# Engineering Probability & Statistics
# (AGE 1150)
## Chapter 6: Some Continuous Probability Distributions

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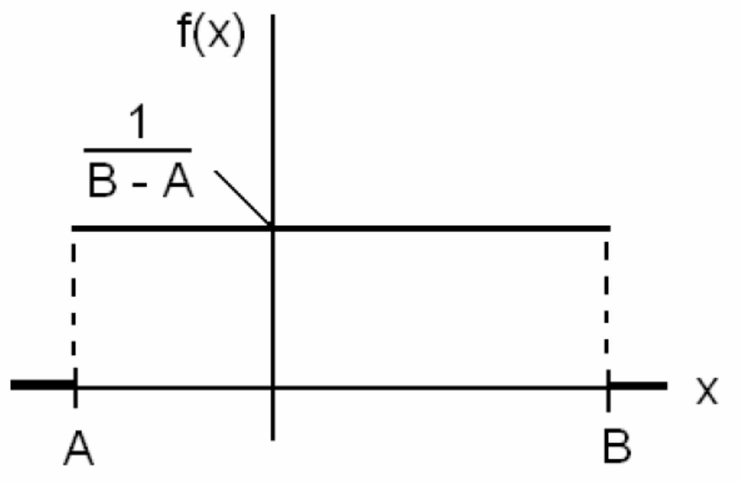
# Continuous Uniform distribution (Rectangular Distribution)

- The probability density function of the continuous uniform random variable X on the interval [A, B] is given by:

$$f(x) = f(x; A, B) = \begin{cases} \dfrac{1}{B - A} \ ; \ A \le x \le B \\ 0 \ ; \ elsewhere \end{cases}$$




- We write X ~ Uniform(A,B).
- Theorem 6.1:
- The mean and the variance of the continuous uniform distribution on the interval [A, B] are:

$$\mu = \frac{A + B}{2}$$

$$\sigma^2 = \frac{(B - A)^2}{12}$$

Example 6.1:

Suppose that, for a certain company, the conference time, X, has a uniform distribution on the interval [0,4] (hours).

(a) What is the probability density function of X?

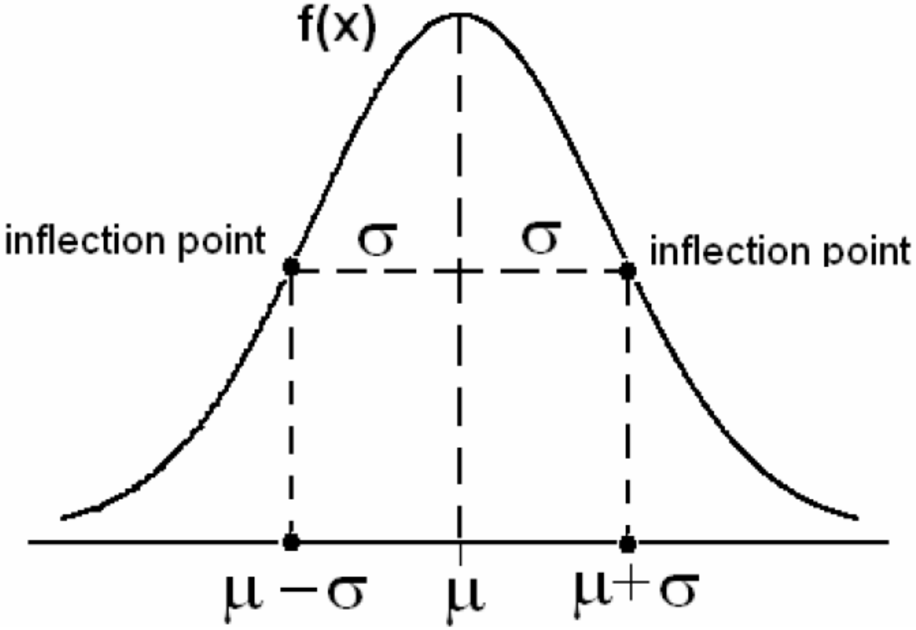
(b) What is the probability that any conference lasts at least 3 hours?

(a) $f(x) = f(x;0,4) = \begin{cases} \frac{1}{4} & ; 0 \le x \le 4 \\ 0 & ; elsewhere \end{cases}$

(b) $P(X \ge 3) = \int_3^4 f(x)\,dx = \int_3^4 \frac{1}{4}\,dx = \frac{1}{4}$

# Normal Distribution

- The normal distribution is one of the most important continuous distributions.
- Many measurable characteristics are normally or approximately normally distributed, such as, height and weight.
- The graph of the probability density function (pdf) of a normal distribution, called the **normal curve**, is a **bell-shaped** curve.



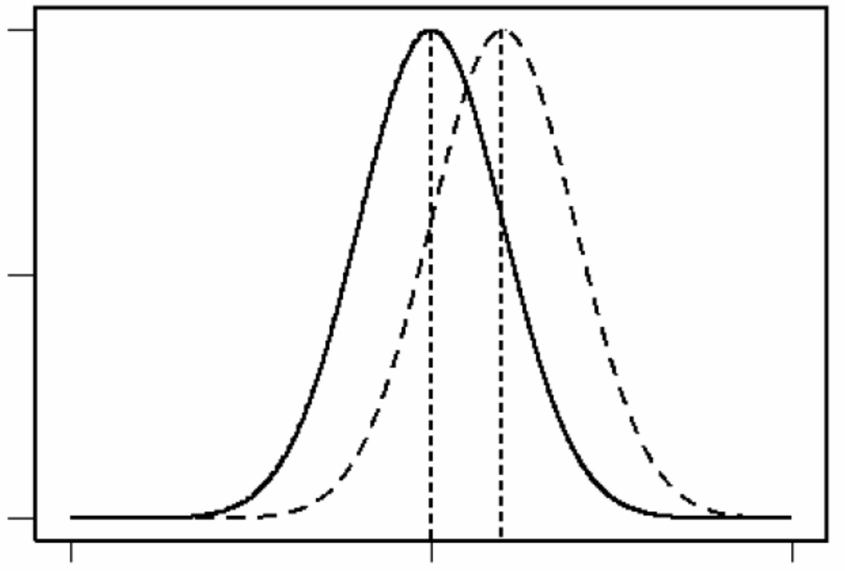
- The pdf of the normal distribution depends on two parameters: mean = E(X)= μ and variance = Var(X) = $\sigma^2$.
- If the random variable X has a normal distribution with mean μ and variance $\sigma^2$, we write:
- X ~ Normal (μ , σ ) or X ~ N( μ , σ)
- The pdf of X ~ Normal (μ , σ) is given by:

$$f(x) = n(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad ; \begin{cases} -\infty < x < \infty \\ -\infty < \mu < \infty \\ \sigma > 0 \end{cases}$$

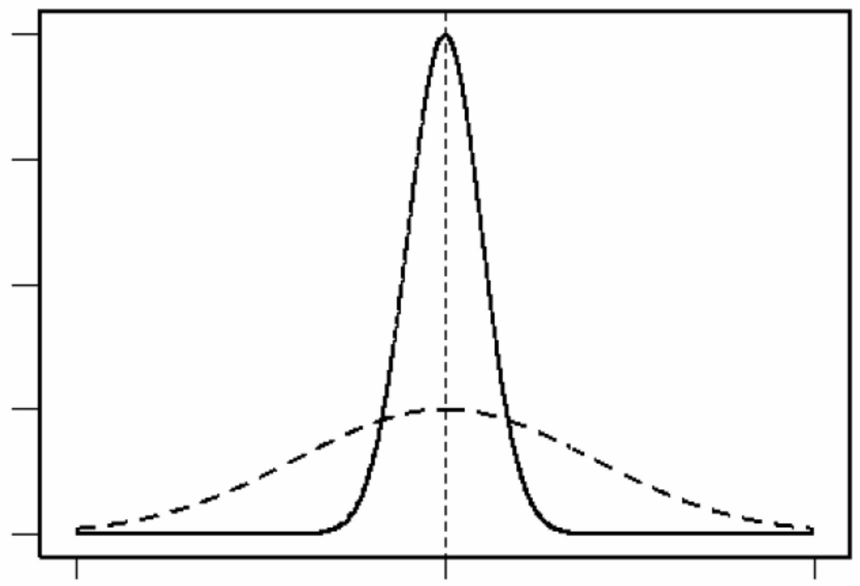
- The location of the normal distribution depends on μ and its shape depends on σ.
- Suppose we have two normal distributions:

________ $N(\mu_1, \sigma_1)$

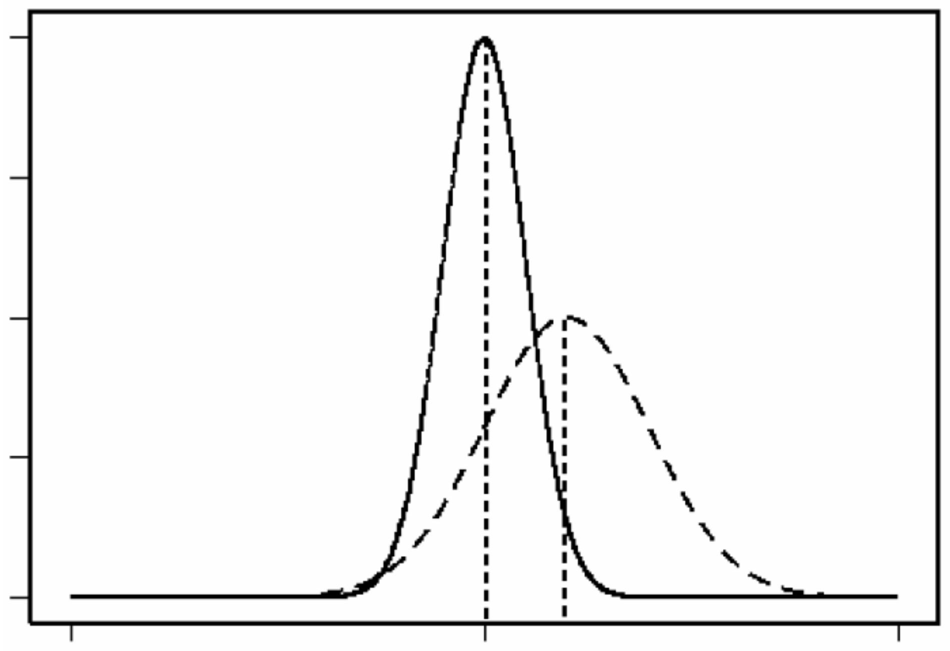
----------- $N(\mu_2, \sigma_2)$



$\mu_1 < \mu_2, \sigma_1 = \sigma_2$



$\mu_1 = \mu_2, \sigma_1 < \sigma_2$



$\mu_1 < \mu_2, \sigma_1 < \sigma_2$

- Some properties of the normal curve f(x) of N(μ , σ):

1. f(x) is symmetric about the mean μ.
2. f(x) has two points of inflection at x = μ ± σ.
3. The total area under the curve of f(x) =1.
4. The highest point of the curve of f(x) at the mean μ.

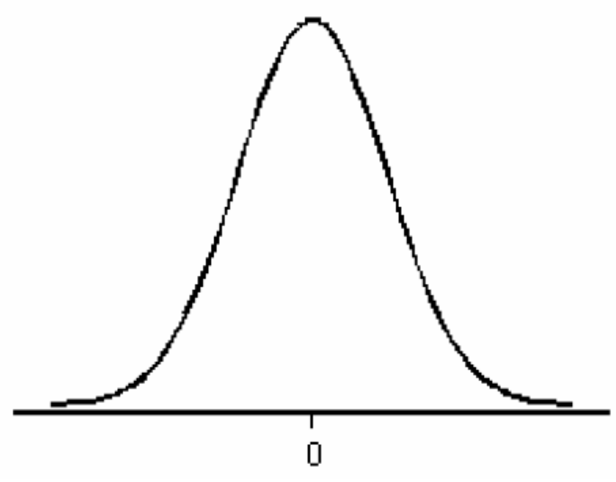
# Areas Under the Normal Curve

Definition 6.1:

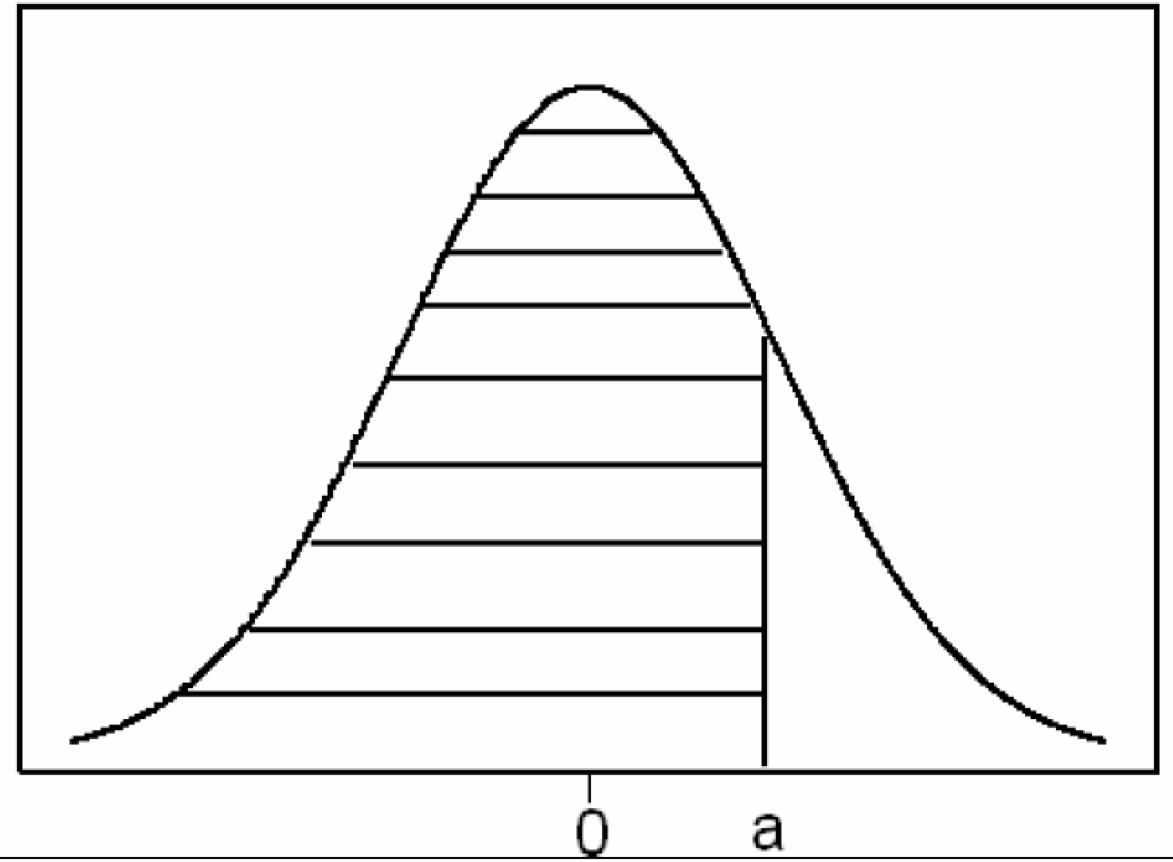
The Standard Normal Distribution:

- The normal distribution with mean $\mu = 0$ and variance $\sigma^2 = 1$ is called the standard normal distribution and is denoted by Normal (0,1) or N(0,1). If the random variable Z has the standard normal distribution, we write Z~Normal (0,1) or Z~N(0,1).
- The pdf of Z~N(0,1) is given by:

$$f(z) = n(z;0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}$$



- The standard normal distribution, $Z \sim N(0,1)$, is very important because probabilities of any normal distribution can be calculated from the probabilities of the standard normal distribution.
- Probabilities of the standard normal distribution $Z \sim N(0,1)$ of the form $P(Z \leq a)$ are tabulated (Table A.3).




$$P(Z \le a) = \int_{-\infty}^{a} f(z)\,dz$$

$$= \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}\,dz$$

$$= \text{from the table}$$

We can transfer any normal distribution X~N(μ,σ) to the standard normal distribution, Z~N(0,1) by using the following result.

- Result: If X~N(μ,σ), then

$$Z = \frac{X-\mu}{\sigma} \sim N(0,1).$$

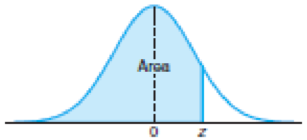



**Table A.3** Areas under the Normal Curve

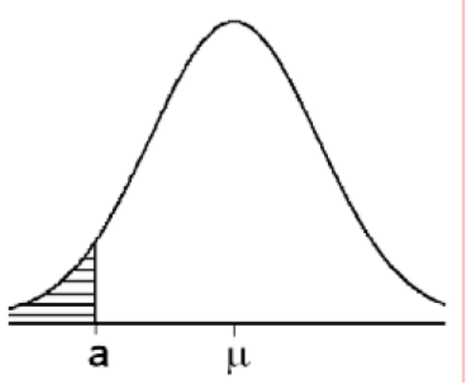
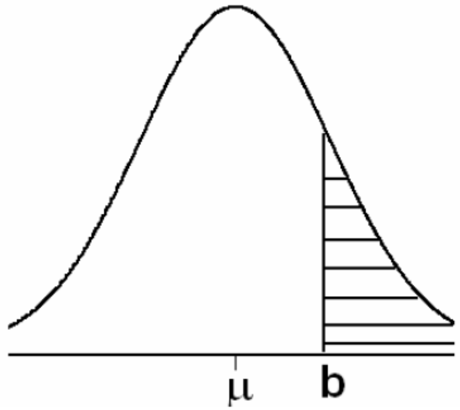
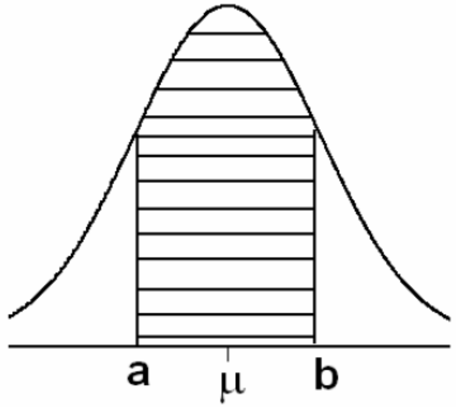
| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| −3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| −3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| −3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| −3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| −2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| −2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| −2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| −2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| −2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| −2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| −2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| −2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| −2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| −2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| −1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| −1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| −1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| −1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| −1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| −1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| −1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| −1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| −1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| −1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| −0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| −0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| −0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| −0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| −0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| −0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| −0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| −0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| −0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| −0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |

**Table A.3** (continued) Areas under the Normal Curve

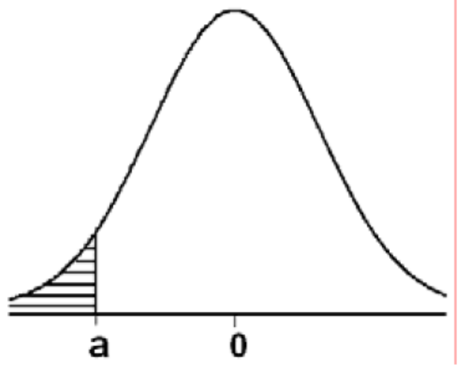
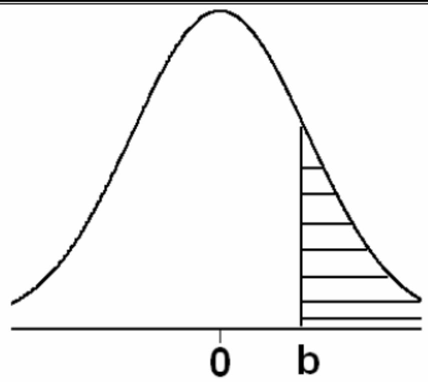
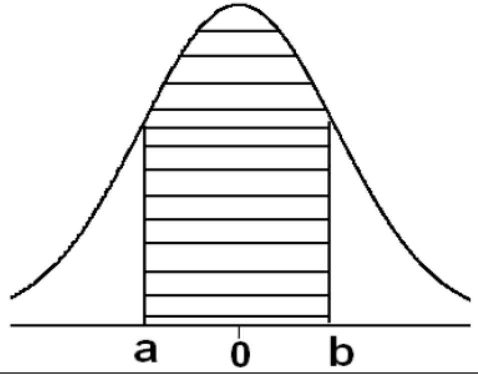
| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

# Areas Under the Normal Curve of X~N(μ,σ)

- The probabilities of the normal distribution N(μ,σ) depends on μ and σ.

| | | |
|---|---|---|
|   |   |   |
| $P(X<a)=\int_{-\infty}^{a} f(x)\,dx$ | $P(X>b)=\int_{b}^{\infty} f(x)\,dx$ | $P(a<X<b)=\int_{a}^{b} f(x)\,dx$ |

**Probabilities of Z~N(0,1):**
Suppose Z ~ N(0,1).

| | | |
|---|---|---|
|   |   |   |
| $P(Z\leq a)$ =From Table (A.3) | $P(Z\geq b) = 1-P(Z\leq b)$ | $P(a\leq Z\leq b) = P(Z\leq b)-P(Z\leq a)$ |

Note: $P(Z=a)=0$ for every a .

# Examples

Suppose Z~N(0,1).

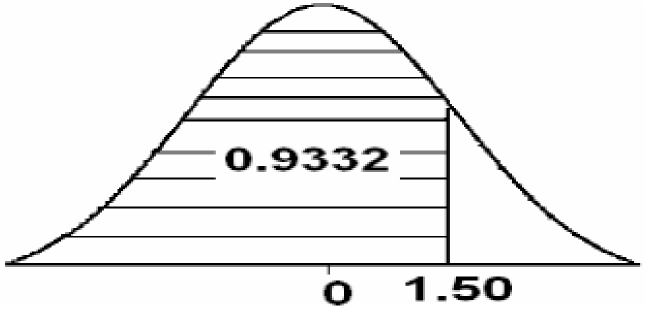
- P(Z≤1.50) = 0.9332

| Z | 0.00 | 0.01 | … |
|---|---|---|---|
| ⋮ | ⇓ | | |
| 1.5 ⇒ | 0.9332 | | |
| ⋮ | | | |

Table A.3 (continued) Areas under the Normal Curve

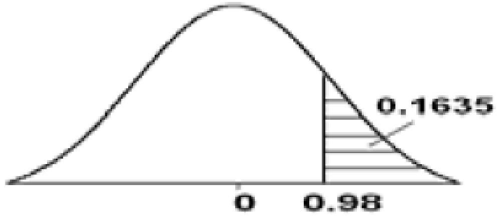
| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | [illegible] | [illegible] | 0.9222 | [illegible] | 0.9251 | [illegible] | [illegible] | 0.9292 | [illegible] | [illegible] |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |

1.5 0.9332



- P(Z≥0.98)=1−P(Z≤0.98)
  =1 − 0.8365
  = 0.1635

| Z | 0.00 | … | 0.08 |
|---|---|---|---|
| ⋮ | ⋮ | ⋮ | ⇓ |
| ⋮ | … | … | ⇓ |
| 0.9⇒ | ⇒ | ⇒ | 0.8365 |



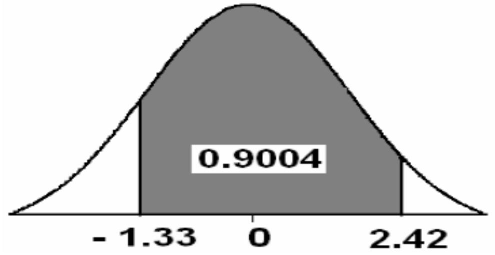

- P(−1.33≤ Z≤2.42)

= P(Z≤2.42) − P(Z≤−1.33)

= 0.9922 − 0.0918

= 0.9004

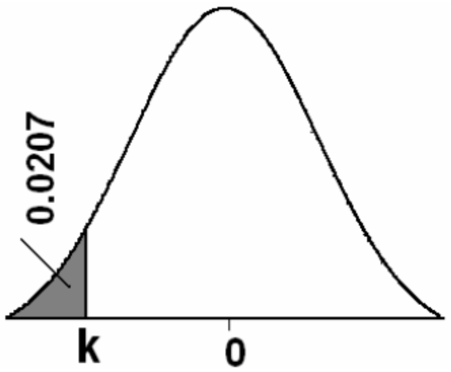
| Z | … | 0.02 | 0.03 |
|---|---|---|---|
| ⋮ | ⋮ | ⇓ | ⇓ |
| −1.3 | ⇒ | | 0.0918 |
| ⋮ | | ⇓ | |
| 2.4 | ⇒ | 0.9922 | |




- P(Z≤0)=P(Z ≥0)=0.5
- Suppose Z~N(0,1). Find the value of *k* such that P(Z≤*k*)= 0.0207.

.*k* = −2.04

| Z | … | 0.04 | |
|---|---|---|---|
| ⋮ | ⋮ | ⇑ | |
| | | ⇑ | |
| −2.0 | ⇐⇐ | 0.0207 | |
| ⋮ | | | |



## Probabilities of X~N(μ,σ):

- Result: $X \sim \mathrm{N}(\mu,\sigma) \Leftrightarrow Z = \dfrac{X-\mu}{\sigma} \sim \mathrm{N}(0,1)$
- $X \le a \Leftrightarrow \dfrac{X-\mu}{\sigma} \le \dfrac{a-\mu}{\sigma} \Leftrightarrow Z \le \dfrac{a-\mu}{\sigma}$

1. $P(X \le a) = P\left(Z \le \dfrac{a-\mu}{\sigma}\right)$
2. $P(X \ge a) = 1 - P(X \le a) = 1 - P\left(Z \le \dfrac{a-\mu}{\sigma}\right)$
3. $P(a \le X \le b) = P(X \le b) - P(X \le a) = P\left(Z \le \dfrac{b-\mu}{\sigma}\right) - P\left(Z \le \dfrac{a-\mu}{\sigma}\right)$
4. P(X=a)=0 for every a.
5. $P(X \le \mu) = P(X \ge \mu) = 0.5$

Example:

Suppose that the hemoglobin level for healthy adults males has a normal distribution with mean μ=16 and variance $\sigma^2 = 0.81$ (standard deviation σ = 0.9).

(a) Find the probability that a randomly chosen healthy adult male has hemoglobin level less than 14.

(b) What is the percentage of healthy adult males who have hemoglobin level less than 14?

- Let *X* = the hemoglobin level for a healthy adult male
- *X* ~ N(μ,σ)= N(16, 0.9).

(a) $P(X \leq 14) = P\left(Z \leq \frac{14-\mu}{\sigma}\right) = P\left(Z \leq \frac{14-16}{0.9}\right)$

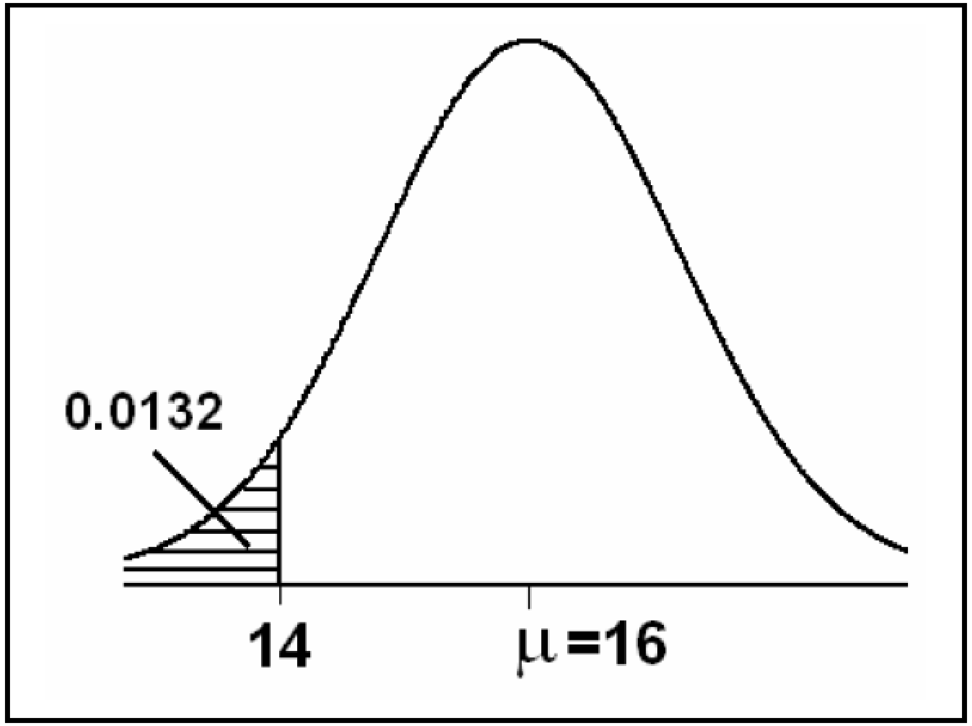
$= P(Z \leq -2.22) = 0.0132$

(b) The percentage of healthy adult males who have hemoglobin level less than 14 is

$P(X \leq 14) \times 100\% = 0.01320 \times 100\%$

$= 1.32\%$

Therefore, 1.32% of healthy adult males have hemoglobin level less than 14.



Example: Suppose that the birth weight of Saudi babies has a normal distribution with mean μ=3.4 and standard deviation σ=0.35.
(a) Find the probability that a randomly chosen Saudi baby has a birth weight between 3.0 and 4.0 kg.
(b) What is the percentage of Saudi babies who have a birth weight between 3.0 and 4.0 kg?

- X = birth weight of a Saudi baby

$\mu = 3.4$ , $\sigma = 0.35$

$X \sim N(3.4, 0.35)$

(a) $P(3.0<X<4.0)=P(X<4.0)-P(X<3.0)$

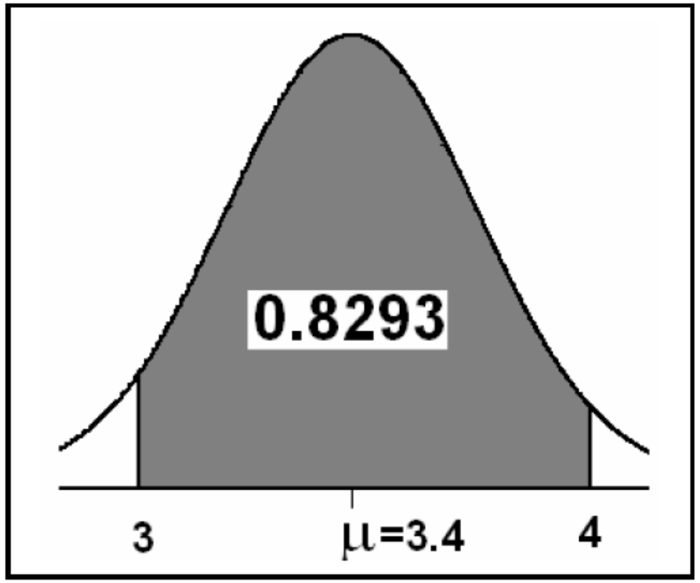
$$= P\left(Z \le \frac{4.0-\mu}{\sigma}\right) - P\left(Z \le \frac{3.0-\mu}{\sigma}\right)$$

$$= P\left(Z \le \frac{4.0-3.4}{0.35}\right) - P\left(Z \le \frac{3.0-3.4}{0.35}\right)$$

$$= P(Z \le 1.71) - P(Z \le -1.14)$$

$$= 0.9564 - 0.1271$$

$$= 0.8293$$



(b) The percentage of Saudi babies who have a birth weight between 3.0 and 4.0 kg is

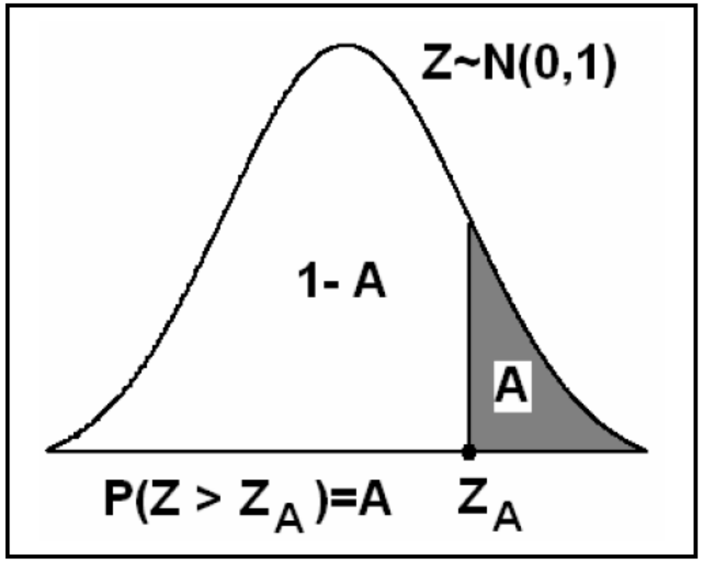
$P(3.0<X<4.0) \times 100\% = 0.8293 \times 100\% = 82.93\%$

**Notation:**

- $P(Z \geq Z_A) = A$

**Result:**

- $Z_A = -Z_{1-A}$
- **Example:**
- $Z \sim N(0,1)$
- $P(Z \geq Z_{0.025}) = 0.025$
- $P(Z \geq Z_{0.95}) = 0.95$
- $P(Z \geq Z_{0.90}) = 0.90$
- **Example:**
- $Z \sim N(0,1)$
- $Z_{0.025} = 1.96$
- $Z_{0.95} = -1.645$
- $Z_{0.90} = -1.285$




| Z | … | 0.06 |
|---|---|---|
| ⋮ | ⋮ | ⇑ ⇑ |
| 1.9 | ⇐⇐ | 0.975 |
| $P(Z \geq Z_{0.025}) = 0.025$ <br> $Z_{0.025} = 1.96$ | | |

# Application of the Normal Distribution

**Example 6.9:** In an industrial process, the diameter of a ball bearing is an important component part. The buyer sets specifications on the diameter to be 3.00±0.01 cm. The implication is that no part falling outside these specifications will be accepted. It is known that, in the process, the diameter of a ball bearing has a normal distribution with mean 3.00 cm and standard deviation 0.005 cm. On the average, how many manufactured ball bearings will be scrapped?

Sol.

- $\mu=3.00$
- $\sigma=0.005$
- X=diameter
- $X \sim N(3.00, 0.005)$
- The specification limits are:
- 3.00±0.01
- $x_1$=Lower limit=3.00−0.01=2.99
- $x_2$=Upper limit=3.00+0.01=3.01
- $P(x_1 < X < x_2)=P(2.99<X<3.01)=P(X<3.01)-P(X<2.99)$

$P(x_1 < X < x_2) = P(2.99 < X < 3.01) = P(X < 3.01) - P(X < 2.99)$

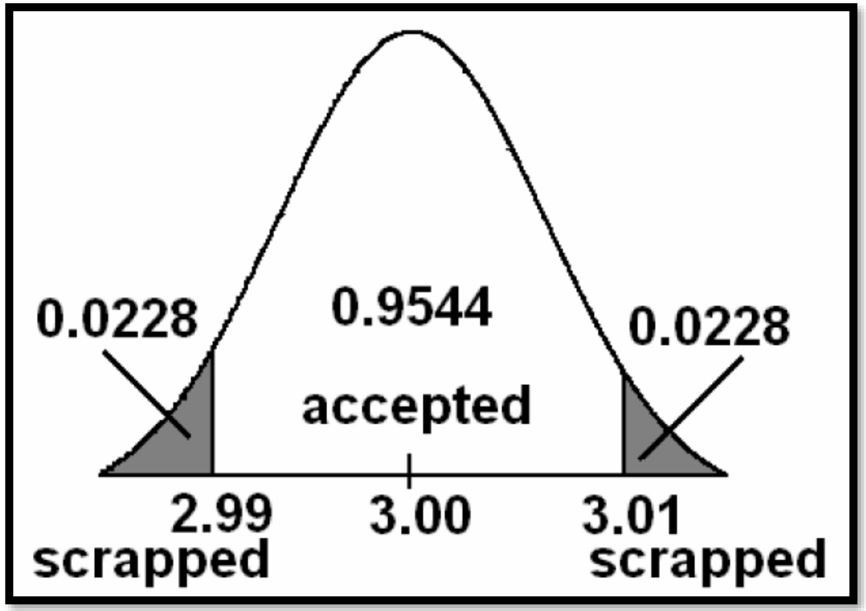
$$= P\left(Z \le \frac{3.01-\mu}{\sigma}\right) - P\left(Z \le \frac{2.99-\mu}{\sigma}\right)$$

$$= P\left(Z \le \frac{3.01-3.00}{0.005}\right) - P\left(Z \le \frac{2.99-3.00}{0.005}\right)$$

$$= P(Z \le 2.00) - P(Z \le -2.00)$$

$$= 0.9772 - 0.0228$$

$$= 0.9544$$

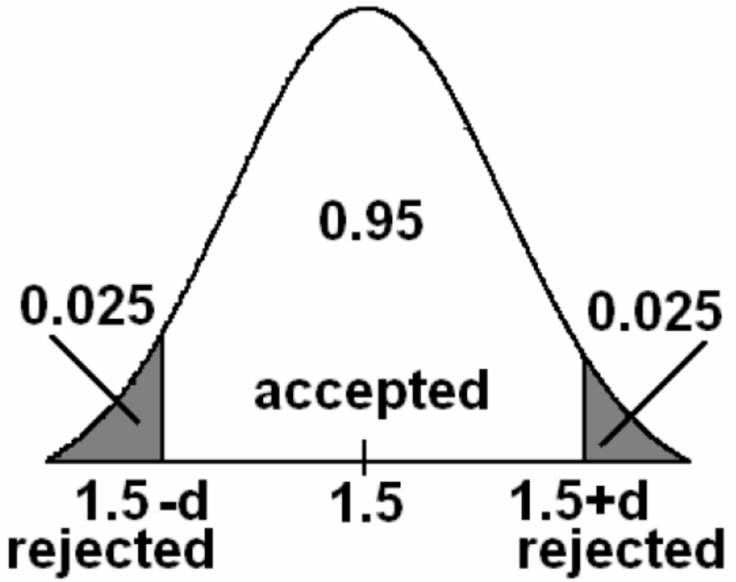



Therefore, on the average, 95.44% of manufactured ball bearings will be accepted and **4.56%** will be scrapped.

**Example 6.10:**

Gauges are use to reject all components where a certain dimension is not within the specifications 1.50±*d*. It is known that this measurement is normally distributed with mean 1.50 and standard deviation 0.20. Determine the value *d* such that the specifications cover 95% of the measurements.

**Solution:**




$\mu=1.5$

$\sigma=0.20$

X= measurement

X~N(1.5, 0.20)

The specification limits are:

$1.5 \pm d$

$x_1$ = Lower limit = $1.5 - d$

$x_2$ = Upper limit = $1.5 + d$

$P(X > 1.5+d) = 0.025 \Leftrightarrow P(X < 1.5+d) = 0.975$

$P(X < 1.5-d) = 0.025$

$$P\left(\frac{X-\mu}{\sigma} \le \frac{(1.5-d)-\mu}{\sigma}\right)=0.025$$

$$P\left(Z \le \frac{(1.5-d)-\mu}{\sigma}\right)=0.025$$

$$P\left(Z \le \frac{(1.5-d)-1.5}{0.20}\right)=0.025$$

$$P\left(Z \le \frac{-d}{0.20}\right)=0.025$$

$$\frac{-d}{0.20} = -1.96$$

$$-d = (0.20)(-1.96)$$

$$d = 0.392$$

| Z | … | 0.06 | |
|---|---|---|---|
| : | : | ⇑ ⇑ | |
| −1.9 | ⇐⇐ | 0.025 | |

$$P(Z \le \frac{-d}{0.20}) = 0.025$$

$$\frac{-d}{0.20} = -1.96$$

Note: $\frac{-d}{0.20} = Z_{0.025}$

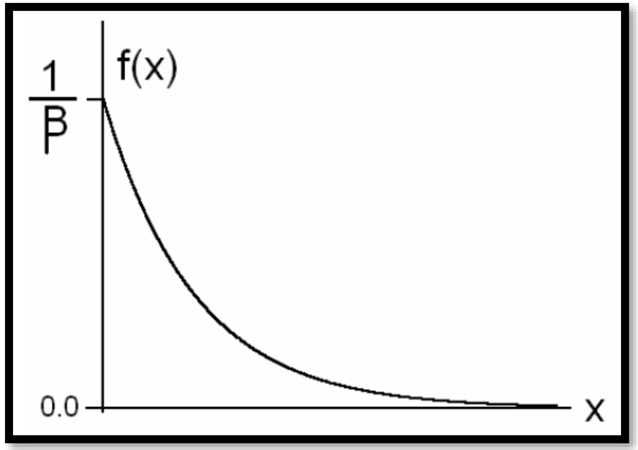
The specification limits are:

$X_1$= Lower limit =1.5 − d = 1.5 − 0.392 = 1.108

$x_2$= Upper limit = 1.5 + d = 1.5 + 0.392 = 1.892

Therefore, 95% of the measurements fall within the specifications (1.108, 1.892).

# Exponential Distribution




**Definition:**

- The continuous random variable X has an exponential distribution with parameter β, if its probability density function is given by:

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta} & ; x > 0 \\ 0 ; \textit{elsewhere} \end{cases}$$

- and we write X~Exp(β)
- **Theorem:**
- If the random variable X has an exponential distribution with parameter β, i.e., X~Exp(β), then the mean and the variance of X are:

$$E(X) = \mu = \beta$$
$$Var(X) = \sigma^2 = \beta^2$$

**Example 6.17:**

Suppose that a system contains a certain type of component whose time in years to failure is given by *T*. The random variable *T* is modeled nicely by the exponential distribution with mean time to failure β=5. If 5 of these components are installed in different systems, what is the probability that at least 2 are still functioning at the end of 8 years?

**Solution:**

- β=5
- *T*~Exp(5)
- The pdf of *T* is

$$f(t)=\begin{cases}\frac{1}{5}\,e^{-t/5}\;;t>0\\ 0\;;elsewhere\end{cases}$$

- The probability that a given component is still functioning after 8 years is given by:

$$P(T>8)=\int_8^\infty f(t)\,dt=\int_8^\infty \frac{1}{5}\,e^{-t/5}\,dt=e^{-8/5}=0.2$$

Now define the random variable:

X= number of components functioning after 8 years out of 5 components

X~ Binomial(5, 0.2) (*n*=5, *p*= P(*T*>8)= 0.2)

$$f(x) = P(X = x) = b(x;5,0.2) = \begin{cases} \binom{5}{x} 0.2^x \ 0.8^{5-x} \ ; & x = 0, 1, \ldots, 5 \\ 0 \ ; & otherwise \end{cases}$$

The probability that at least 2 are still functioning at the end of 8 years is:

$$P(X \geq 2) = 1 - P(X < 2) = 1 - [P(X=0) + P(X=1)]$$

$$= 1 - [\binom{5}{0} 0.2^0 \ 0.8^{5-0} + \binom{5}{1} 0.2^1 \ 0.8^{5-1}]$$

$$= 1 - [\ 0.8^5 + 5 \times 0.2 \ \times 0.8^4]$$

$$= 1 - 0.7373$$

$$= 0.2627$$