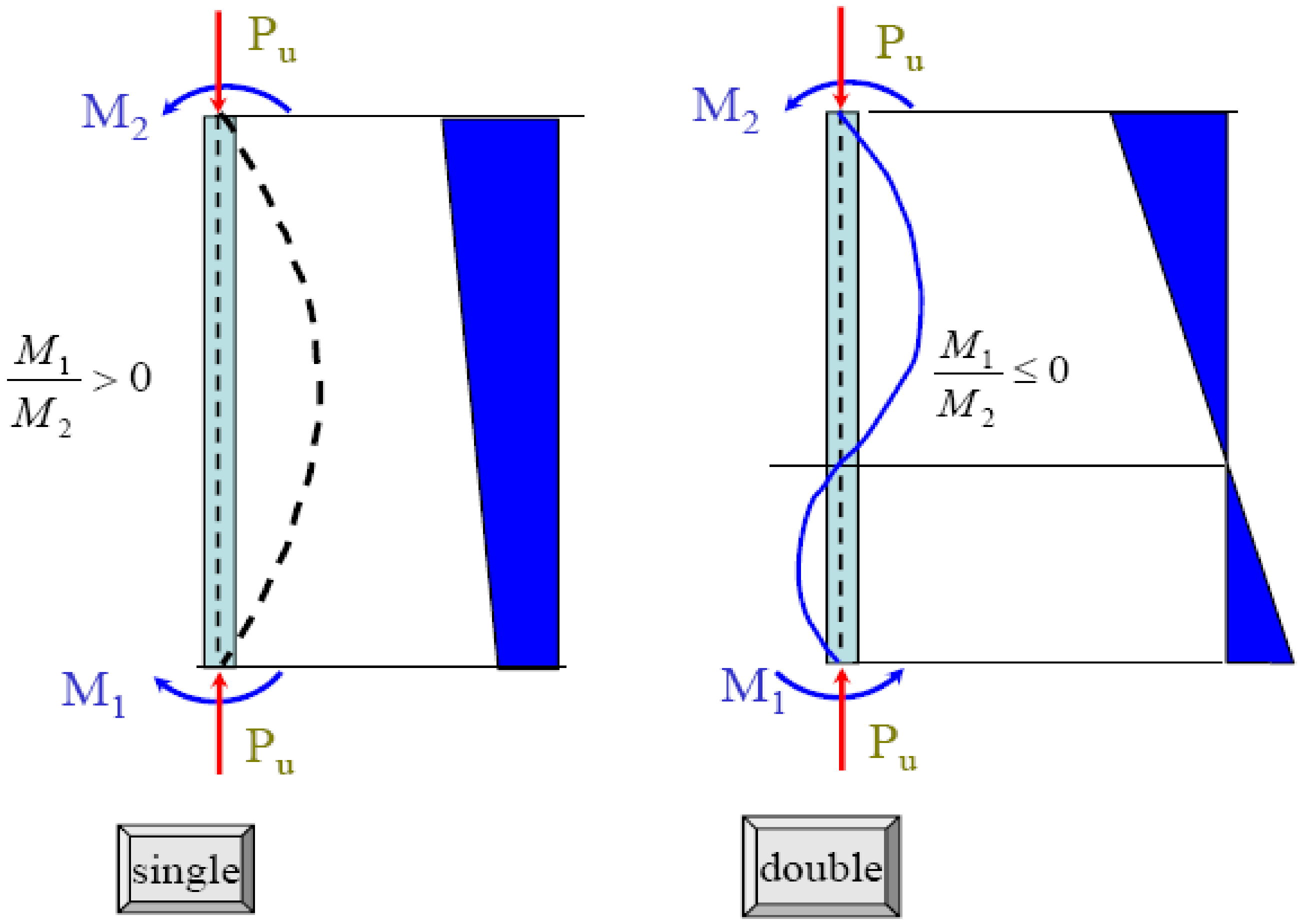


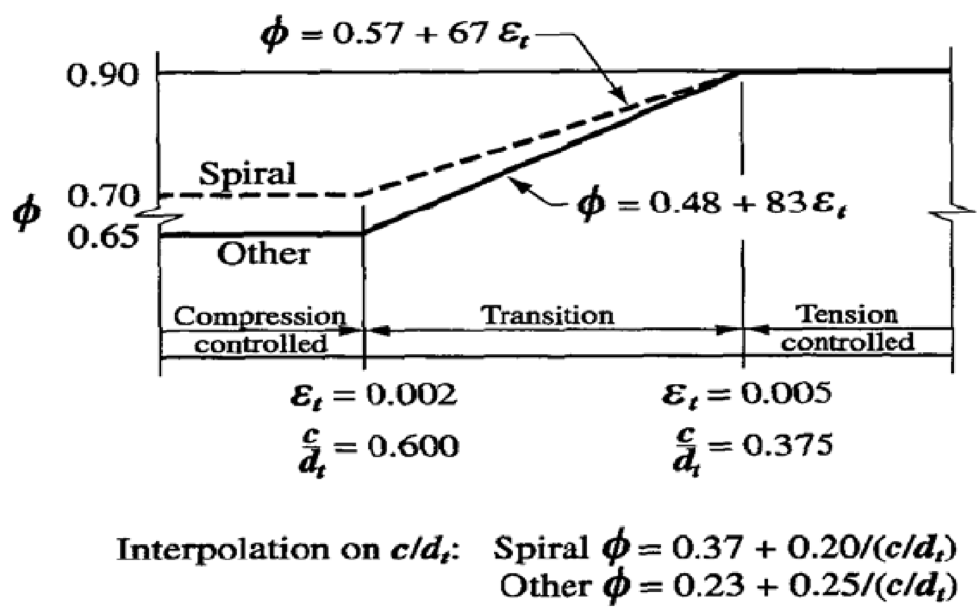


Fig. R9.3.2-Variation of ϕ with net tensile ε_t and c/d_t for Grade 420 reinforcement and for prestressing steel.

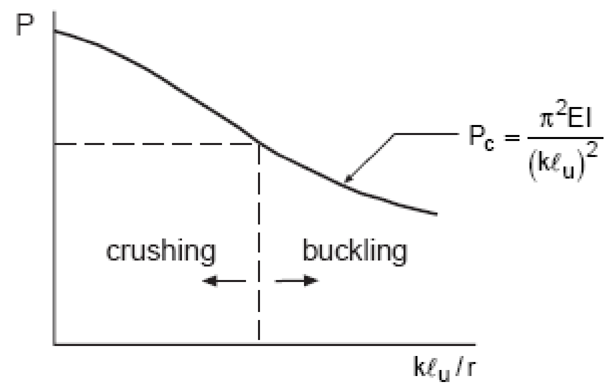


Figure 11-2 Failure Load as a Function of Column Slenderness

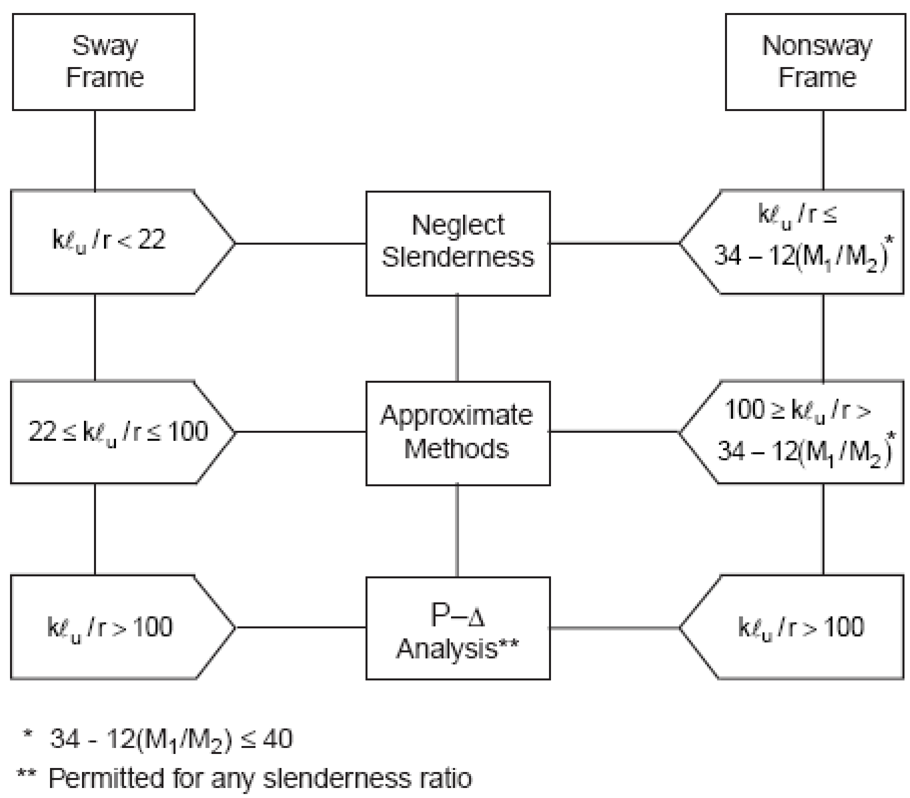


Figure 11-4 Consideration of Column Slenderness

Consideration of column slenderness:-

$$1 - \text{if}\left\{\frac{kL_u}{r} \leq 34 - 12\left(\frac{M_1}{M_2}\right)\right\} \Rightarrow \left\{\begin{matrix}\text{Yes} \ \therefore \ \text{Short column} \\ \text{No} \ \therefore \ \text{Long column}\end{matrix}\right\}$$

$$\left\{34 - 12\left(\frac{M_1}{M_2}\right)\right\} \leq 40$$

Where,

k = effective length =1 for nonsway.

L_u= unsupported length of column.

r= radius of gyration =0.3*dimensions

M_1= smallest moment.

M_2= largest moment.

$$C_m = 0.6 + 0.4\left(\frac{M_1}{M_2}\right) \geq 0.4$$

$$E_c = 4700\sqrt{f_c'}$$

$$EI = \frac{0.4 * E_c * I_g}{1 + \beta_d}$$

$$P_c = \frac{\pi^2 EI}{(kL_u)^2}$$

$M_{2_{min}} = p_u * (15 + 0.03h)$, where h in (mm)

$$M_2 = \max(M_2, M_{2_{min}})$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75 * P_c}} \geq 1$$

$$M_c = \delta_{ns} * M_2$$

E_c= modules of elasticity of concrete. P_c=critical buckling load. EI= rigidity factor.

I_g= moment of inertia. $\beta_d = \frac{\text{maximum factor sustained axial load}}{\text{maximum factor axial load}} \approx 0.6$

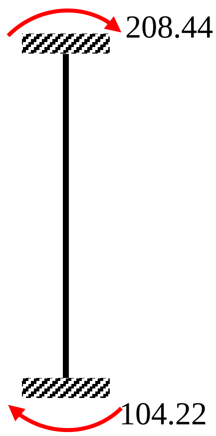
δ_{ns}= moment magnification factor. M_c= magnification moment.

Example 1:-

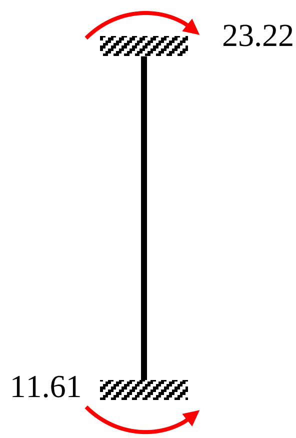
Given data:

$$f_c' = 25\ \text{MPa} \quad , \quad f_y = 420\ \text{Mpa} \quad , \quad L = 6.2\text{m} \quad , \quad P_u = 494\ \text{Kn}$$

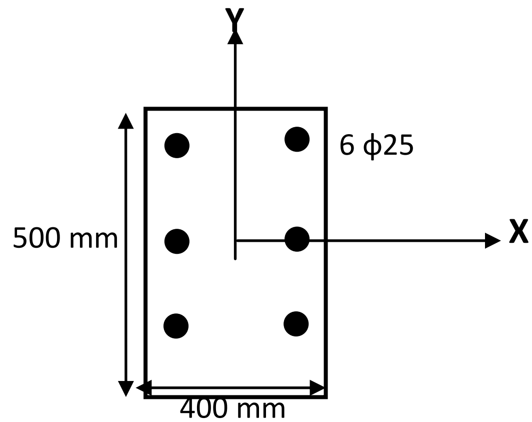
Determine the magnified moment and check column safety ?



Moment about X-axis



Moment about Y-axis



Solution:

Slenderness about X-axis:-

$$\frac{1 * 6200}{0.3 * 500} \leq 34 - 12\left(\frac{104.22}{-208.44}\right) \Rightarrow 41.33 \nleq 40 \Rightarrow \text{slender column(Long column)}$$

$$C_m = 0.6 + 0.4\left(\frac{104.22}{-208.44}\right) \geq 0.4$$

$$C_m = 0.4$$

$$I_g = \frac{b * h^3}{12} = \frac{400 * 500^3}{12} = 4.1667X10^9 mm^4$$

$$E_c = 4700\sqrt{f_c'} = 4700\sqrt{25} = 23500\ \text{Mpa}$$

$$\beta_d = 0.6$$

$$EI = \frac{0.4 * E_c * I_g}{1 + \beta_d} = \frac{0.4 * 23500 * 4.1667X10^9}{1 + 0.6} = 2.448X10^{13} N.mm^2$$

$$P_c = \frac{\pi^2 EI}{(kL_u)^2} = \frac{\pi^2(2.448X10^{13})}{(1 * 6200)^2} * 10^{-3} = 6278.74\ Kn$$

$$M_{2_{min}} = p_u * (15 + 0.03h) = 494 * (15 + 0.03 * 500)10^{-3} = 14.82\ Kn.m$$

$$M_2 = \max(M_2, M_{2_{min}}) = 208.44\ Kn.m$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75 * P_c}} \geq 1 \Rightarrow \frac{0.4}{1 - \frac{494}{0.75 * 6278.74}} = 0.447$$

$$\therefore \delta_{ns} = 1$$

$$M_c = 1 * M_2 = 1 * 208.44 = 208.44\ Kn.m$$

$$A_{s1} = 981.25\ mm^2 \quad , \quad A_{s2} = 981.25\ mm^2 \quad , \quad A_{s3} = 981.25\ mm^2$$

$$A_g = 200 * 10^3\ mm^2 \quad , \quad A_{st} = 2943.75\ mm^2$$

a- Pure compression point:-

$$P_o = \left[0.85\ f_c`\left(A_g - A_{st}\right) + \ f_y * A_{st}\ \right] * 10^{-3}$$

$$P_o = [0.85 * 25 * (200 * 10^3 - 2943.75) + \ 420 * 2943.75\ ] * 10^{-3}$$

$$P_o = 5423.82\ Kn$$

$$\emptyset P_{n,max} = 0.80 * 0.65 * \left[0.85\ f_c`\left(A_g - A_{st}\right) + \ f_y * A_{st}\ \right] * 10^{-3}$$

$$\emptyset P_{n,max} = 0.80 * 0.65 * [0.85 * 25 * (200 * 10^3 - 2943.75) + \ 420 * 2943.75\ ] * 10^{-3}$$

$$\emptyset P_{n,max} = 2820.39\ Kn$$

$$d_1 = 62.5\ mm \quad , \quad d_2 = 250\ mm \quad , \quad d_3 = 437.5\ mm$$

b – Balanced point $\boldsymbol{\varepsilon_t = 0.0021}$

$$C = \frac{0.003}{0.003 + 0{,}0021} * 437.5 = 257.35\ mm$$

$$\varepsilon_i = \frac{d_i - C}{C} * 0.003$$

$$\varepsilon_1 = \frac{62.5 - 257.35}{257.35} * 0.003 = -0.0023$$

$$\varepsilon_2 = \frac{250 - 257.35}{257.35} * 0.003 = -8.5681 * 10^{-5}$$

$$\varepsilon_3 = \frac{437.5 - 257.35}{257.35} * 0.003 = 0.0021$$

$$f_{si} = \varepsilon_i * E = \qquad -420 < \quad f_{si} < 420$$

$$f_{s1} = -0.0023 * 200{,}000 = -460 \quad use - 420\ Mpa$$

$$f_{s2} = -8.5681 * 10^{-5} * 200{,}000 = -17.1362\ Mpa$$

$$f_{s3} = 0.0021 * 200{,}000 = 420\ Mpa$$

$$a_b = \beta * C$$

$$a_b = 0.85 * 257.35 = 218.75\ mm$$

$$a_b > d_i \quad \rightarrow compression\ zone \quad , \quad a_b < d_i \quad \rightarrow tension\ zone$$

$$F_{si} = f_{si} * A_{si} * 10^{-3} \ \rightarrow Tension\ \ Zone$$

$$F_{si} = (f_{si} + 0.85 * f_c`) * A_{si} * 10^{-3} \ \rightarrow Compression\ \ Zone$$

$$F_{s1} = (-420 + 0.85 * 25) * 981.25 * 10^{-3} = -391.27\ Kn$$

$$F_{s2} = -17.1362 * 981.25 * 10^{-3} = -16.815\ Kn$$

$$F_{s3} = 420 * 981.25 * 10^{-3} = 412.125\ Kn$$

$$C_c = 0.85 * f_c` * b * \beta * c * 10^{-3}$$

$$C_c = 0.85 * 25 * 400 * 0.85 * 257.35 * 10^{-3} = 1859.35\ Kn$$

$$P_n = C_c - \Sigma F_s$$

$$P_n = 1859.35 - (-391.27 - 16.815 + 412.125) = 1855.31\ Kn$$

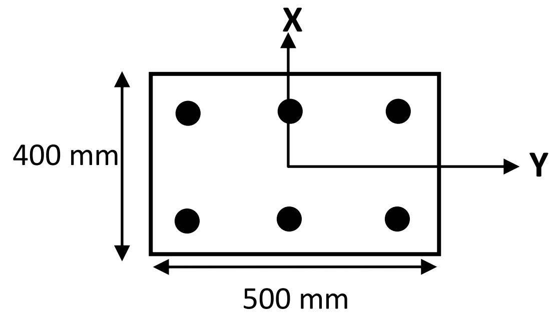
$$M_n = C_c\left[\frac{h}{2} - \frac{a}{2}\right] + F_{s1}\left[d_1 - \frac{h}{2}\right] + F_{s2}\left[d_2 - \frac{h}{2}\right] + F_{s3}\left[d_3 - \frac{h}{2}\right] * 10^{-3}$$

$$M_n = 1859.35\left[\frac{500}{2} - \frac{218.75}{2}\right] - 391.27\left[62.5 - \frac{500}{2}\right] - 16.815\left[250 - \frac{500}{2}\right] + 412.125\left[437.5 - \frac{500}{2}\right] * 10^{-3} = 412.11\ Kn.m$$

$$\emptyset P_n = 0.65 * 1859.35 = 1208.58\ Kn$$

$$\emptyset M_n = 0.65 * 309.08 = 276.87\ Kn.m$$

point	P_n	M_n	$\emptyset P_n$	$\emptyset M_n$
Pure compression $\emptyset = 0.65$	5423.82	0	2820.39	0
Balanced $\emptyset = 0.65$	1855.31	412.11	1208.58	276.87
0.005 strain $\emptyset = 0.9$	808.45	355.39	727.60	319.85
Pure bending $\emptyset = 0.9$	0	247.45	0	222.71



Slenderness about Y-axis:-

$$\frac{1 * 6200}{0.3 * 400} \leq 34 - 12\left(\frac{11.61}{23.22}\right) \Rightarrow 51.67 \nleq 40$$

$$\Rightarrow \text{slender column(Long column)}$$

$$C_m = 0.6 + 0.4\left(\frac{11.61}{23.22}\right) \geq 0.4$$

$$C_m = 0.8$$

$$I_g = \frac{b * h^3}{12} = \frac{500 * 400^3}{12} = 2.6667X10^9 mm^4$$

$$E_c = 4700\sqrt{f_c'} = 4700\sqrt{25} = 23500 \text{ Mpa}$$

$$\beta_d = 0.6$$

$$EI = \frac{0.4 * E_c * I_g}{1 + \beta_d} = \frac{0.4 * 23500 * 2.6667X10^9}{1 + 0.6} = 1.5667X10^{13} N.mm^2$$

$$P_c = \frac{\pi^2 EI}{(kL_u)^2} = \frac{\pi^2(1.5667X10^{13})}{(1 * 6200)^2} * 10^{-3} = 4018.4\ Kn$$

$$M_{2_{min}} = p_u * (15 + 0.03h) = 494 * (15 + 0.03 * 400)10^{-3} = 13.34\ Kn.m$$

$$M_2 = \max\left(M_2, M_{2_{min}}\right) = 23.22\ Kn.m$$

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75 * P_c}} \geq 1 \Rightarrow \frac{0.8}{1 - \frac{494}{0.75 * 4018.4}} = 0.957$$

$$\therefore \delta_{ns} = 1$$

$$M_c = 1 * M_2 = 1 * 23.22 = 23.22\ Kn.m$$

$$A_{s1} = 1471.875\ mm^2 \quad , \quad A_{s2} = 1471.875\ mm^2 \quad ,$$

$$A_g = 200 * 10^3\ mm^2 \quad , \quad A_{st} = 2943.75\ mm^2$$

a- Pure compression point:-

$$P_o = \left[0.85\ f_c`\left(A_g - A_{st}\right) + \ f_y * A_{st}\ \right] * 10^{-3}$$

$$P_o = [0.85 * 25 * (200 * 10^3 - 2943.75) + \ 420 * 2943.75\ ] * 10^{-3}$$

$$P_o = 5423.82\ Kn$$

$$\emptyset P_{n,max} = 0.80 * 0.65 * \left[0.85\ f_c`\left(A_g - A_{st}\right) + \ f_y * A_{st}\ \right] * 10^{-3}$$

$$\emptyset P_{n,max} = 0.80 * 0.65 * [0.85 * 25 * (200 * 10^3 - 2943.75) + \ 420 * 2943.75\ ] * 10^{-3}$$

$$\emptyset P_{n,max} = 2820.39\ Kn$$

$$d_1 = 62.5\ mm \quad , \quad d_3 = 337.5\ mm$$

$\mathbf{b - Balanced\ point} \quad \boldsymbol{\varepsilon_t = 0.0021}$

$$C = \frac{0.003}{0.003 + 0{,}0021} * 337.5 = 198.53\ mm$$

$$\varepsilon_i = \frac{d_i - C}{C} * 0.003$$

$$\varepsilon_1 = \frac{62.5 - 198.53}{198.53} * 0.003 = -0.00206$$

$$\varepsilon_2 = \frac{337.5 - 257.35}{257.35} * 0.003 = 0.0021$$

$$f_{si} = \varepsilon_i * E = \qquad -420 < \quad f_{si} < 420$$

$$f_{s1} = -0.0023 * 200{,}000 = -412$$

$$f_{s2} = 0.0021 * 200{,}000 = 420\ Mpa$$

$$a_b = \beta * C$$

$$a_b = 0.85 * 198.53 = 168.75\ mm$$

$$a_b > d_i \quad \to compression\ zone \quad , \quad a_b < d_i \quad \to tension\ zone$$

$$F_{si} = f_{si} * A_{si} * 10^{-3} \rightarrow Tension\ \ Zone$$

$$F_{si} = (f_{si} + 0.85 * f_c`) * A_{si} * 10^{-3} \rightarrow Compression\ \ Zone$$

$$F_{s1} = (-420 + 0.85 * 25) * 1471.875 * 10^{-3} = -586.91\ Kn$$

$$F_{s2} = 420 * 1471.875 * 10^{-3} = 618.1875\ Kn$$

$$C_c = 0.85 * f_c` * b * \beta * c * 10^{-3}$$

$$C_c = 0.85 * 25 * 500 * 0.85 * 198.35 * 10^{-3} = 1792.97\ Kn$$

$$P_n = C_c - \Sigma F_s$$

$$P_n = 1792.97 - (-586.91 + 618.1875) = 1761.70\ Kn$$

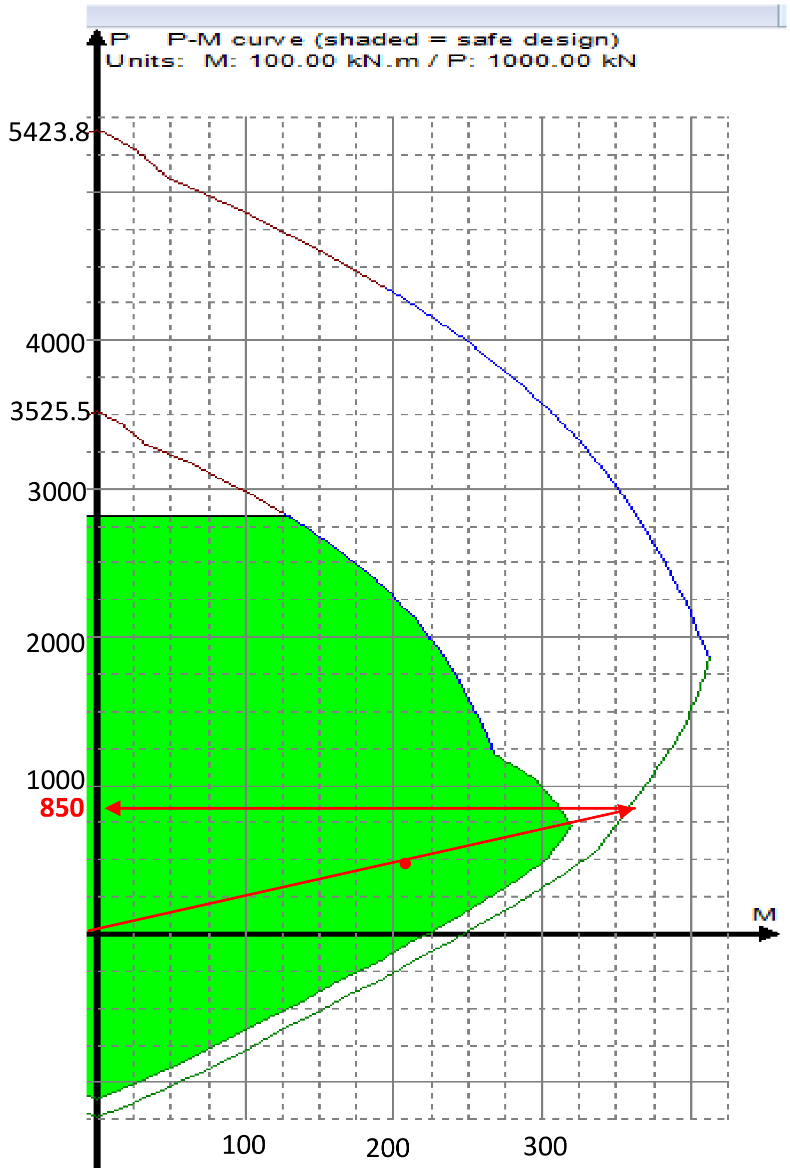
$$M_n = C_c\left[\frac{h}{2} - \frac{a}{2}\right] + F_{s1}\left[d_1 - \frac{h}{2}\right] + F_{s2}\left[d_2 - \frac{h}{2}\right] * 10^{-3}$$

$$M_n = 1792.97\left[\frac{400}{2} - \frac{168.75}{2}\right] - 586.91\left[62.5 - \frac{400}{2}\right] + 618.1875\left[337.5 - \frac{400}{2}\right] * 10^{-3} = 373.01\ Kn.m$$

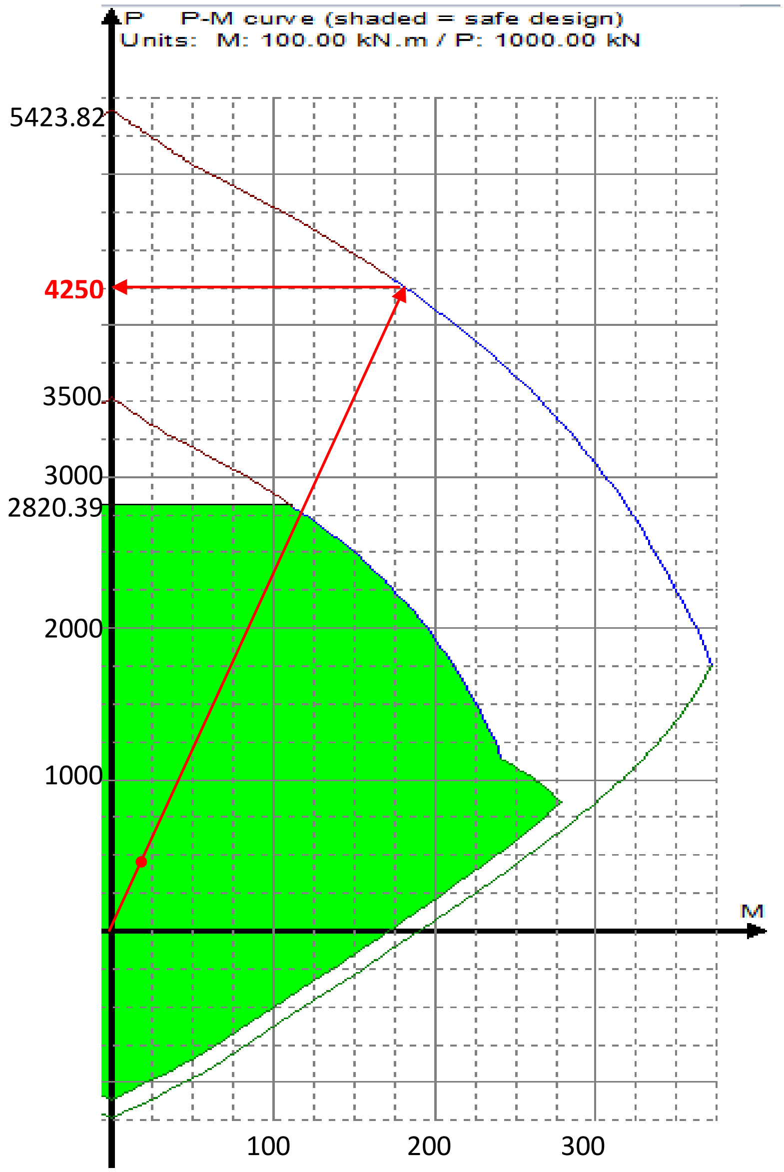
$$\emptyset P_n = 0.65 * 1761.97 = 1145.28\ Kn$$

$$\emptyset M_n = 0.65 * 373.01 = 242.45\ Kn.m$$

point	P_n	M_n	$\emptyset P_n$	$\emptyset M_n$
Pure compression $\emptyset = 0.65$	5423.82	0	2820.39	0
Balanced $\emptyset = 0.65$	1855.31	412.11	1208.58	276.87
0.005 strain $\emptyset = 0.9$	940.56	309.28	846.51	278.36
Pure bending $\emptyset = 0.9$	0	190.47	0	171.42


P P-M curve (shaded = safe design)
Units: M: 100.00 kN.m / P: 1000.00 kN
5423.8
4000
3525.5
3000
2000
1000
850
M
100
200
300

About X-axis



About Y-axis

Reciprocal method :-

All points are located in shaded area (safe zone)

$$\frac{1}{P_n} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_{no}} = \frac{1}{850} + \frac{1}{4250} - \frac{1}{5423.82} = 1.2274 * 10^{-3} \ \ 1/Kn$$

$$P_n = 814.74 \, Kn$$

$$\emptyset P_n = 0.65 * 814.74 = 529.58 \, Kn > P_u \quad \Rightarrow ok\ safe$$