Chapter 4 problems without solutions:

Problem 1: Prove that

If $W_1, W_2, \ldots, W_r$ are subspaces of a vector space V, then the intersection of these subspaces is also a subspace of V.

Problem 2:

Prove that *The solution set of a homogeneous linear system $A\mathbf{x} = \mathbf{0}$ of m equations in n unknowns is a subspace of R^n.*

Problem 3:

11. In each part, determine whether the vectors span R^3.

(a) $\mathbf{v}_1 = (2, 2, 2),\ \mathbf{v}_2 = (0, 0, 3),\ \mathbf{v}_3 = (0, 1, 1)$

(b) $\mathbf{v}_1 = (2, -1, 3),\ \mathbf{v}_2 = (4, 1, 2),\ \mathbf{v}_3 = (8, -1, 8)$

12. Suppose that $\mathbf{v}_1 = (2, 1, 0, 3)$, $\mathbf{v}_2 = (3, -1, 5, 2)$, and $\mathbf{v}_3 = (-1, 0, 2, 1)$. Which of the following vectors are in $\text{span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

(a) $(2, 3, -7, 3)$ (b) $(0, 0, 0, 0)$

(c) $(1, 1, 1, 1)$ (d) $(-4, 6, -13, 4)$

13. Determine whether the following polynomials span P_2.

$$\mathbf{p}_1 = 1 - x + 2x^2, \quad \mathbf{p}_2 = 3 + x,$$
$$\mathbf{p}_3 = 5 - x + 4x^2, \quad \mathbf{p}_4 = -2 - 2x + 2x^2$$

14. Let $\mathbf{f} = \cos^2 x$ and $\mathbf{g} = \sin^2 x$. Which of the following lie in the space spanned by $\mathbf{f}$ and $\mathbf{g}$?

(a) $\cos 2x$ (b) $3 + x^2$ (c) 1 (d) $\sin x$ (e) 0

Question 3 : [9pts]

1. Let $E = \{(x, y, z) \in \mathbb{R}^3;\ ax + y + 2z = b^2 - 4\}$.
   Find $a, b \in \mathbb{R}$ such that E is a sub-space of $\mathbb{R}^3$.

2. Let F be the subspace of $\mathbb{R}^3$ generated by the vectors $v_1 = (1, -1, 2)$, $v_2 = (0, 1, -1)$, $v_3 = (1, 0, 1)$, and $v_4 = (1, 1, 0)$.
   Is the vector $v = (1, 1, 1)$ in F? (Justify your answer.)

3. Let $W = \{(x, y, z, t) \in \mathbb{R}^4;\ x - 2z = 0,\ y + z = 0\}$.
   (a) Find a basis for W.
   (b) Which of the following vectors belong to W.
   $u = (0, 1, -1, 1)$, $v = (2, 0, -1, 5)$, $w = (-2, 1, -1, -7)$. (Justify your answer.)