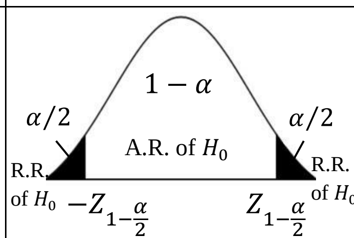
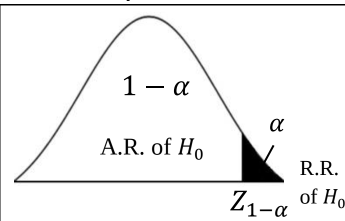
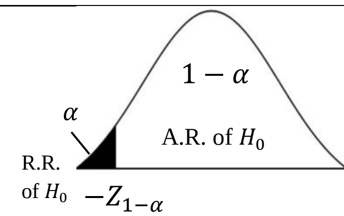
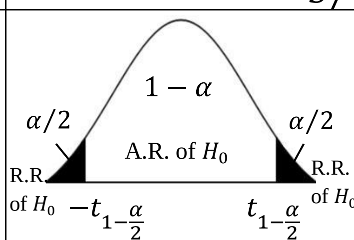
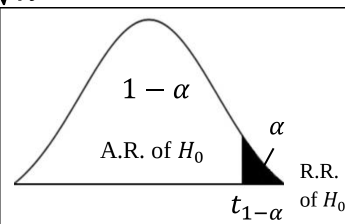
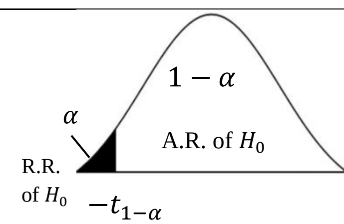
Chapter 7: Test Hypotheses

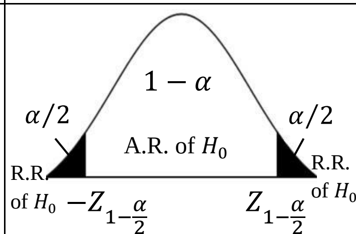
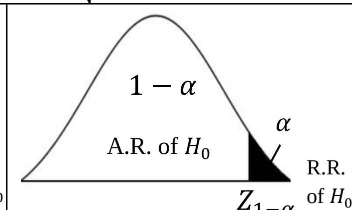
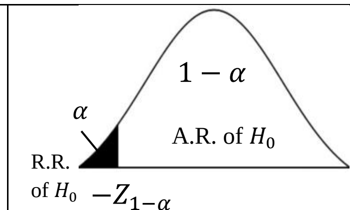
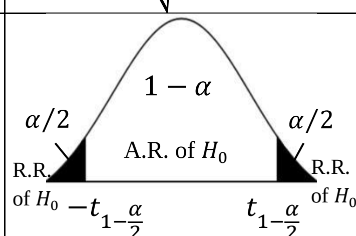
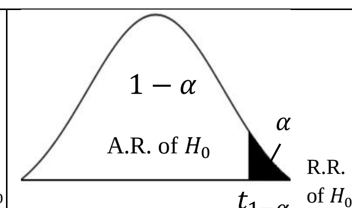
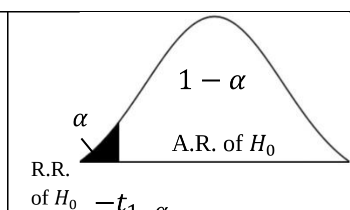
1. Test Hypotheses for the population Mean (μ):

Test Procedures:

<table>
<tr><td>Hypotheses</td><td>$H_0: \mu = \mu_0 \ \ vs$
$H_A: \mu \neq \mu_0$</td><td>$H_0: \mu \leq \mu_0 \ \ vs$
$H_A: \mu > \mu_0$</td><td>$H_0: \mu \geq \mu_0 \ \ vs$
$H_A: \mu < \mu_0$</td></tr>
<tr><td>First Case</td><td colspan="3">σ^2 is known; Normal or Non-normal Distribution</td></tr>
<tr><td>Test Statistic (T.S.)</td><td colspan="3">$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$</td></tr>
<tr><td>Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Critical Value</td><td>$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$</td><td>$Z_{1-\alpha}$</td><td>$-Z_{1-\alpha}$</td></tr>
<tr><td rowspan="2">Decision</td><td colspan="3">We reject H_0 (and accept H_A) at the significance level α if:</td></tr>
<tr><td>$Z < -Z_{1-\frac{\alpha}{2}}$ or
$Z > Z_{1-\frac{\alpha}{2}}$</td><td>$Z > Z_{1-\alpha}$</td><td>$Z < -Z_{1-\alpha}$</td></tr>
<tr><td>Second Case</td><td colspan="3">σ^2 is unknown; Normal Distribution</td></tr>
<tr><td>Test Statistic (T.S.)</td><td colspan="3">$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t(n-1), \ \ df = v = n-1$</td></tr>
<tr><td>Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Critical Value</td><td>$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$</td><td>$t_{1-\alpha}$</td><td>$-t_{1-\alpha}$</td></tr>
<tr><td rowspan="2">Decision</td><td colspan="3">We reject H_0 (and accept H_A) at the significance level α if:</td></tr>
<tr><td>$T < -t_{1-\frac{\alpha}{2}}$ or
$T > t_{1-\frac{\alpha}{2}}$</td><td>$T > t_{1-\alpha}$</td><td>$T < -t_{1-\alpha}$</td></tr>
</table>

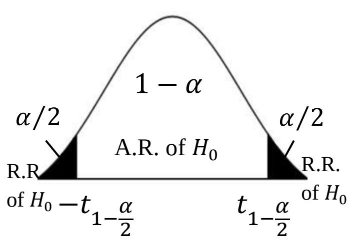
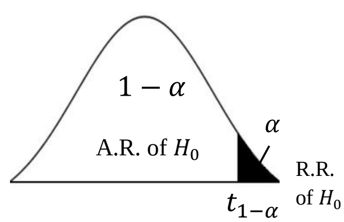
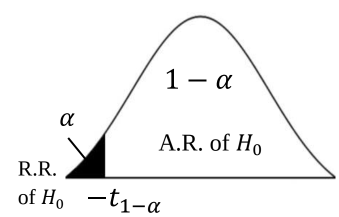
2. Test Hypotheses for the Difference Between Two Population Means $(\mu_1 - \mu_2)$(Independent Populations):

Test Procedures:

Hypotheses	$H_0: \mu_1 = \mu_2 \ \ vs$ $H_A: \mu_1 \neq \mu_2$	$H_0: \mu_1 \leq \mu_2 \ \ vs$ $H_A: \mu_1 > \mu_2$	$H_0: \mu_1 \geq \mu_2 \ \ vs$ $H_A: \mu_1 < \mu_2$
First Case	σ_1^2 and σ_2^2 are known		
Test Statistic (T.S.)	$Z = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}} \sim N(0,1)$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0			
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
Decision	We reject H_0 (and accept H_A) at the significance level α if:		
	$Z < -Z_{1-\frac{\alpha}{2}}$ or $Z > Z_{1-\frac{\alpha}{2}}$	$Z > Z_{1-\alpha}$	$Z < -Z_{1-\alpha}$
Second Case	σ_1^2 and σ_2^2 are known but equal $(\sigma_1^2 = \sigma_2^2 = \sigma^2)$		
Test Statistic (T.S.)	$T = \dfrac{\bar{X}_1 - \bar{X}_2}{\sqrt{\dfrac{S_p^2}{n_1} + \dfrac{S_p^2}{n_2}}} \sim t(n_1 + n_2 - 2), \ \ df = v = n_1 + n_2 - 2$ $S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0			
Critical Value	$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$	$t_{1-\alpha}$	$-t_{1-\alpha}$
Decision	We reject H_0 (and accept H_A) at the significance level α if:		
	$T < -t_{1-\frac{\alpha}{2}}$ or $T > t_{1-\frac{\alpha}{2}}$	$T > t_{1-\alpha}$	$T < -t_{1-\alpha}$

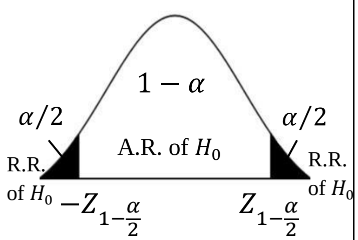
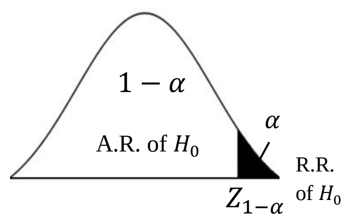
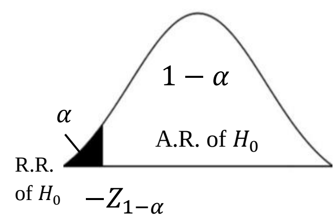
3. Confidence Interval and Test Hypotheses for the Difference Between Two Population Means ($\mu_1 - \mu_2 = \mu_D$) (Dependent/Related Populations):

The Procedure:

Calculate the Quantities	• The differences (D-observations): $D_i = X_i - Y_i\ ,\ \ i = 1,2,\dots,n$ • Sample Mean of the D-observations: $\overline{D} = \frac{\sum_{i=1}^{n} D_i}{n}$ • Sample Variance of the D-observations: $S_D^2 = \frac{\sum_{i=1}^{n}(D_i - \overline{D})^2}{n-1}$ • Sample Standard Deviation of the D-observations: $S_D = \sqrt{S_D^2}$		
Confidence Interval for $\mu_D = \mu_1 - \mu_2$			
$100(1-\alpha)\%$ Confidence Interval for μ_D	$\overline{D} \pm t_{1-\frac{\alpha}{2}} \frac{S_D}{\sqrt{n}}\ ,\quad df = v = n-1$		
Test Hypotheses for $\mu_D = \mu_1 - \mu_2$			
Hypotheses	$H_0: \mu_1 = \mu_2\ \ vs$ $H_A: \mu_1 \neq \mu_2$ or $H_0: \mu_D = 0\ \ vs$ $H_A: \mu_D \neq 0$	$H_0: \mu_1 \leq \mu_2\ \ vs$ $H_A: \mu_1 > \mu_2$ or $H_0: \mu_D \leq 0\ \ vs$ $H_A: \mu_D > 0$	$H_0: \mu_1 \geq \mu_2\ \ vs$ $H_A: \mu_1 < \mu_2$ or $H_0: \mu_D \geq 0\ \ vs$ $H_A: \mu_D < 0$
Test Statistic (T.S.)	$T = \frac{\overline{D}}{S_D/\sqrt{n}} \sim t(n-1)\ ,\quad df = v = n-1$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0			
Critical Value	$-t_{1-\frac{\alpha}{2}}$ and $t_{1-\frac{\alpha}{2}}$	$t_{1-\alpha}$	$-t_{1-\alpha}$
Decision	We reject H_0 (and accept H_A) at the significance level α if:		
	$T < -t_{1-\frac{\alpha}{2}}$ or $T > t_{1-\frac{\alpha}{2}}$	$T > t_{1-\alpha}$	$T < -t_{1-\alpha}$

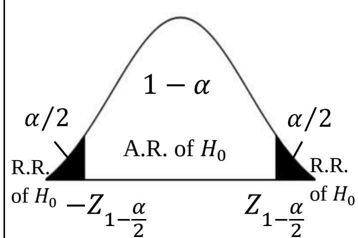
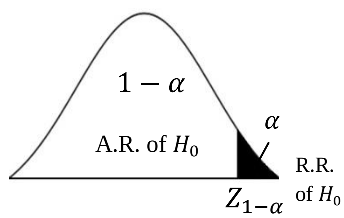
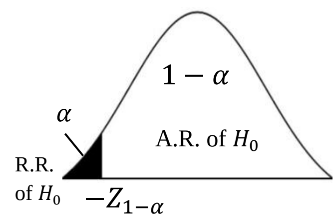
4. Test Hypotheses for the Population Proportion (p):

Test Procedure:

Hypotheses	$H_0: p = p_0 \ \ vs$ $H_A: p \neq p_0$	$H_0: p \leq p_0 \ \ vs$ $H_A: p > p$	$H_0: p \geq p_0 \ \ vs$ $H_A: p < p_0$
Test Statistic (T.S.)	$Z = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}} \sim N(0,1)$, $\quad \hat{p} = \dfrac{X}{n}$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0			
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
Decision	We reject H_0 (and accept H_A) at the significance level α if:		
	$Z < -Z_{1-\frac{\alpha}{2}}$ or $Z > Z_{1-\frac{\alpha}{2}}$	$Z > Z_{1-\alpha}$	$Z < -Z_{1-\alpha}$

5. Test Hypotheses for the Difference Between Two Population Proportions $(p_1 - p_2)$:

Test Procedure:

Hypotheses	$H_0: p_1 = p_2 \ \ vs$ $H_A: p_1 \neq p_2$	$H_0: p_1 \leq p_2 \ \ vs$ $H_A: p_1 > p_2$	$H_0: p_1 \geq p_2 \ \ vs$ $H_A: p_1 < p_2$
Test Statistic (T.S.)	$Z = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n_1} + \dfrac{\bar{p}(1-\bar{p})}{n_2}}} \sim N(0,1)$ $\hat{p}_1 = \dfrac{X_1}{n_1}, \quad \hat{p}_2 = \dfrac{X_2}{n_2}, \quad \bar{p} = \dfrac{X_1}{n_1} + \dfrac{X_2}{n_2}$		
Rejection Region (R.R.) & Acceptance Region (A.R.) of H_0			
Critical Value	$-Z_{1-\frac{\alpha}{2}}$ and $Z_{1-\frac{\alpha}{2}}$	$Z_{1-\alpha}$	$-Z_{1-\alpha}$
Decision	We reject H_0 (and accept H_A) at the significance level α if:		
	$Z < -Z_{1-\frac{\alpha}{2}}$ or $Z > Z_{1-\frac{\alpha}{2}}$	$Z > Z_{1-\alpha}$	$Z < -Z_{1-\alpha}$