Problems of Laminar boundary Layer.

4.1 Boundary Layer on a Flat Plate with

$$\frac{u}{U} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 \quad \delta = \text{boundary layer Thickness}$$

B.C $u = 0$ $y = 0$ $u(0) = 0$ $u(\delta) = U$ $\left(\frac{\partial u}{\partial y}\right)_\delta = 0$

$\frac{\partial^2 u}{\partial y^2}(0) = 0$

Calculate δ θ δ^* c_f C_D Compare with Blasius Solution

$$\theta = \int_0^\delta \left(\frac{u}{U}\right)\left(1 - \left(\frac{u}{U}\right)\right) dy \qquad \eta = y/\delta \quad dy = \delta\, d\eta$$

$y = 0$ $\eta = 0$ $y = \delta$ $\eta = 1$

$$\theta = \int_0^1 \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)\left(1 - \left(\frac{3}{2}\eta - \frac{\eta^3}{2}\right)\right)\delta\, d\eta \equiv \frac{39}{280}\delta$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy = \delta \int_0^1 \left[1 - \left(\frac{3}{2}\eta^2 + \frac{\eta^3}{2}\right)\right] d\eta \equiv \frac{3\delta}{8}$$

$$H = \frac{\delta^*}{\theta} = \frac{\frac{3\delta}{8}}{\frac{39\delta}{280}} = 2.69$$

$$\frac{u}{U} = 1.5\, y/\delta - 0.5\frac{y^3}{\delta^3}$$

$$\frac{\partial u}{dy} = U\left[\frac{1.5}{\delta} - \frac{1.5\, y^3}{\delta^3}\right)$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = \frac{1.5U}{\delta}$$

$$c_f = \frac{2\tau_w}{\rho U^2} = \frac{2\mu\left(\partial u/\partial y\right)_{y=0}}{\rho U^2} =$$

$$c_f = \frac{\mu 2 \times \frac{1.5\cancel{U}}{\delta}}{\rho U^2} = \frac{3\mu}{\rho\delta U}$$

$$\frac{2\mu U}{\delta} \times \frac{1}{\rho U^2}$$

def $\boxed{c_f = 2\frac{d\theta}{dx}} = \cancel{2}\frac{d}{dx}\left(\frac{39\delta}{\cancel{280}_{140}}\right) = \frac{78}{\cancel{140}}\frac{d\delta}{dx}$.

$$\frac{78}{\cancel{140}}\frac{d\delta}{dx} = \frac{3\mu}{\rho U\delta} \quad \delta\, d\delta = \frac{3 \times \cancel{140}^{70}\mu}{\cancel{78}_{26_{13}}\rho U}dx = \frac{70\mu}{13\rho U}$$

$$\frac{\delta^2}{2} = \frac{70\mu x}{13\rho U} \quad \delta^2 = \frac{\cancel{140}\,\mu x}{13\rho U} \quad \frac{\delta^2}{x^2} = \frac{140\mu}{13\rho U x} = \frac{\delta}{x} = \frac{?}{\sqrt{Re}}$$

$$\theta = \frac{39}{280}\delta$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = 2\frac{d\theta}{dx} = 2\frac{d}{dx}\left(\frac{39\delta}{280}\right) = \frac{78}{280}\frac{d\delta}{dx}$$

$$\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0} \qquad \frac{u}{U} = 1.5\frac{y}{\delta} - 0.5\left(\frac{y}{\delta}\right)^3$$

$$\tau_w = \frac{1.5\mu U}{\delta} \qquad \frac{du}{dy} = \left[\frac{1.5}{\delta} - 1.5\frac{y^2}{\delta}\right]U$$

$$\frac{du}{dy} \text{ at } y=0 = \frac{1.5U}{\delta}$$

$$\therefore c_f = \frac{1.5\mu U}{0.5\rho U^2\delta}$$

$$= \frac{3\mu}{\rho\delta U} \equiv \frac{78}{280}\frac{d\delta}{dx}$$

$$\delta\, d\delta = \frac{280\times 3}{78}\frac{\mu\, dx}{\rho U} \qquad \begin{matrix}\delta=0\\ x=0\end{matrix}$$

$$\frac{\delta^2}{2} = \frac{280}{26}\frac{\mu x}{\rho U}$$

$$\frac{\delta^2}{x^2} = \frac{280}{13}\frac{\mu}{\rho U x}$$

$$\frac{\delta}{x} = \sqrt{\frac{280}{13}}\frac{1}{\sqrt{Re_x}} = \frac{4.64}{\sqrt{Re_x}} \qquad \frac{\delta}{x} = \frac{4.64}{\sqrt{Re_x}}$$

$$\frac{\theta}{x} = \frac{39}{280}\delta \qquad \frac{\theta}{x} = \frac{39}{280}\frac{\delta}{x} = \frac{39\times 4.64}{280\times\sqrt{Re_x}} = \frac{0.646}{\sqrt{Re_x}}$$

$$\frac{\delta^*}{x} = \frac{3}{8}\delta \qquad \frac{\delta^*}{x} = \frac{3}{8}\frac{\delta}{x} = \frac{3}{8}\times\frac{4.64}{\sqrt{Re_x}} = \frac{1.74}{\sqrt{Re_x}}$$

See the difference with Blasius solution

4.2

Given $\frac{T-T_e}{T_w-T_e} = 1-2\xi+2\xi^3+\xi^4$

what boundary conditions does profile satisfy

using $\frac{u}{U_\infty} = 1-2\,y/_\delta + 2\frac{y^3}{\delta^3} - y/_\delta^4$ and assume $\xi = \frac{\delta_T}{\delta} \ll 1$

find heat transfer relations

$\Leftarrow$ B.C are $T(y=0)=T_w$ $\quad \frac{\partial T}{\partial y}$ at $y=\delta = 0$ $\quad T$ at $y=\delta = T_e$

$\frac{\partial^2 T}{\partial y^2}$ at $y=0$ $=0$ $\quad \frac{\partial^2 T}{\partial y^2}$ at $y=\delta = 0$

$$q_w = \frac{2k(T_w-T_e)}{3\delta} = \frac{d}{dx}\left(\int_0^{\delta_T} \rho c_p U (T_w-T_e)(2\xi\eta - \xi^2\eta^2)(1-2\eta+2\eta^3-\eta^4)\,\delta_T\,\eta \right\}$$

$$q_w = \frac{d}{dx}\left[\rho c_p U (T_w-T_e)\,\delta\left(\frac{2\xi^2}{15} - \frac{\xi^3}{42}\right)\right] \qquad \xi = \frac{\delta_T}{\delta} \ll 1$$

Also $\frac{d\delta}{dx} = \frac{15\nu}{U\delta}$

$$\xi^3 - \frac{5}{28}\xi^4 = \frac{1}{Pr} \qquad \xi^3\left(1-\frac{5\xi}{28}\right) = \frac{1}{Pr} \simeq \xi^3 = \frac{1}{Pr}$$

$$\xi = \frac{\delta_T}{\delta} = Pr^{-\frac{1}{3}}$$

$$N_u = \frac{q_w x}{k(T_w-T_e)} = \frac{x}{k\Delta T}\,\frac{2k\Delta T}{\xi\delta} = \frac{2}{4.64} Re^{1/2} Pr^{\frac{1}{3}}$$

$$N_u = \frac{1}{2.32}\sqrt{Re}\,\sqrt[3]{Pr}$$

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(3) Given $\frac{u}{U} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$ $\quad u(0)=0$ $\quad u(\delta)=U$ $\quad \left(\frac{\partial u}{\partial y}\right)_\delta = 0$ $\quad$ zero pressure gradient $\frac{\partial^2 u}{\partial y^2}(0)=0$

Calculate δ, δ^* θ, C_f

$$\theta = \int_0^\delta \frac{u}{U}\left(1-\frac{u}{U}\right)dy \qquad \frac{y}{\delta} = \eta \quad dy = \delta\,d\eta \quad y=0\ \eta=0 \quad y=\delta\ \eta=1$$

$$\theta = \int_0^{1.0} \delta \sin\left(\frac{\pi}{2}\eta\right)\left(1-\sin\frac{\pi\eta}{2}\right)d\eta = 0.13668$$

$$\tau_w = \mu\frac{du}{dy}\Big|_{y=0} = \frac{\pi\mu U}{2\delta} \qquad C_f = \frac{2\tau_w}{\rho U^2} = \frac{2\pi\mu U}{2\delta\rho U^2} \simeq \frac{\pi\nu}{U\delta}$$

$$q = 2\frac{d\theta}{dx} = 2\frac{d}{dx}(0.1366\delta) = .2732\frac{d\delta}{dx}$$

$$\therefore \frac{\pi\nu}{\delta U} = 0.2732\frac{d\delta}{dx} \qquad \delta\, d\delta = \frac{\pi\nu}{0.2732\,U}dx$$

integral 0 – 1 $\quad \frac{\delta^2}{2} = \frac{\pi\nu}{0.2732\,U}x$

$$\delta^2 = \frac{\pi}{0.1366}\frac{\nu x}{U} \qquad \frac{\delta^2}{x^2} = \frac{\pi}{0.1366}\frac{\nu}{xU}.$$

$$\frac{\delta}{x} = \left(\frac{\pi}{0.1366}\right)^{\frac{1}{2}}\frac{1}{\sqrt{Re_x}} = \frac{4.8}{\sqrt{Re_x}}$$

$$\theta = 0.1366\,\delta$$

$$\frac{\theta}{x} = 0.1366\frac{\delta}{x} = \frac{0.1366\times 4.8}{\sqrt{Re_x}} =$$

$$q = \frac{0.655}{\sqrt{Re_x}}$$

$$\int \sin^2 ax = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right)dy = \int_0^1 \delta\left(1 - \sin\frac{\pi\eta}{2}\right)d\eta$$

$$= \delta\left[\int_0^1 d\eta - \int_0^1 \sin\frac{\pi\eta}{2}\,d\eta\right] = \left|\delta\eta - \delta\cos\frac{\pi\eta}{2}\times\frac{\eta}{2}\right|_0^1$$

$$\delta - \frac{\delta}{2}\left[\cos\frac{\pi}{2} - \cos\right.$$

$$\int \sin ax\,dx = \frac{\cos ax}{a}$$

$$\delta^* = \sim\delta \qquad \frac{\delta^*}{x} = \sim\delta/x$$

(4) Unheated starting length $\xi = 0 \quad x = 0$

$$\frac{T - T_e}{T_w - T_e} = 1 - \frac{2y}{\delta_T} + \frac{2y^3}{\delta_T^3} - \frac{y^4}{\delta_T^4}$$

$$\frac{u}{U} = 1.5\frac{y}{\delta} - 0.5\frac{y^2}{\delta^2}$$

Substitute energy integral equation

$$\xi\,\frac{d}{dx}\left[\delta\left(\frac{2\xi^2}{15} - \frac{\xi^3}{42}\right)\right] = \frac{2\alpha}{U} \qquad \xi = \frac{\delta_T}{\delta} < 1 \qquad \xi = 0\ \delta_{x=0}$$

$$\xi^3 + \frac{4}{3}x\frac{d}{dx}\xi^3 = \frac{1}{P_T} \qquad \xi^3 = \frac{1}{P_T}\left(1 - \left(\frac{x}{x_0}\right)^{0.75}\right)$$

4.10 Clausser Parameter $= \frac{\delta^*}{\tau_w}\frac{dP}{dx}$ (16)

For Falkner Scan flow (see Book)

$$\eta = y\left[(m+1)\frac{U}{2\nu x}\right]^{\frac{1}{2}} \qquad f(\eta) = \frac{u}{U} \qquad \eta = y/\delta$$

$$\delta^* = \int_0^\delta \left(1-\frac{u}{U}\right)dy = \int_0^1\left(1-\frac{u}{U}\right)\delta\, d\eta$$

If $\eta^* \triangleq \int_0^1 (1-f'(\eta))\,d\eta \qquad \delta^* = \sqrt{\frac{2\nu x}{(m+1)U}}\;\eta^*$

$$\tau_w = \mu\left(\frac{du}{\partial y}\right)_{y=0} = \mu U f'(0)\,\frac{(m+1)U}{2\nu x}$$

outside free stream velocity

$$U = k x^m \qquad \frac{dP}{dx} = -\rho U\frac{dU}{dx}$$

$$\frac{dP}{dx} = -\frac{\rho m U^2}{x}$$

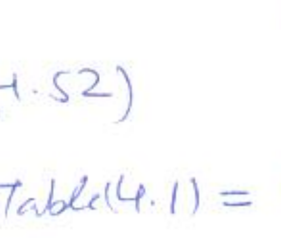
$\therefore \frac{\delta^*}{\tau_w}\frac{dP}{dx} \equiv \frac{m\,\eta^*}{(m+1)f'(0)}$ which does not depend on x

and hence is constant

4.12 $T = 20^\circ C \quad P = 100\,kPa$

$\rho = ?$

$\tau_w = f(x) = \left(\mu\frac{du}{dy}\right)_{y=0}$

$\rightarrow$ 

$\tau_w = 0.332\sqrt{\frac{\rho U}{x}}\,U^{3/2}$

strip area $L\left(1-\frac{x}{L}\right)dx = (L-x)\,dx$

$L = 2\sin 60 = 1.732\,m$

$F = \int_0^L \tau_w\, dA = 0.39N$

τ_w is given in equation (4.52)

$$\tau_w = \frac{\mu U f''(0)}{\sqrt{\frac{2\nu x}{U}}}\;,\; f''(0) \text{ from Table(4.1)} = 0.4690$$

Questions <u>Extra</u>

(1) The velocity Profile of laminar Boundary layer is given by

$$\frac{u}{U_1} = f(\eta) + \lambda\, G(\eta) \qquad \eta = \frac{y}{\delta}$$

Prove if
$$f(\eta) = 0 \rightarrow G(\eta) \equiv 0$$
$$f(1) = 1 \rightarrow G(1) \equiv 0$$
$$f''(0) = 0 \rightarrow G''(0) = 1$$
$$f'(0) = 0 \rightarrow G'(0) = 0$$

(2) If for the velocity Profile $\frac{u}{U_1} = f(\eta) + \lambda G(\eta)$

(a) $f(\eta) = \sin\frac{\pi\eta}{2}$ Then $G(\eta) = \frac{2}{\pi^2}\left[\sin\frac{\pi\eta}{2} - \sin^2\frac{\pi\eta}{2}\right]$

(b) If $f(\eta) = \sin\frac{\pi}{2}\eta$ Then at separation $\lambda = -\frac{\pi^2}{2}$

(3) For Laminar boundary Layer on a flat plate find δ, δ^*, θ, H, δ_E τ and C_F if the velocity Profile is given as under. Take $U_1 = 2.5x$

i $\frac{u}{U_1} = \sin\left(\frac{\pi}{2}\cdot\frac{y}{\delta}\right)$ iv $= \frac{u}{U_1} = 2\frac{y}{\delta} - \frac{y^2}{\delta^2}$

ii $\frac{u}{U_1} = 2\frac{y}{\delta} - \frac{y^2}{2\delta^2}$

iii $\frac{U}{U_1} = \frac{1}{2}\frac{y^2}{\delta^2}$

iv $\frac{u}{U_1} = \frac{y}{\delta}$

(4) Given the external velocity Profile leading to Similar solutions is $U_1 = C x^m$. Find the value of m for which boundary layer is always on the point of separation. Assume velocity Profile is

$$\frac{u}{U_1} = f(\eta) + \lambda G(\eta)$$

Given at separation $H = 3.5$ and λ at separation is $\frac{\delta^2}{\nu}\frac{dU_1}{dx}$

(5) Given velocity Profile for laminar flow as

$$\frac{u}{U_1} = f(\eta) + \lambda G(\eta)$$

if $f(\eta) = \sin\frac{\pi\eta}{2}$ then $G(\eta) = \frac{2\lambda}{\pi^2}\left(\sin\frac{\pi\eta}{2} - \sin^2\frac{\pi\eta}{2}\right)$

Prove for separation $\lambda = -\frac{\pi^2}{2}$

⇒ Velocity Profile for a laminar boundary Layer is given as →

$$\frac{u}{U_1} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$$

For a flat plate at zero angle of attack Evaluate $\frac{\delta}{x}$, $\frac{\delta^*}{x}$, $\frac{\theta}{x}$, H, C_f and C_F

Solution

Momentum integral Equation

$$\frac{d\theta}{dx} + \frac{1}{U_1}\frac{dU_1}{dx}(H+2) = \frac{\tau_w}{\rho U_1^2}$$

For flat plate $\frac{dU_1}{dx_1} = 0$

$$\therefore \frac{d\theta}{dx} = \frac{\tau_w}{\rho U_1^2}$$

Given $\frac{u}{U_1} = \sin\left(\frac{\pi}{2}\frac{y}{\delta}\right)$

$$\tau_w = \mu\left(\frac{du}{dy}\right)_{y=0} \qquad \frac{du}{dy} = U_1\frac{\partial}{\partial y}\sin\left(\frac{\pi}{2}y/\delta\right)$$

$$\frac{du}{dy} = U_1\cos\left(\frac{\pi}{2}y/\delta\right)\frac{\pi}{2\delta} = \frac{\pi U_1}{2\delta}\cos\left(\frac{\pi y}{2\delta}\right)$$

$$\frac{du}{dy} \text{ at } y=0 = \frac{\pi U_1}{2\delta}$$

$$\tau_w = \mu\frac{du}{dy}_{y=0} = \frac{\mu\pi U_1}{2\delta}$$

$$\therefore \frac{d\theta}{dx} = \frac{\mu \pi U_1}{2\delta} \times \frac{1}{\rho U_1^2} = \frac{\pi \nu}{2\delta U_1}$$

$$\theta = \int_0^{\delta} \frac{u}{U_1}\left(1 - \frac{u}{U_1}\right) dy$$

$$\theta = \int_0^{\delta} \sin\frac{\pi y}{2\delta}\left(1 - \sin\frac{\pi y}{2\delta}\right) dy$$

$$\theta \triangleq 0.136\delta$$

$$\frac{d\theta}{dx} = 0.136 \frac{d\delta}{dx} = \frac{\pi \nu}{2\delta U_1} =$$

$$0.272\, \delta\, d\delta = \pi \frac{\nu}{U_1} dx$$

$$\frac{0.272}{\pi} \frac{\delta^2}{2} = \frac{\nu x}{U_1}$$

$$\frac{\delta^2}{x} = \frac{2\pi}{0.272} \frac{\nu}{U_1}$$

$$\frac{\delta^2}{x^2} = \frac{2\pi}{0.272} \frac{\nu}{U_1 x}$$

$$\frac{\delta}{x} = \sqrt{\frac{2\pi}{0.272}} \sqrt{\frac{\nu}{U_1 x}}$$

$$\boxed{\frac{\delta}{x} = \frac{4.79}{\sqrt{R_x}}}$$

$$\theta = 0.136\,\delta$$

$$\frac{\theta}{x} = 0.136\,\frac{\delta}{x} = \frac{0.136 \times 4.79}{\sqrt{R_x}}$$

$$\frac{\theta}{x} = \frac{0.65}{\sqrt{R_x}}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy \qquad \eta = \frac{y}{\delta}$$

$$= \int_0^\delta \left(1 - \sin\frac{\pi y}{2\delta}\right) dy \qquad dy = \delta\, d\eta$$

$$= \int_0^1 \delta\left(1 - \sin\frac{\pi}{2}\eta\right) d\eta$$

$$= \delta\left[\int_0^1 d\eta - \int_0^1 \sin\left(\frac{\pi}{2}\eta\right) d\eta\right]$$

$$\delta\left[1 - \left[\frac{\cos\frac{\pi}{2}\eta}{\pi/2}\right]_0^1\right]$$

$$\delta^* = \delta\left(1 - \left(0 - \frac{2}{\pi}\right)\right] = \left(1 + \frac{2}{\pi}\right)\delta = 1.637\,\delta$$

$$\delta^* = 1.637\,\delta$$

$$\frac{\delta^*}{x} = 1.637\,\frac{\delta}{x} = \frac{1.637 \times 4.79}{\sqrt{R_x}}$$

Solution (4) $U_1 = c x^m$

$$\frac{u}{U_1} = f(\eta) + \lambda G(\eta)$$

$$H = 3.5 \quad \lambda = \frac{\delta^2}{\nu}\frac{dU_1}{dx}$$

Momentum Integral Equation

$$\frac{d\theta}{dx} + \frac{\theta}{U_1}\frac{dU_1}{dx}(H+2) = \frac{\tau_\omega}{\rho U_1^2}$$

At separation $$\frac{d\theta}{dx} + \frac{\theta}{U_1}\frac{dU_1}{dx}(3.5+2) = \frac{\tau_\omega}{\rho U_1^2}$$

$$\tau_\omega = \mu\left(\frac{du}{dy}\right)_{y=0} = 0 \text{ at separation}$$

$$\frac{d\theta}{dx} + \frac{5.5\theta}{U_1}\frac{dU_1}{dx} = 0 \quad (1)$$

Multiply (1) by $U_1^{5.5}$

$$U_1^{5.5}\frac{d\theta}{dx} + \frac{5.5 U_1^{5.5}}{U_1}\theta\frac{dU_1}{dx} = 0$$

$$U_1^{5.5}\frac{d\theta}{dx} + \theta\, 5.5 U_1^{5.5-1}\frac{dU_1}{dx} = 0$$

$$\frac{d}{dx}\left(U_1^{5.5}\theta\right) = 0$$

$$U_1^{5.5}\theta = \text{Constant}$$

At separation

$$\frac{u}{U_1} = f(\eta) + \lambda G(\eta)$$

$$\frac{\partial(u/U_1)}{\partial\eta} = f'(\eta) + \lambda G'(\eta)$$

At separation $\frac{\partial u}{\partial y} = 0$ $y = 0$

$\therefore \frac{\partial u/U_1}{\partial \eta} = 0$ at $\eta = 0$

$f'(\eta) + \lambda G'(\eta) = 0 \quad \therefore \lambda = -\frac{f'(\eta)}{G'(\eta)}$

for one parameter $\frac{\delta}{\theta} = f(\lambda) = \text{constant} \approx k_1$

$\frac{\delta}{\theta} = k_1 \quad \delta = k_1 \theta$

At separation $\lambda = \frac{\delta^2}{\nu} \frac{dU_1}{dx} = k_2$ ②

$\frac{\delta}{\theta} = k_1 \quad \delta^2 = k_1 \theta^2$

from 2 $\delta^2 = \frac{k_2 \nu}{\frac{dU_1}{dx}} = k_1 \theta^2$

$\therefore \theta^2 = \frac{k \nu}{\frac{dU_1}{dx}} \quad \left(\frac{k_2}{k} = k\right)$

we have $U_1^{5.5} \theta = \text{Constant} = C_1$

Squaring $U_1^{11} \theta^2 = C$

$U_1^{11} \frac{k\nu}{\frac{dU_1}{dx}} = C \rightarrow U_1^{11} k\nu = \frac{dU_1}{dx}$

$U_1^{-11} dU = \frac{k\nu}{c} dx = C_2 \nu dx$

Integrate $\frac{U_1^{-10}}{-10} = C_2 \nu x$

$U_1^{-10} = -10 C_2 \nu x \rightarrow C_3 x$ ~~~~

$U_1 = C_4 x^{-1/10}$ ④ Compare ④ with $U_1 = C x^m$

$\therefore m = -1/10$

Q= ~~Quiz 4 Home Question Solution~~

Given $U_1 = 2.5x$ $\frac{u}{U} = \frac{y}{\delta}$ calcul. $\delta\ \delta^*\ \theta\ H$

$$\frac{u}{U} = \frac{y}{\delta} \to \frac{du}{dy} = \frac{U}{\delta} \to \tau = \mu\frac{du}{dy} = \frac{\mu U}{\delta} \quad \tau = \frac{\mu U}{\delta} \quad (1)$$

$$\frac{u}{U} = \frac{y}{\delta}.\ \delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right)dy = \int_0^\delta \left(1 - \frac{y}{\delta}\right)dy = \frac{\delta}{2}$$

$$\frac{\delta^*}{\delta} = \frac{1}{2} \quad (2)$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right)dy \to \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right)dy = \frac{\delta}{6},\ \frac{\theta}{\delta} = \frac{1}{6} \quad (3)$$

$$H = \frac{\delta^*}{\theta} = 3 \longrightarrow (4)$$

Momentum Equation

$$\frac{\tau_w}{\rho U^2} = \frac{d\theta}{dx} + (H+2)\frac{dU}{dx}\frac{\theta}{U} \to \frac{d\theta}{dx} + 5\frac{\theta}{U}\frac{dU}{dx}$$

$$\frac{\mu U}{\rho \delta U^2} = \frac{\nu}{\delta U} = \frac{d\theta}{dx} + 5\frac{\theta}{U}\frac{dU}{dx} \quad (5)$$

Multiply (5) by U^4

$$\frac{\nu}{\delta} = U\frac{d\theta}{dx} + \frac{5\theta}{U}\frac{dU}{dx} \quad (5)$$

$$\frac{\nu U^4}{\delta} = U^5\frac{d\theta}{dx} + 5U^4\theta\frac{dU}{dx} = \frac{d}{dx}(U^5\theta)$$

$$\frac{d}{dx}(U^5\theta) = \frac{\nu U^4}{\delta} \quad (6)$$ Integrate (6)

$$U^5\theta = \int_0^x \frac{\nu}{\delta}U^4 dx = \frac{\nu}{\delta}\int_0^x (2.5x)^4 dx$$

$$\theta = \frac{\nu}{\delta U^5}2.5^4\int x^4 dx = \frac{\nu\, 2.5^4 x^5}{5\delta U^5}$$

$$\theta = \frac{\nu\, 2.5^4\, x^5}{5\delta\, 2.5^5 x^5} = \frac{\nu}{12.5\delta} \quad (7)$$

$$\theta = \frac{\delta}{6} = \frac{\nu}{12.5\delta} \qquad \delta^2 = \frac{6\nu}{12.5}$$

$$\delta = 0.693\nu^{0.5} \quad (8)$$

$$\frac{\delta}{x} = 0.693\frac{\nu^{0.5}}{x} = 0.693\frac{\nu^{0.5}\, 2.5^{0.5}}{x^{0.5}\, 2.5^{0.5} x^{0.5}} \qquad U = 2.5x$$

$$\frac{\delta}{x} = 1.523\sqrt{\frac{\nu}{xU}} = \underline{\frac{1.523}{\sqrt{R_x}}} \qquad R_x = \sqrt{\frac{Ux}{\nu}}$$

$$\theta = \frac{\delta}{6} \quad \frac{\theta}{x} = \frac{1}{6}\frac{\delta}{x} = \underline{\frac{\theta}{x} = \frac{0.254}{\sqrt{R_x}}}$$

$$\delta^* = \frac{\delta}{2} \quad \frac{\delta^*}{x} = \frac{1}{2}\frac{\delta}{x} \Rightarrow \frac{\delta^*}{x} = \frac{0.7615}{\sqrt{R_x}}$$

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{2\tau_w}{\rho U^2} = \frac{2\mu U}{\rho\delta U^2} = \frac{2\nu}{\delta U} \quad \text{use } 8$$

$$C_f = \frac{2\nu}{0.693\nu^{0.5}U} = 2.886\frac{\nu^{0.5}}{U}$$

$$\frac{C_f}{x} = \frac{2.886}{\nu^{0.5}}\frac{\nu}{xU} = \frac{2.886}{\sqrt{\nu}}\frac{1}{\sqrt{R_x}}$$

$$\boxed{\begin{array}{ll} \dfrac{\delta}{x} = \dfrac{1.523}{\sqrt{R_x}} & \dfrac{\delta^*}{x} = \dfrac{0.7615}{\sqrt{R_x}} \\[2ex] \dfrac{\theta}{x} = \dfrac{0.254}{\sqrt{R_x}} & \dfrac{C_f}{x} = \dfrac{2.886}{\nu^{0.5}}\dfrac{1}{\sqrt{R_x}} \end{array}}$$

Given $\frac{u}{U} = 2\eta - \frac{\eta^2}{2}$ & $\frac{T-T_e}{T_w-T_e} = 1 - \frac{y}{\delta_T}$

$\underline{Q_3}$ calculate δ_h

$$\delta_h = \int_0^{\delta} \frac{T-T_e}{T_w-T_e} \frac{U}{U} dy$$

$$\delta_h = \int_0^{\delta} \left[1 - 2\frac{y}{\delta_T}\right]\left[2\frac{y}{\delta} - 0.5\frac{y^2}{\delta^2}\right] dy$$

$\delta_T = \xi \delta$

$$\delta_h = \int_0^{\delta} \left(1 - \frac{2y}{\xi\delta}\right)\left(\frac{2y}{\delta} - \frac{y^2}{2\delta^2}\right) dy$$

$$\delta_h = \int_0^{\delta} \frac{2y}{\delta} - \int_0^{\delta} \frac{y^2}{2\delta^2} dy - \int_0^{\delta} \frac{4y^2}{\xi\delta^2} + \int^{\delta} \frac{y^3}{\xi\delta^3} dy$$

$$\delta_h = \frac{2\delta^2}{2\delta} - \frac{\delta^3}{6\delta} - \frac{4\delta^3}{3\xi\delta^2} + \frac{\delta^4}{4\xi\delta^3}$$

$$\delta_h = \delta - \frac{\delta}{6} - \frac{4}{3\xi}\delta + \frac{\delta}{4\xi}$$

$$\delta_h = \delta\left(1 - \frac{1}{6}\right) - \frac{\delta}{\xi}\left(\frac{4}{3} - \frac{1}{4}\right)$$

$$\frac{5\delta}{6} - \frac{13\delta}{12\xi} = \delta\left(\frac{5}{6} - \frac{13}{12\xi}\right)$$

$$\boxed{\frac{\delta_h}{\delta} = \frac{5}{6} - \frac{13}{12\xi}}$$

Q_2 $\frac{\delta^*}{\delta} = \frac{1}{6}$	$\frac{\theta}{\delta} = 0.221$	$\frac{\delta_E}{\delta} = -0.415$

Sol (5)

$$\frac{u}{U_1} = \sin\frac{\pi\eta}{2} + \frac{2\lambda}{\pi^2}\left(\sin\frac{\pi\eta}{2} - \sin^2\frac{\pi\eta}{2}\right)$$

$$\frac{u}{U_1} = \left(1+\frac{2\lambda}{\pi^2}\right)\sin\frac{\pi\eta}{2} - \frac{2\lambda}{\pi^2}\sin^2\frac{\pi\eta}{2}$$

At separation $\frac{\partial u}{\partial y} = 0 \rightarrow \left(\frac{\partial u/U_1}{\partial y/\delta}\right)_{y=0} = 0$

$$\frac{\partial(u/U_1)}{\partial\eta} = \left(1+\frac{2\lambda}{\pi^2}\right)\frac{\pi}{2}\cos\frac{\pi\eta}{2} - \frac{2\lambda}{\cancel{\pi^2}}\,2\sin\frac{\pi\eta}{2}\times\frac{\pi}{2}\cos\frac{\pi\eta}{2}$$

$$2\sin\frac{\pi\eta}{2}\cos\frac{\pi\eta}{2} = \sin\pi\eta$$

$$\frac{\partial u/U_1}{\partial\eta} = \left(1+\frac{2\lambda}{\pi^2}\right)\frac{\pi}{2}\cos\frac{\pi\eta}{2} - \frac{2\lambda}{\pi}\sin\pi\eta -$$

at $\eta = 0$

$$\frac{\partial(u/U_1)}{\partial\eta} = \left(1+\frac{2\lambda}{\pi^2}\right)\frac{\pi}{2} = 0$$

$$1+\frac{2\lambda}{\pi^2} = 0 \qquad \frac{2\lambda}{\pi^2} = -1 \qquad \boxed{\lambda = -\frac{\pi^2}{2}}$$