex. The model structure J_B is integrable if and only if $B = 0$.

For $\zeta \in \partial\mathbb{H}$, one can check that the map $\Psi_B^\zeta(z) := \zeta *_B z$ is an J_B-automorphism of $\mathbb{H}$, where

$$\zeta *_B z = (z_0 + \zeta_0 - 2 < {}'z, {}'\zeta >_{\mathbb{C}} + iReB({}'z, {}'\xi),\ {}'z + {}'\zeta),$$

$< .,. >_{\mathbb{C}}$ is the standard Hermitian inner product in $\mathbb{C}^n$ and $B({}'z, {}'\xi) = \sum_{j,k=1}^n b_{j,k} z_j \xi_k$. Note that $H_B = (\partial\mathbb{H}, *_B)$ is a Lie group. When $B = 0$, it is the usual Heisenberg group. A brief description of the automorphisms of $(\mathbb{H}, J_B)$ is summarized in the following proposition (for details, we refer the reader to [19]).

Proposition 1 *The automorphism group of $(\mathbb{H}, J_B)$ admits the following decomposition:*

$$Aut(\mathbb{H}, J_B) = Aut_s(\mathbb{H}, J_B) \circ \mathcal{D} \circ H_B$$

where $Aut_s(\mathbb{H}, J_B)$ is the isotropy group of $s = (-1, {}'0)$ and $\mathcal{D} = \{\Lambda_\tau, \tau > 0\}$ with $\Lambda_\tau(z_0, {}'z) = \left(\frac{z_0}{\tau}, \frac{{}'z}{\sqrt{\tau}}\right)$.

Recently, it was proved in [4], that the automorphism groups of $(\mathbb{H}, J_B)$ are isomorphically embedded in the automorphism group of the unit ball. More precisely, the authors proved the following.

Theorem 3 (Byun et al. [4]) *There exists a diffeomorphism $T : \mathbb{H} \to \mathbb{B}$ such that $T \circ Aut(\mathbb{H}, J_B) \circ T^{-1}$ is a subgroup of the automorphism group of the unit ball $\mathbb{B}$ with the standard structure.*

Remark 2 In order to prove Theorem 3, the authors found all almost complex structures on the model domain $(\mathbb{H}, J_B)$ which are invariant under the action of $Aut(\mathbb{H}, J_B)$. Among such structures, they proved that there are integrable structures for which $\mathbb{H}$ is strongly pseudoconvex. Now by using the Wong–Rosay theorem, it follows that there is a diffeomorphism from $\mathbb{H}$ onto the unit ball $\mathbb{B}$ which induces a conjugate isomorphism from $Aut(\mathbb{H}, J_B)$ onto $Aut(\mathbb{B}, J_{st})$.

Proof of Claim 4 Let k_0 be an arbitrary integer. In view of Claim 3, the map $F = f_{k_0} \circ \hat{g}^{-1} : (\mathbb{H}, J_B) \to (D', J')$ is a pseudo-holomorphic covering. According to [12], $F : \mathbb{H} \to D'$ is a Galois covering, (i.e., F is a connected covering and the group $\Gamma = \{\gamma \in Hom(\mathbb{H}) : F \circ \gamma = F\}$ acts transitively on each fiber $F^{-1}(b), b \in D'$, $Hom(\mathbb{H})$ denotes the set of homomorphisms of $\mathbb{H}$). Moreover, the group Γ is isomorphic to $\pi_1(D', b)/F_*(\pi_1(\mathbb{H}, x))$, $b \in D'$ and $x \in F^{-1}(b)$. But $\mathbb{H}$ is simply connected, then Γ is isomorphic to $\pi_1(D', b)$. The mapping F is locally (J_B, J')-biholomorphic, so all elements of Γ are automorphisms of $(\mathbb{H}, J_B)$ (i.e., $\Gamma \subset Aut(\mathbb{H}, J_B)$)). Since F is a finite covering, the group Γ is finite. We need to prove the following lemma.

Lemma 1 *Let (D, J) and (D', J') be connected almost complex manifolds of the same dimension. Assume that (D, J) is $\mathcal{C}^\infty$-smooth and Kobayashi hyperbolic. If $f : (D, J) \to (D', J')$ is a peudoholomorphic mapping factored by a finite subgroup Γ of automorphisms of $Aut(D, J)$ (i.e., for all $z \in D$, $f^{-1}(f(z)) = \{\gamma(z), \gamma \in \Gamma\}$). Then the critical set of f is given by*

$$V_f = \cup_{\{\gamma \in \Gamma, \gamma \neq Id_D\}} \{z \in D : \gamma(z) = z\}.$$

Proof Let $\gamma \in \Gamma$, $\gamma \neq Id_D$ and let $z_0 \in \{\gamma \in \Gamma,\ \gamma(z) = z\}$. By differentiation of the equality $f \circ \gamma = f$ at z_0, we obtain : $df(z_0)(d\gamma(z_0) - Id) = 0$. If $z_0 \notin V_f$, $df(z_0)$ is an isomorphism and so $d\gamma(z_0) = Id$. It follows from Cartan's theorem in almost complex manifold [20], that γ is the identity mapping. This is a contradiction.

Conversely, assume that $z_0 \in V_f$ and let us show that there exists $\gamma \in \Gamma$ such that $\gamma(z_0) = z_0$. For all k, the mapping f is not one to one on the ball $B(z_0, \frac{1}{k})$. Then there exist two sequences $\{z_1^k\}$ and $\{z_2^k\}$ in $B(z_0, \frac{1}{k})$ such that $z_1^k \neq z_2^k$ and $f(z_1^k) = f(z_2^k)$ for all k. The group Γ acts transitively on the fiber, then for all k, there exists $\gamma_k \in \Gamma$ such that $\gamma_k(z_1^k) = z_2^k$. As Γ is finite, there exists $\gamma \in \Gamma$ such that $\gamma(z_1^k) = z_2^k$ for an infinite number of k. Passing to the limit, it follows that $\gamma(z_0) = z_0$. This finishes the proof of the lemma. □

Now, we continue with the proof of Claim 4. By Theorem 3, the group Γ is conjugate isomorphic to a finite subgroup of the automorphism group of the unit ball with the standard structure. According to Theorem 3.1 in [30], for any finite subgroup Γ_1 of the automorphism of the unit ball $\mathbb{B}$ in $\mathbb{C}^{n+1}$, there exists a point in $\mathbb{B}$ that is fixed by all automorphisms of Γ_1 (note that this result holds for any compact group of isometries of simply connected complete Riemannian manifolds with negative curvature; Theorem 13.5 page 75 in [13]). It follows that there exists $z_0 \in \mathbb{H}$ such that $\gamma(z_0) = z_0$ for all $\gamma \in \Gamma$. From Lemma 1, it follows that z_0 is a critical point for F. This contradicts the fact that F is unbranched. Therefore, $\Gamma = \{Id_{\mathbb{H}}\}$. This proves that F is globally one to one and completes the proof of Claim 4 and also Theorem 1. □

3.2 Proof of Theorem 2

If (D, J) and (D', J') are both biholomorphic to the unit ball $(\mathbb{B}, J_{st})$, then obviously the set $\mathcal{P}_{(J,J')}(D, D')$ is noncompact.

Assume now that D' is defined by $\{\rho' < 0\}$, where ρ' is a $\mathcal{C}^2$-regular defining function that is strongly J'-plurisubharmonic on $\bar{D}'$. The noncompactness of $\mathcal{P}_{(J,J')}(D, D')$ is equivalent to the existence of a sequence $\{f_k\}$ in $\mathcal{P}_{(J,J')}(D, D')$ and a point $p \in D$ such that $\{f_k(p)\}$ converges to some boundary point of D'. For an arbitrary integer k_0, the function $\rho := \rho' \circ f_{k_0}$ is a bounded strongly J-plurisubharmonic exhaustion function for D. It follows from Theorem 1 that the domains (D, J) and (D', J') are both biholomorphic to a simple model domain $(\mathbb{H}, J_B)$. Since (D', J') is relatively compact in (M', J') and strongly pseudoconvex, then according to [3], the structure J_B is integrable (i.e., $B = 0$). This proves that (D, J) and (D', J') are both biholomorphic to the unit ball $(\mathbb{B}, J_{st})$. $\square$

3.3 Proof of Corollary 1

Let $f : (D, J) \to (D', J')$ be a proper (J, J')-holomorphic map. Since the boundary of D' contains a J'-strongly pseudoconvex point q and the map f is surjective, there exists a sequence $\{p_k\}$ in D such that $f(p_k) \to q$. But (D, J) is homogeneous, then for some $p \in D$, there exists a sequence of automorphisms $\{\varphi_k\} \subset Aut(D, J)$ such that $\varphi_k(p) = p_k$. It follows that $\{f \circ \varphi_k(p)\}$ converges to q. If the critical set of f is empty, then according to Theorem 1, both (D, J) and (D', J') are equivalent to a model domain. This contradiction completes the proof.

Acknowledgements I would like to thank Pr. Hervé Gaussier for helpful discussions and the referees for valuable remarks.

References

1. Bell, S.: A generalization of Cartan's theorem to proper holomorphic mappings. J. Math. Pures Appl. (9) **67**(1), 85–92 (1988)
2. Blanc-Centi, L.: Proper pseudo-holomorphic maps between strictly pseudoconvex regions. Mich. Math. J. **56**, 183–203 (2008)
3. Byun, J., Gaussier, H., Lee, K.H.: On the automorphism group of strongly pseudoconvex domains in almost complex manifolds. Ann. Inst. Fourier (Grenoble) **59**(1), 291–310 (2009)
4. Byun, J., Lee, K.H., Lee, S.: On the upper-semicontinuity of the automorphism groups of model domains in almost complex manifolds. J. Math. Anal. Appl. **421**, 118–130 (2015)
5. Coupet, B., Gaussier, H., Sukhov, A.: Fefferman's mapping theorem on almost complex manifolds in complex dimension two. Math. Z. **250**, 59–90 (2005)
6. Diederich, K., Sukhov, A.: Plurisubharmonic exhaustion functions and almost complex Stein structures. Mich. Math. J. **56**(2), 331–355 (2008)
7. Federer, H.: Geometric Measure Theory. Springer, New York (1969)
8. Gaussier, H., Kim, K.T.: Compactness of certain families of pseudo-holomorphic mappings into $\mathbb{C}^n$. Int. J. Math. **15**, 1–12 (2004)
9. Gaussier, H., Kim, K.T., Krantz, S.G.: A note on the Wong–Rosay theorem in complex manifolds. Complex Var. Theory Appl. **47**(9), 761–768 (2002)
10. Gaussier, H., Sukhov, A.: Estimates of the Kobayashi–Royden metric in almost complex manifolds. Bull. Soc. Math. Fr. **133**(2), 259–273 (2005)
11. Gaussier, H., Sukhov, A.: On the geometry of model almost complex manifolds with boundary. Math. Z. **254**, 567–589 (2006)
12. Godbillon, C.: Éléments de topologie algébrique. Hermann, Collection méthodes, Paris (1971)
13. Helgason, S.: Differential Geometry, Lie Groups and Symmetric Spaces. Academic, New York (1978)
14. Ivashkovich, S., Rosay, J.P.: Boundary values and boundary uniqueness of J-holomorphic mappings. Int. Math. Res. Not. IMRN **17**, 3839–3857 (2011)
15. Ivashkovich, S., Rosay, J.P.: Schwarz-type lemmas for solutions of $\overline{\partial}$-inequalities and complete hyperbolicity of almost complex structures. Ann. Inst. Fourier **54**, 2387–2435 (2004)
16. Kobayashi, S., Nomizu, K.: Foundations of Differential Geometry. Interscience Publishers, New York (1969)

17. Lee, K.H.: Domains in almost complex manifolds with an automorphism orbit accumulating at a strongly pseudoconvex boundary point. Mich. Math. J. **54**(1), 179–205 (2006)
18. Lee, K.H.: Automorphism groups of almost complex manifolds. Thesis, Pohang University of Science and Technology (2005)
19. Lee, K.H.: Strongly pseudoconvex homogeneous domains in almost complex manifolds. J. Reine Angew. Math. **623**, 123–160 (2008)
20. Lee, K.H.: Almost complex manifolds and Cartan's uniqueness theorem. Trans. Am. Math. Soc. **358**(5), 2057–2069 (2006)
21. Lee, K.H.: Complex Analysis and Geometry. Springer Proceedings in Mathematics and Statistics, vol. 144, pp. 221–233. Springer, Tokyo (2015)
22. Lin, E.B., Wong, B.: Curvature and proper holomorphic mappings between bounded domains in $\mathbb{C}^n$. Rocky Mt. J. Math. **20**(1), 179–197 (1990)
23. Opshtein, E.: A Wong–Rosay type theorem for proper holomorphic self-maps. Ann. Fac. Sci. Toulouse Math. (6) **19**(3–4), 513–524 (2010)
24. Ourimi, N.: A local version of Wong–Rosay's theorem for proper holomorphic mappings. Proc. AMS **128**(3), 831–836 (2000)
25. Ourimi, N.: Some compactness theorems of families of proper holomorphic correspondences. Publ. Mat. **47**(1), 31–43 (2003)
26. Peyron, M.: Théorème de Poincaré–Alexander pour les domaines modèles. C. R. Math. Acad. Sci. Paris **351**(9–10), 353–356 (2013)
27. Pinchuk, S .: The scaling method and holomorphic mappings. Several complex variables and complex geometry, Part 1 (Santa Cruz, CA, 1989), pp. 151–161. In: Proceedings of Symposia in Pure Mathematics, vol. 52, Part 1. American Mathematical Society, Providence (1991)
28. Rosay, J.P.: Sur une caractérisation de la boule parmi les domaines de $\mathbb{C}^n$ par son groupe d'automorphismes. Ann. Inst. Fourier Grenoble **29**, 91–97 (1979)
29. Royden, H.: Remarks on the Kobayashi metric. In: Proceedings of Maryland Conference on Several Complex Variables. Lecture Notes in Mathematics, vol. 185, pp. 369–383 (1971)
30. Rudin, W.: Proper holomorphic maps and finite reflection groups. Indiana Univ. Math. J. **31**(5), 701–720 (1982)
31. Sikorav, J.C.: Some properties of holomorphic curves in almost complex manifolds. In: Audin, M., Lafontaine, J. (eds.) Holomorphic Curves in Symplectic Geometry. Progress in Mathematics, vol. 117, pp. 165–189. Birkhäuser, Basel (1994)
32. Spivak, M.: A Comprehensive Introduction to Differential Geometry, vol. 1, 2nd edn. Publish or Perish, Wilmington (1979)
33. Wong, B.: Characterization of the unit ball in $\mathbb{C}^n$ by its automorphism group. Invent. Math. **41**, 253–257 (1977)