Chapter 5

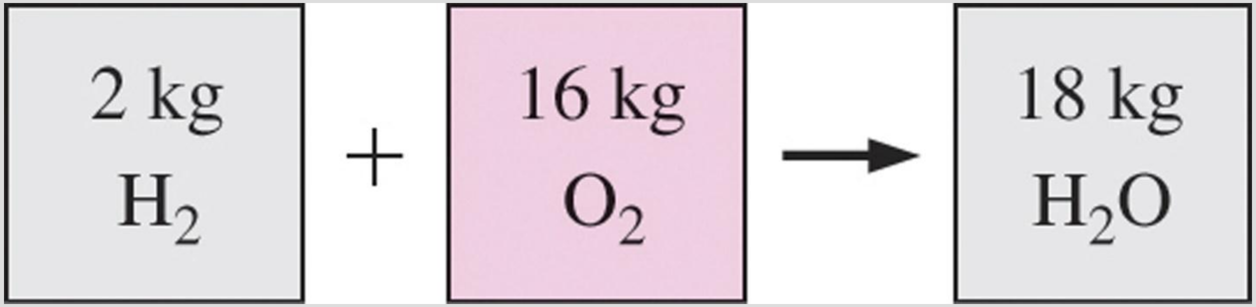
MASS AND ENERGY ANALYSIS OF CONTROL VOLUMES

Conservation of Mass

Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

Closed systems: The mass of the system remain constant during a process.

Control volumes: Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.



Conservation of Mass Principle

- ***The conservation of mass principle*** asserts that mass is a conserved property, and it cannot be created or destroyed during a process.
- For a control volume, the conservation of mass principle can be expressed as:

$$\begin{pmatrix}\text{Total mass entering}\\ \text{the CV during } \Delta t\end{pmatrix} - \begin{pmatrix}\text{Total mass leaving}\\ \text{the CV during } \Delta t\end{pmatrix} = \begin{pmatrix}\text{Net change in mass}\\ \text{within the CV during } \Delta t\end{pmatrix}$$

Conservation of Mass Principle

- In mathematical form, the conservation of energy principle for a control volume can be expressed as:

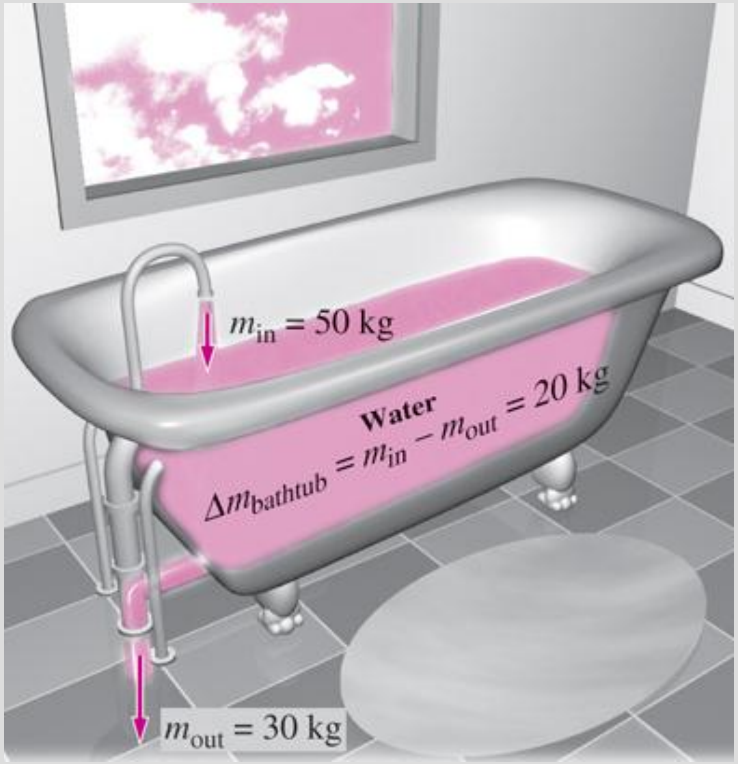
$$m_{\text{in}} - m_{\text{out}} = \Delta m_{\text{CV}}$$

$$\sum m_{in} - \sum m_{out} = \Delta\, m_{\text{CV}}$$

- In rate form,

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = dm_{\text{CV}}/dt$$

$$\sum \dot{m}_{in} - \sum \dot{m}_{out} = dm_{\text{CV}}/dt$$



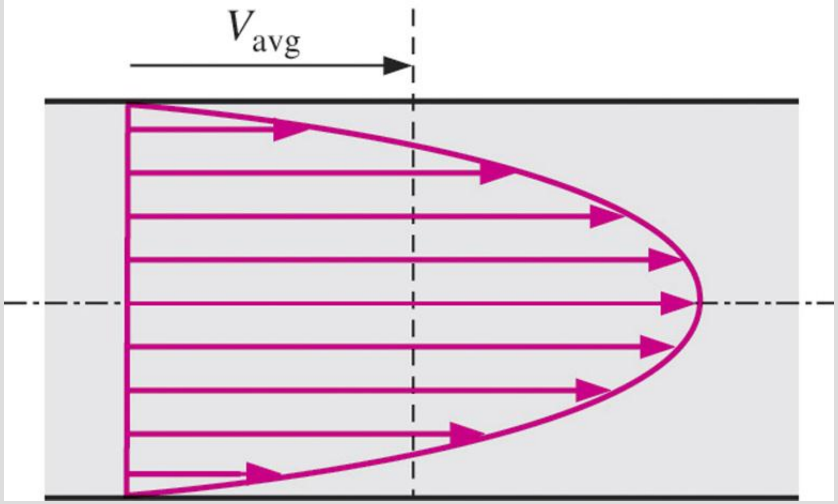
Mass Flow Rate

- $\dot{m}$ is called the mass flow rate and it is given by:

$$\delta\dot{m} = \rho V_n \, dA_c \Rightarrow \dot{m} = \int_{A_c} \delta\dot{m} = \int_{A_c} \rho V_n \, dA_c$$

- A_c is the cross-sectional area
- V_{avg} is defined as the average speed through the cross section.

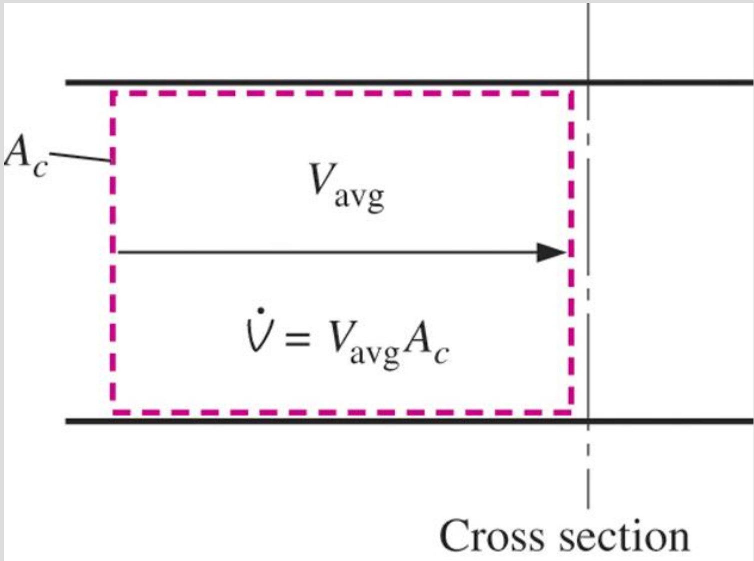
$$\dot{m} = \rho V_{\text{avg}} A_c$$



Volume Flow Rate

- $\dot{V}$ is called the volume flow rate and it is given by:

$$\dot{V} = \int_{A_c} V_n \, dA_c = V_{\text{avg}} A_c = V A_c$$



- $\dot{m}$ and $\dot{V}$ are related:

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v}$$

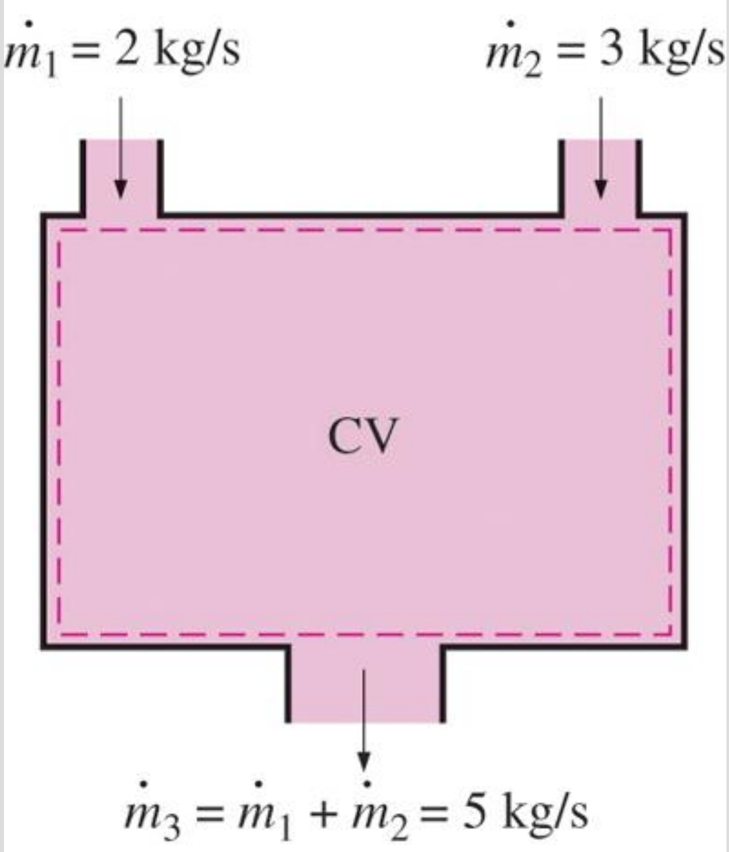
Mass Balance for Steady Flow Processes

- During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant).
- In this case, $\Delta m_{CV} = 0$
- This leads to:

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m}$$

- If there is only a single inlet and single outlet,

$$\dot{m}_1 = \dot{m}_2 \rightarrow \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$



Special Case: Incompressible Flow

- ***Incompressible Flow***. A flow in which the specific volume (and density) remain constant.

- The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.

$$\sum_{\text{in}} \dot{V} = \sum_{\text{out}} \dot{V} \qquad (\mathrm{m^3/s})$$

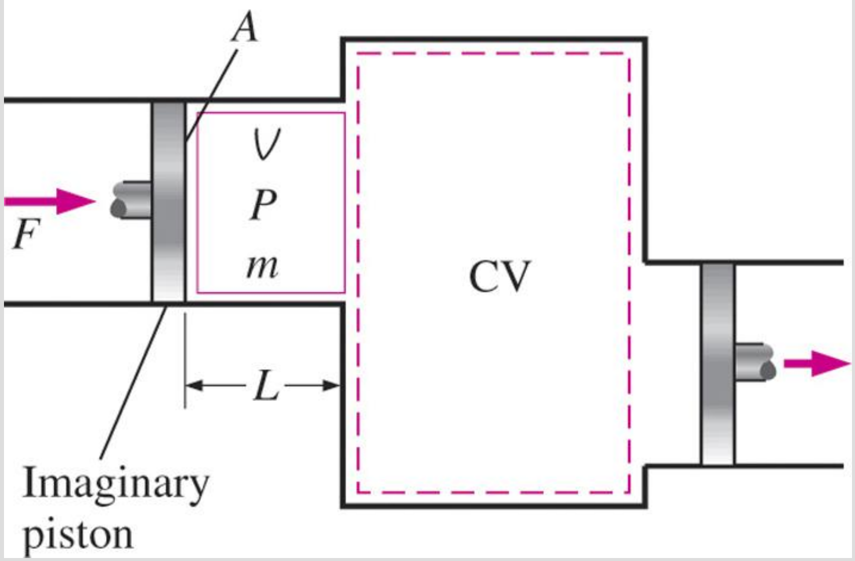
Steady, incompressible

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

Steady, incompressible flow (single stream)

Flow Work (Flow Energy)

- **Flow work, or flow energy** is the work (or energy) required to push the mass into or out of the control volume.
- This work is necessary for maintaining a continuous flow through a control volume.



$$W_{\text{flow}} = FL = PAL = PV$$

$$w_{\text{flow}} = Pv$$

Energy Transfer by Mass

- As the fluid crosses the boundary, the *energy* contained in it enters the system.
- This energy is:

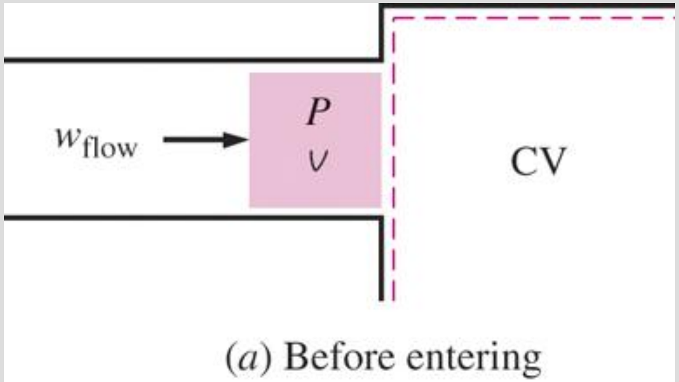
$$e = u + \text{ke} + \text{pe} = u + \frac{V^2}{2} + gz$$

- Also, the flow work that "pushes" the fluid enters the system as well.
- The combined energy entering the system with the mass of the fluid is:

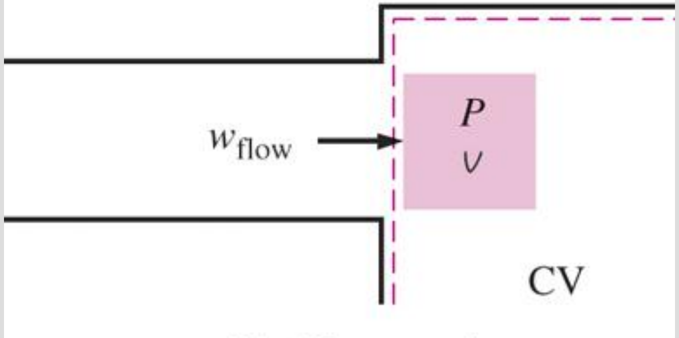
$$\theta = P\text{v} + e = P\text{v} + (u + \text{ke} + \text{pe})$$

OR

$$\theta = h + \text{ke} + \text{pe} = h + \frac{V^2}{2} + gz$$



(*a*) Before entering



(*b*) After entering

Energy Transfer by Mass

- The total energy transferred by mass is denoted by E_{mass} and is equal to:

$$E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right)$$

- In rate form,

$$\dot{E}_{\text{mass}} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

Final Form of the First Law

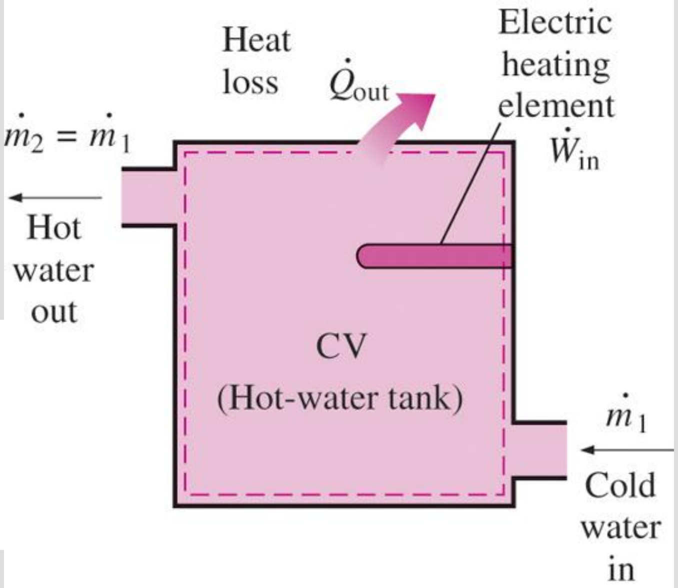
$$Q_{\text{in}} + W_{\text{in}} + \sum_{\text{in}} m\left(h + \frac{V^2}{2} + gz\right) - \left[Q_{\text{out}} + W_{\text{out}} + \sum_{\text{out}} m\left(h + \frac{V^2}{2} + gz\right)\right] = \Delta U + \Delta KE + \Delta PE$$

Or in rate form:

$$\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum_{\text{in}} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \left[\dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum_{\text{out}} \dot{m}\left(h + \frac{V^2}{2} + gz\right)\right] = \frac{dU}{dt} + \frac{dKE}{dt} + \frac{dPE}{dt}$$

Energy Balance for Steady Flow Processes

- During a steady-flow process, the total amount of energy contained within a control volume also does not change with time (E_{CV} = constant).
- In this case, $\Delta E_{CV} = 0$
- This leads to:



$$\underbrace{\dot{E}_{in}}_{\text{Rate of net energy transfer in by heat, work, and mass}} = \underbrace{\dot{E}_{out}}_{\text{Rate of net energy transfer out by heat, work, and mass}}$$

- Expanding both terms,

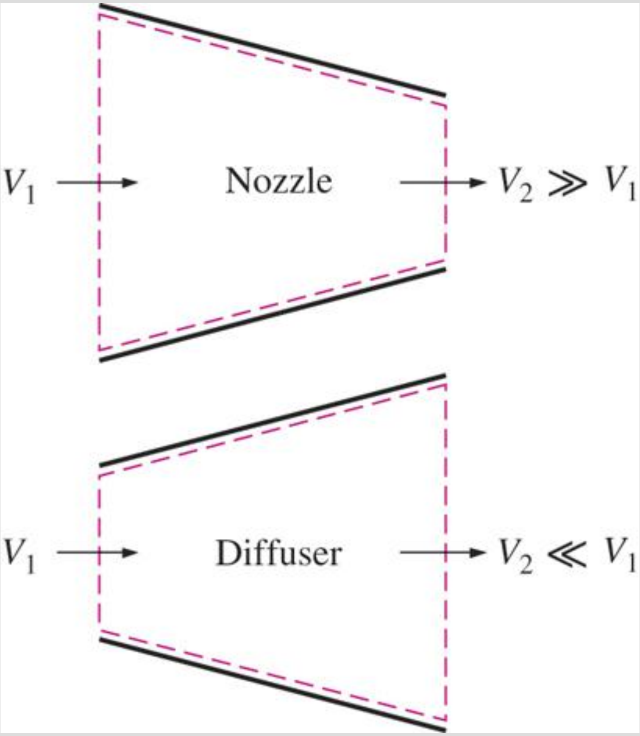
$$\dot{Q}_{in} + \dot{W}_{in} + \underbrace{\sum_{in} \dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each inlet}} = \dot{Q}_{out} + \dot{W}_{out} + \underbrace{\sum_{out} \dot{m}\left(h + \frac{V^2}{2} + gz\right)}_{\text{for each exit}}$$

Some Steady Flow Engineering Devices

- Many engineering devices operate under the same conditions for long periods of time.
- Examples: turbines, compressors, heat exchangers, pumps.
- These devices can be conveniently analyzed as steady-flow devices.
- We will cover the application of the first law (energy balance) on some of these devices, namely:
  - **Nozzles and diffusers**
  - **Turbines and compressors**
  - **Throttling valves**
  - **Mixing chambers**
  - **Heat exchangers**
  - **Pipe and duct flow**

Nozzles and Diffusers

- A **nozzle** is a device that increases the velocity of a fluid (the pressure decreases as a result).
- A **diffuser** is a device that increases the pressure of a fluid by slowing it down.
- The cross-sectional area of a nozzle decreases in the flow direction.
- The reverse is true for diffusers.



$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

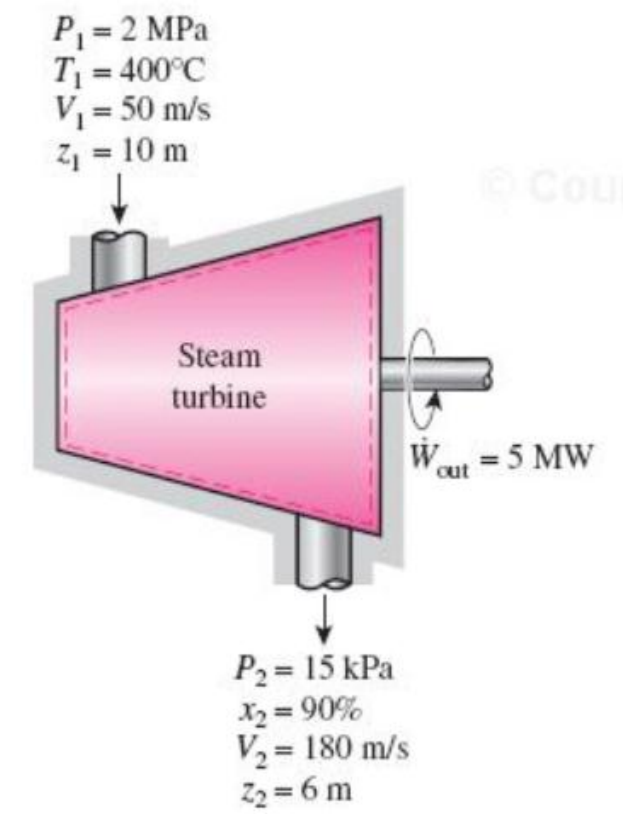
$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

Common Assumptions

- Heat transfer is often neglected
- No work input or output
- Changes in potential energy are usually neglected

Turbines

- **Turbine** drives the electric generator In steam, gas, or hydroelectric power plants.
- The fluid passes through the turbine.
- Work is done against the blades, which are attached to the shaft.
- The shaft rotates, and the turbine produces work.



Common Assumptions

- Heat transfer is often neglected
- No work input
- Changes in kinetic and potential energy of the entering and leaving fluid are sometimes neglected

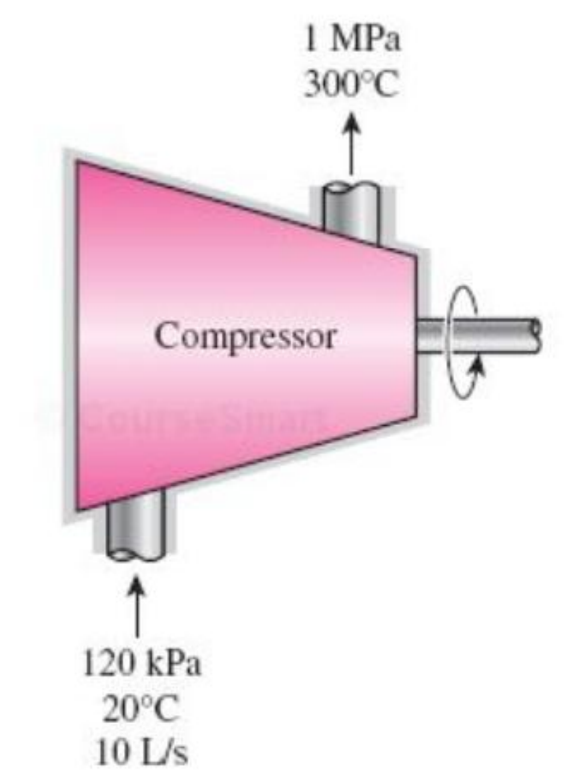
$$\dot{m}h_1 - \dot{W}_{\text{out}} = \dot{m}h_2$$

$$\Downarrow$$

$$\dot{W}_{\text{out}} = \dot{m}(h_1 - h_2)$$

Compressors

- **Compressors** are used to increase the pressure of a gas.
- Work is supplied from an external source through a rotating shaft.
- **Pumps** are similar to compressors except that they handle liquids instead of gases.



Common Assumptions

- heat transfer is often neglected
- No work output
- Changes in kinetic and potential energy of the entering and leaving fluid are sometimes neglected

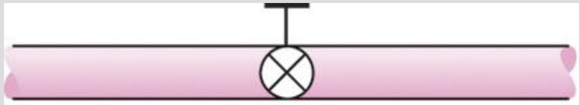
$$\dot{W}_{\text{in}} + \dot{m}h_1 = \dot{m}h_2$$

$$\Downarrow$$

$$\dot{W}_{\text{in}} = \dot{m}(h_2 - h_1)$$

Throttling Valves

- **Throttling valves** are devices restricting the flow, causing significant pressure drop in the fluid.
- Pressure drop is often accompanied by a large drop in temperature.
- Throttling devices are commonly used in refrigeration and air-conditioning applications.



(*a*) An adjustable valve

(*b*) A porous plug

(*c*) A capillary tube

Common Assumptions

- heat transfer is often neglected
- No work input or output
- Changes in kinetic and potential energy of the entering and leaving fluid are usually neglected.

$$h_2 \cong h_1$$

Mixing Chambers

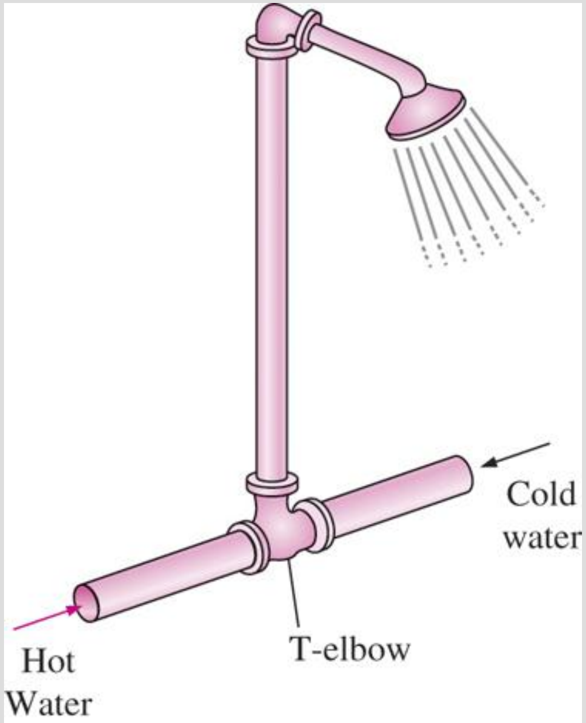
- In engineering applications, the section where the mixing process takes place is commonly referred to as a **mixing chamber.**

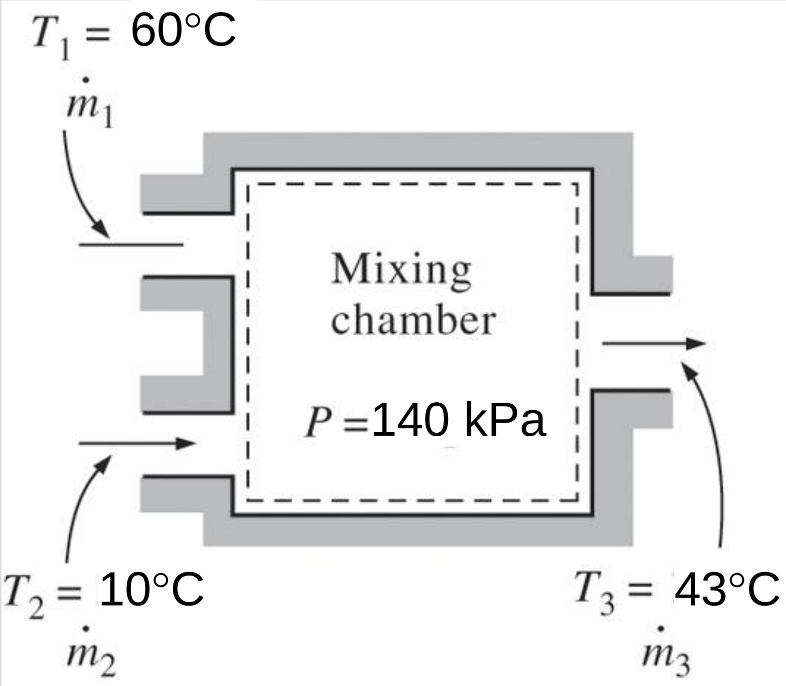
Common Assumptions

- Heat transfer is often neglected
- No work input or output
- Changes in kinetic and potential energy of the entering and leaving fluid are usually neglected

$$\dot{E}_{\mathrm{in}} = \dot{E}_{\mathrm{out}}$$

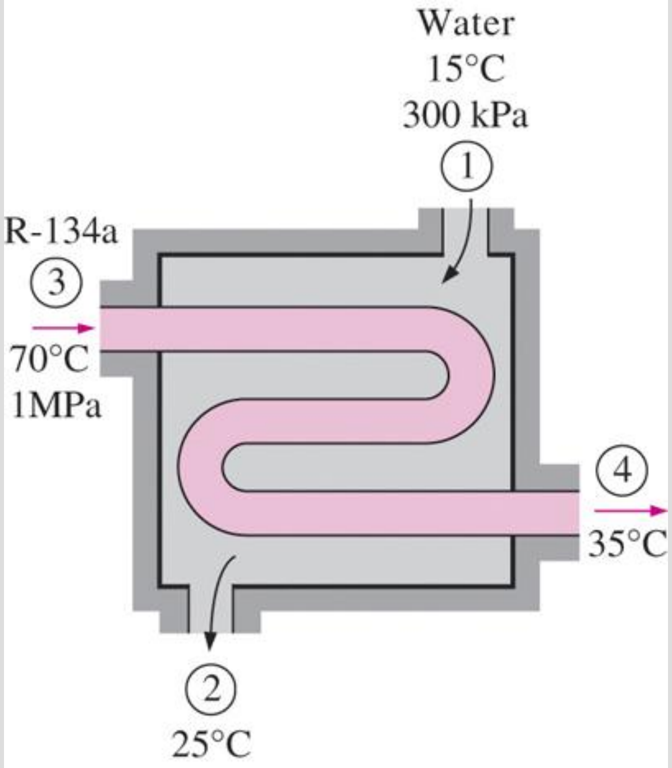
$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$





Heat Exchangers

- **Heat exchangers** are devices where two moving fluid streams exchange heat without mixing.
- Heat exchangers are widely used in various industries.
- In refrigeration, the condenser and evaporator and heat exchangers.



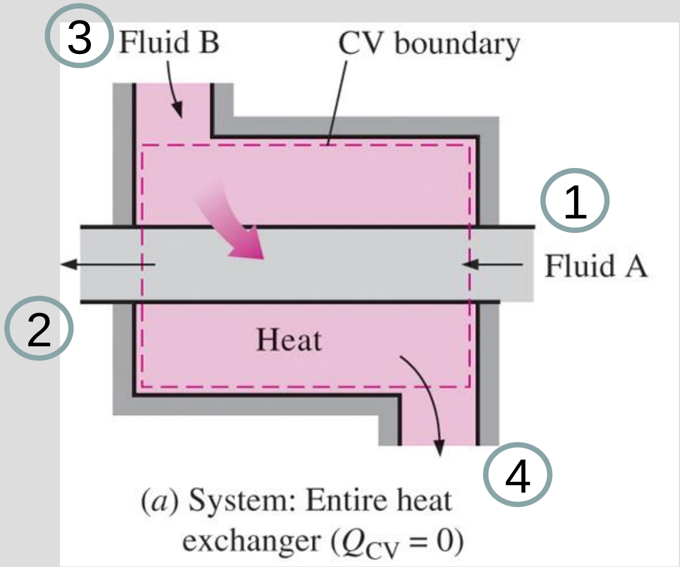
Common Assumptions

- No work input or output
- Changes in kinetic and potential energy of the entering and leaving fluid are usually neglected.
- *What about heat transfer?*

Heat Exchangers

- In Case (a), the entire heat exchanger is the control volume.
- In this case, heat transfer to the surroundings is negligible

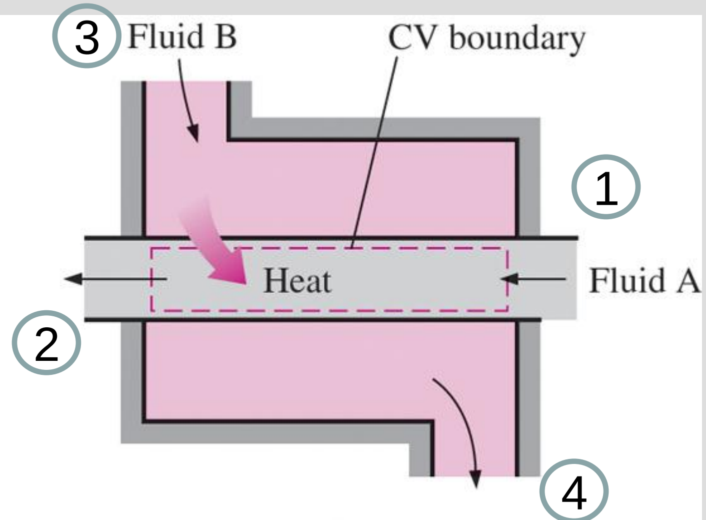
$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4$$



(*a*) System: Entire heat exchanger ($Q_{CV} = 0$)

- In Case (b), the tube(s) alone is the control volume.
- In this case, heat transfer cannot be neglected.

$$\dot{Q}_{\text{in}} = \dot{m}(h_2 - h_1)$$



(*b*) System: Fluid A ($Q_{CV} \neq 0$)

Pipe and Duct Flow

- Many liquids flow through **pipes**
- Many gases flow through **ducts**
- Flow through a pipe or a duct usually satisfies the steady-flow conditions.
- There are no common assumptions
- Each case has its own assumptions.