

Chapter 4

ENERGY ANALYSIS OF CLOSED SYSTEMS

First Law of Thermodynamics

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$

$$\Rightarrow (Q_{\text{in}} + W_{\text{in}} + E_{\text{mass,in}}) - (Q_{\text{out}} + W_{\text{out}} + E_{\text{mass,out}}) = \Delta E_{\text{system}}$$

$$\Rightarrow (Q_{\text{in}} + W_{\text{in}} + E_{\text{mass,in}}) - (Q_{\text{out}} + W_{\text{out}} + E_{\text{mass,out}}) = \Delta U + \Delta KE + \Delta PE$$

$$\Rightarrow (Q_{\text{in}} + W_{\text{in}} + E_{\text{mass,in}}) - (Q_{\text{out}} + W_{\text{out}} + E_{\text{mass,out}}) = m (u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

First Law of Thermodynamics for Closed Systems

$$(Q_{\text{in}} + W_{\text{in}}) - (Q_{\text{out}} + W_{\text{out}}) = \\ m (u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

$$\Rightarrow (Q_{\text{in}} - Q_{\text{out}}) + (W_{\text{in}} - W_{\text{out}}) = \\ m (u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

$$\Rightarrow Q_{\text{net,in}} - W_{\text{net,out}} = \\ m (u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

First Law of Thermodynamics for Closed Systems

Special Case: Stationary System

$$(Q_{in} + W_{in}) - (Q_{out} + W_{out}) =$$
$$m (u_2 - u_1) + \cancel{\frac{1}{2} m (V_2^2 - V_1^2)} + \cancel{m g (z_2 - z_1)}$$

$\xrightarrow{\quad 0 \qquad \qquad \qquad 0}$

⇒ $(Q_{in} + W_{in}) - (Q_{out} + W_{out}) = m (u_2 - u_1)$

⇒ $(Q_{in} + W_{in}) - (Q_{out} + W_{out}) = \Delta U$

If the stationary system is undergoing a cycle:

$$(Q_{in} + W_{in}) - (Q_{out} + W_{out}) = \cancel{m (u_2 - u_1)}$$

$\xrightarrow{\quad 0}$

⇒ $Q_{in} - Q_{out} = W_{out} - W_{in}$

First Law of Thermodynamics for Closed Systems

$$(Q_{\text{in}} + W_{\text{in}}) - (Q_{\text{out}} + W_{\text{out}}) = \\ m (u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

- In thermodynamics, details of Q are not studied.
- W can be calculated from equations given earlier for different types of work (shaft, electrical, etc.)
 - **Only one type of work has not been covered yet (moving boundary work)**
- E_{mass} will be introduced in Chapter 5
- u can be obtained from tables for phase-changing materials
 - **In this chapter, obtaining (u) for ideal gases will be covered.**

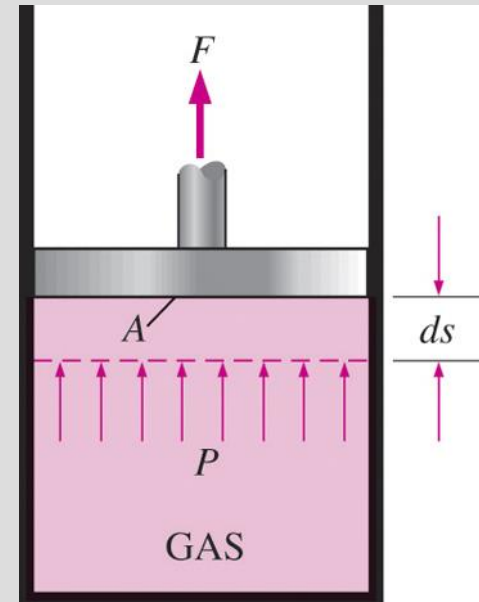
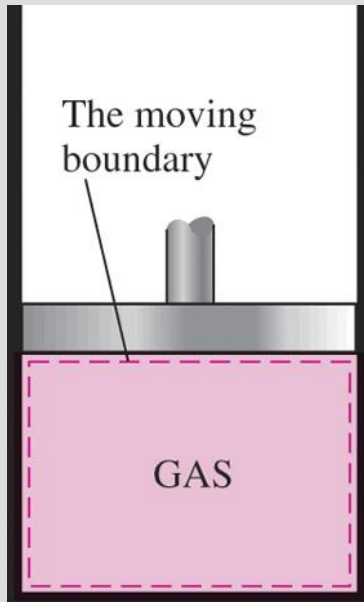
Moving Boundary Work

- The work associated with a moving boundary is called *boundary work*.
- Work is the result of a force (F) acting on an object for a distance (s):

$$\delta W_b = F ds = PA ds = P dV$$

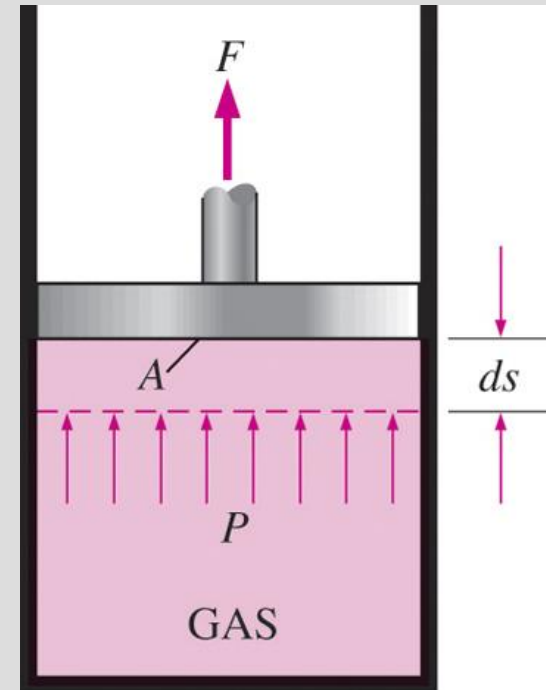


$$W_b = \int_1^2 P dV \quad (\text{kJ})$$



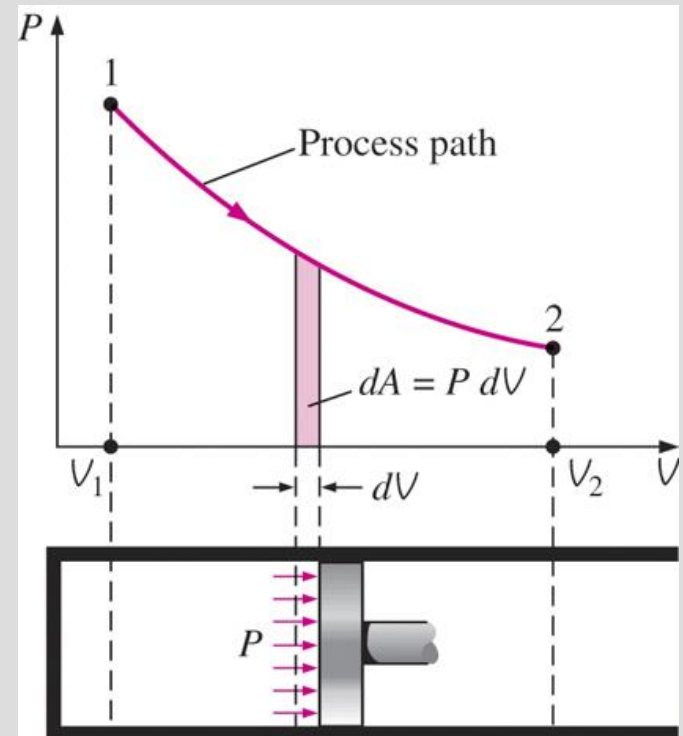
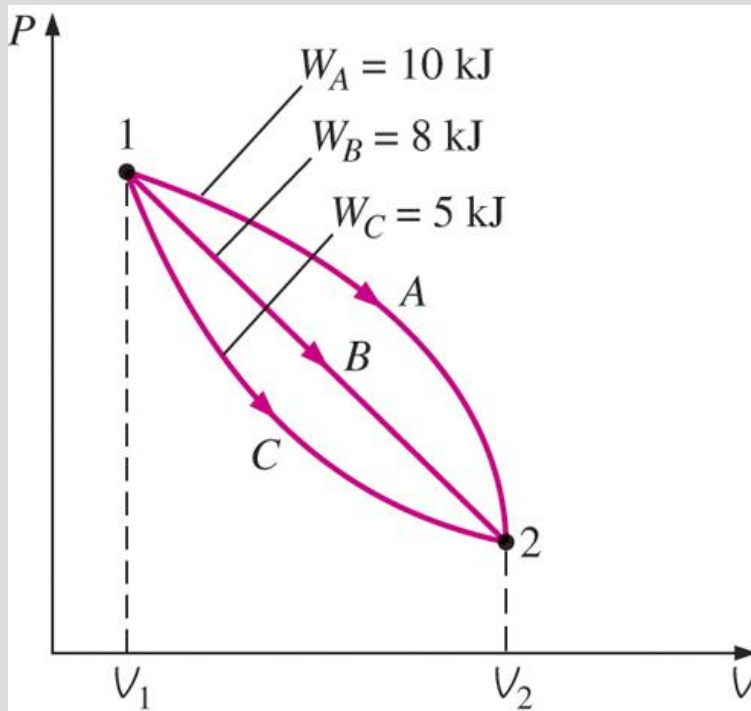
Moving Boundary Work

- If the system is expanding, the fluid inside the system is pushing *out* the boundary
 - The system is exerting work on the surroundings
 - Work is going **out** of the system
- If the system is being compressed, the surroundings are pushing the boundary *in*
 - The surroundings are exerting work on the system
 - Work is going **into** the system



Moving Boundary Work

- The area under the process curve on a P - V diagram represents the boundary work.
- The boundary work done during a process depends on the path followed as well as the end states.



Moving Boundary Work

Polytropic Process

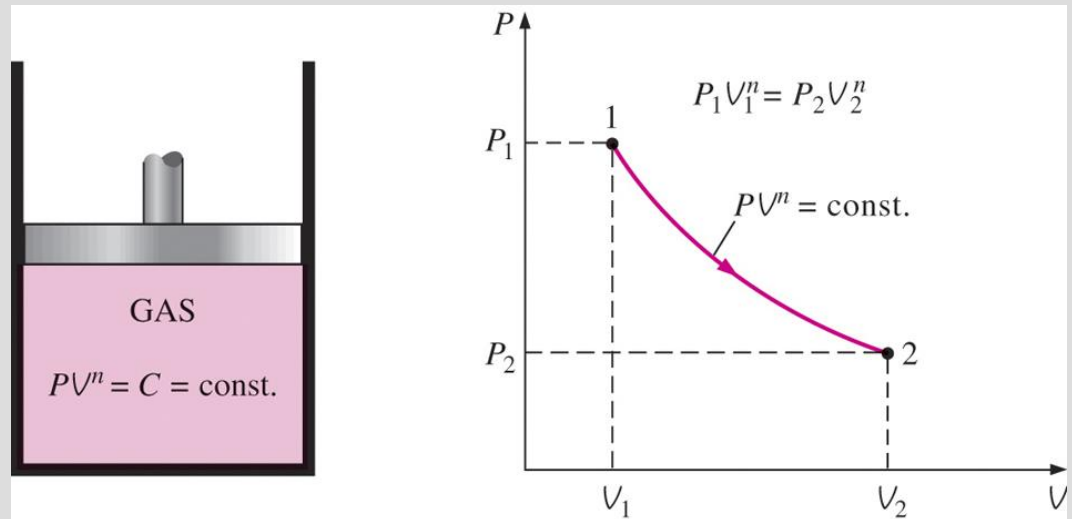
A polytropic process is a process that follows the following form:

$$P = CV^{-n} \quad \text{where } C \text{ and } n \text{ are constants}$$

$$W_b = \int_1^2 P dV = \int_1^2 CV^{-n} dV = C \frac{V_2^{-n+1} - V_1^{-n+1}}{-n + 1} = \frac{P_2 V_2 - P_1 V_1}{1 - n}$$

For an ideal gas, this equations becomes:

$$W_b = \frac{mR(T_2 - T_1)}{1 - n}$$



Moving Boundary Work

Isobaric Process

$$W_b = \int_1^2 P dV = P_0 \int_1^2 dV = P_0(V_2 - V_1)$$

Moving Boundary Work

Isothermal Process for an Ideal Gas in a Closed System

The ideal gas equation of state is: $P v = R T$, or $P V = m R T$

$$P_1 V_1 = m R T_1$$

$$P_2 V_2 = m R T_2$$

$$P V = \text{constant} = m R T$$

$$W_b = \int_1^2 P dV = \int_1^2 \frac{mRT}{V} dV = mRT \ln \frac{V_2}{V_1} = PV \ln \frac{V_2}{V_1}$$

First Law of Thermodynamics for Closed Systems

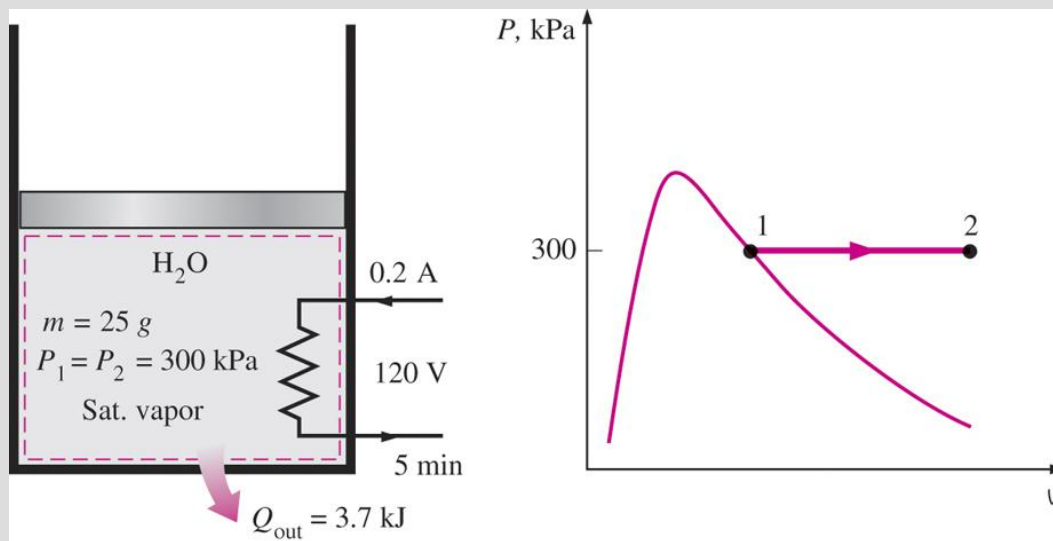
Special Case: Constant Pressure Process (Stationary)

$$(Q_{\text{in}} + W_{\text{in}}) - (Q_{\text{out}} + W_{\text{out}}) = 0$$

$$m(u_2 - u_1) + \cancel{\frac{1}{2} m (V_2^2 - V_1^2)} + m g (\cancel{z_2 - z_1})$$

$$\Rightarrow (Q_{\text{in}} - Q_{\text{out}}) + W_{\text{in}} - W_{\text{out}} = \Delta U$$

$$W_{\text{out}} = \int_1^2 P dV \quad \Rightarrow \quad W_{\text{out}} = P (V_2 - V_1)$$

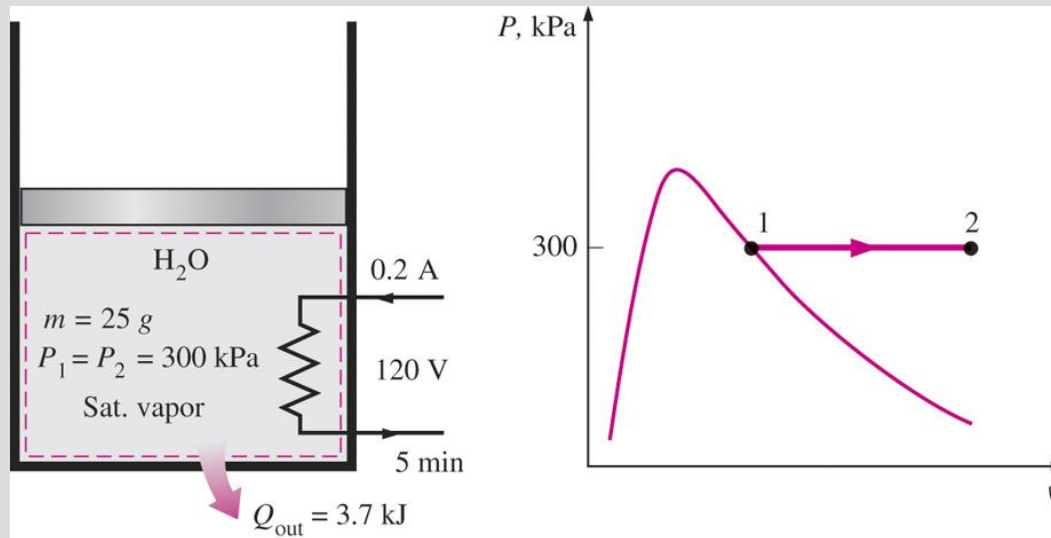


First Law of Thermodynamics for Closed Systems

Special Case: Constant Pressure Process (Stationary)

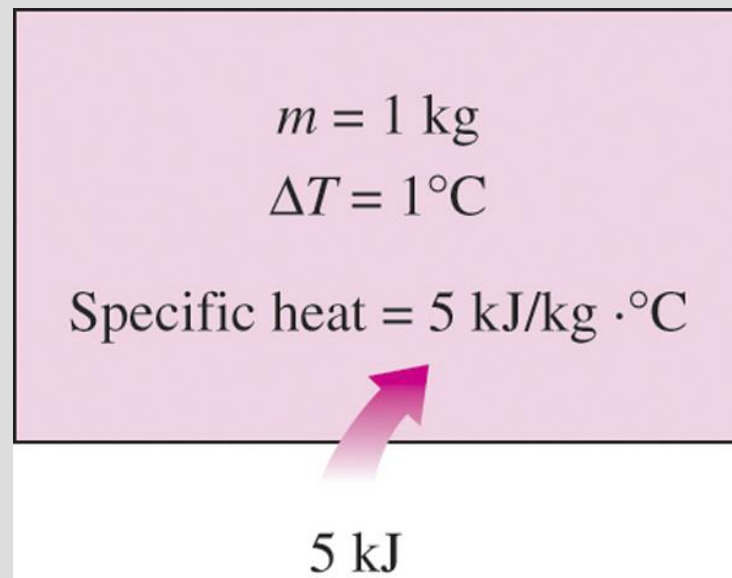
$$\Rightarrow (Q_{\text{in}} - Q_{\text{out}}) + W_{\text{in}} = P (V_2 - V_1) + (U_2 - U_1)$$

$$\Rightarrow (Q_{\text{in}} - Q_{\text{out}}) + W_{\text{in}} = H_2 - H_1$$



Specific Heat

- **Specific heat** is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.



Specific Heat at Constant Volume

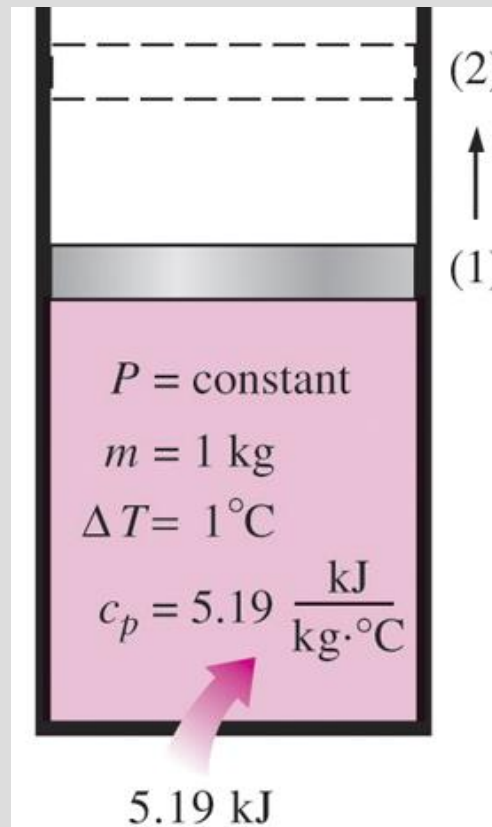
- **Specific heat at constant volume (C_v)** is the energy required to raise the temperature of a unit mass of a substance by one degree as the volume is maintained constant.

$V = \text{constant}$
 $m = 1 \text{ kg}$
 $\Delta T = 1^\circ\text{C}$
 $c_v = 3.12 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}}$

3.12 kJ

Specific Heat at Constant Pressure

- **Specific heat at constant pressure (C_p)** is the energy required to raise the temperature of a unit mass of a substance by one degree as the pressure is maintained constant.



Formal Definition of C_v and C_p

$$C_v = \left(\frac{\partial u}{\partial T} \right)_v \qquad C_p = \left(\frac{\partial h}{\partial T} \right)_p$$

- For C_v , the amount of energy added is equivalent to the change in internal energy
- For C_p , the amount of energy added is equivalent to the change in enthalpy
- Applying the first law of thermodynamics explains the difference.

Dependence of C_v on Internal Energy

$$C_v = \left(\frac{\partial u}{\partial T} \right)_v$$

- The first law is given as follows:

$$(Q_{\text{in}} + W_{\text{in}} + E_{\text{mass,in}}) - (Q_{\text{out}} + W_{\text{out}} + E_{\text{mass,out}}) = m(u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

- For a stationary system undergoing a constant volume process, the first law is simplified to:

$$Q_{\text{in}} = m(u_2 - u_1) \quad \text{or} \quad q_{\text{in}} = u_2 - u_1$$

A pink rectangular box represents a control volume. Inside the box, the following text is written:
 $V = \text{constant}$
 $m = 1 \text{ kg}$
 $\Delta T = 1^\circ\text{C}$
 $c_v = 3.12 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}}$
A pink arrow points from the text '3.12 kJ' located below the box to the value '3.12' in the equation for c_v .

Dependence of C_p on Enthalpy

$$C_p = \left(\frac{\partial h}{\partial T} \right)_p$$

- The first law is given as follows:

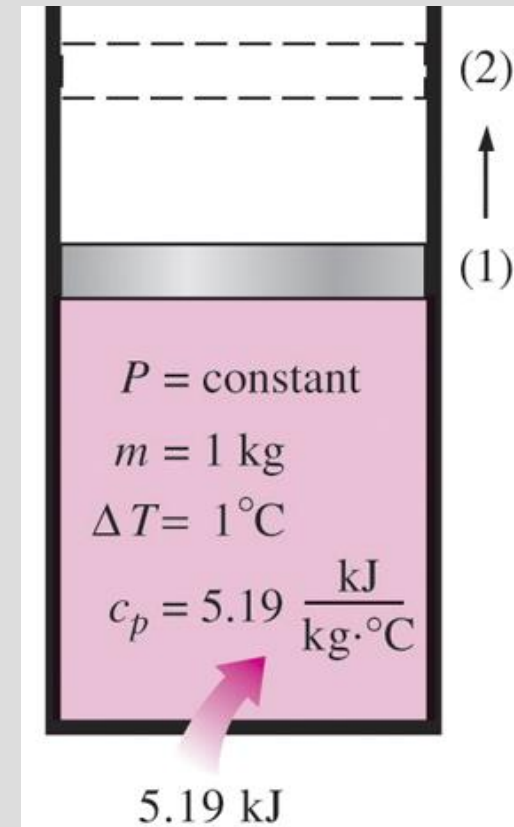
$$(Q_{\text{in}} + W_{\text{in}} + E_{\text{mass,in}}) - (Q_{\text{out}} + W_{\text{out}} + E_{\text{mass,out}}) = m(u_2 - u_1) + \frac{1}{2} m (V_2^2 - V_1^2) + m g (z_2 - z_1)$$

- For a stationary system undergoing a constant pressure process, the first law is simplified to:

$$Q_{\text{in}} - P (V_2 - V_1) = m (u_2 - u_1)$$

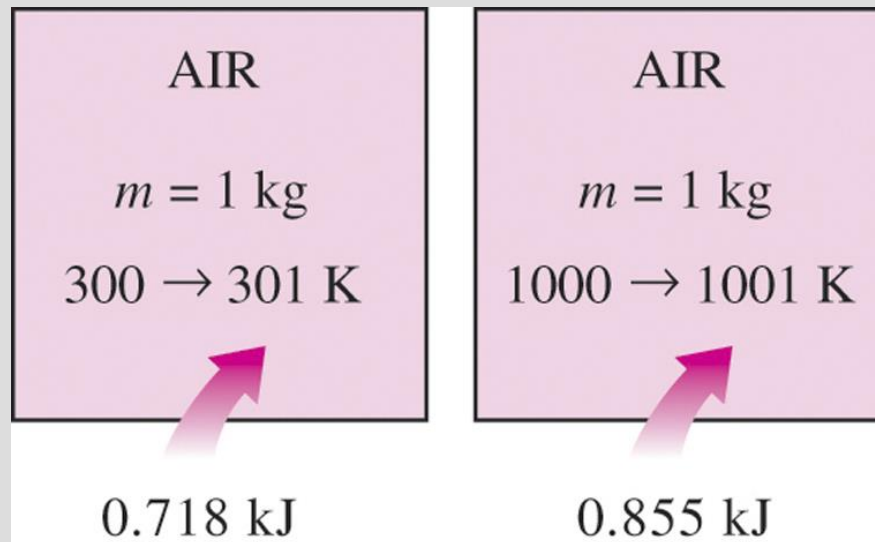
$$Q_{\text{in}} - m P (v_2 - v_1) = m (u_2 - u_1)$$

$$Q_{\text{in}} = m [(u_2 + P v_2) - (u_1 + P v_1)] = m (h_2 - h_1)$$



Notes on C_v and C_p

- C_v and C_p are properties.
- C_p is always larger than C_v
- The specific heat of a substance changes with temperature



Specific Heats for Ideal Gases

- Joule showed in 1843 that the internal energy of ideal gases is a function of temperature only, i.e.

$$u = u(T)$$

- For an ideal gas:

$$\left. \begin{array}{l} h = u + Pv \\ Pv = RT \end{array} \right\} h = u + RT \implies h = h(T)$$

- This means that C_v and C_p are functions of temperature only

$$du = c_v(T) dT \implies \Delta u = u_2 - u_1 = \int_1^2 c_v(T) dT$$

$$dh = c_p(T) dT \implies \Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT$$

Finding u and h for Ideal Gases

- For an ideal gas,

$$u = u(T)$$

$$h = h(T)$$

- There are tables showing values of u and h for different temperatures:

TABLE A-17

Ideal-gas properties of air

T K	h kJ/kg	P_r	u kJ/kg	v_r	s° kJ/kg·K	T K	h kJ/kg	P_r	u kJ/kg	v_r	s° kJ/kg·K
200	199.97	0.3363	142.56	1707.0	1.29559	580	586.04	14.38	419.55	115.7	2.37348
210	209.97	0.3987	149.69	1512.0	1.34444	590	596.52	15.31	427.15	110.6	2.39140
220	219.97	0.4690	156.82	1346.0	1.39105	600	607.02	16.28	434.78	105.8	2.40902
230	230.02	0.5477	164.00	1205.0	1.43557	610	617.53	17.30	442.42	101.2	2.42644
240	240.02	0.6355	171.13	1084.0	1.47824	620	628.07	18.36	450.09	96.92	2.44356
250	250.05	0.7329	178.28	979.0	1.51917	630	638.63	19.84	457.78	92.84	2.46048
260	260.09	0.8405	185.45	887.8	1.55848	640	649.22	20.64	465.50	88.99	2.47716
270	270.11	0.9590	192.60	808.0	1.59634	650	659.84	21.86	473.25	85.34	2.49364
280	280.13	1.0889	199.75	738.0	1.63279	660	670.47	23.13	481.01	81.89	2.50985
285	285.14	1.1584	203.33	706.1	1.65055	670	681.14	24.46	488.81	78.61	2.52589

Finding u and h for Ideal Gases

- An alternative is to find C_v and C_p and apply:

$$\Delta u = u_2 - u_1 = \int_1^2 c_v(T) dT$$

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T) dT$$

- There are tables showing the values of C_v and C_p for different gases at different temperatures:

TABLE A-2

Ideal-gas specific heats of various common gases (*Continued*)

(b) At various temperatures

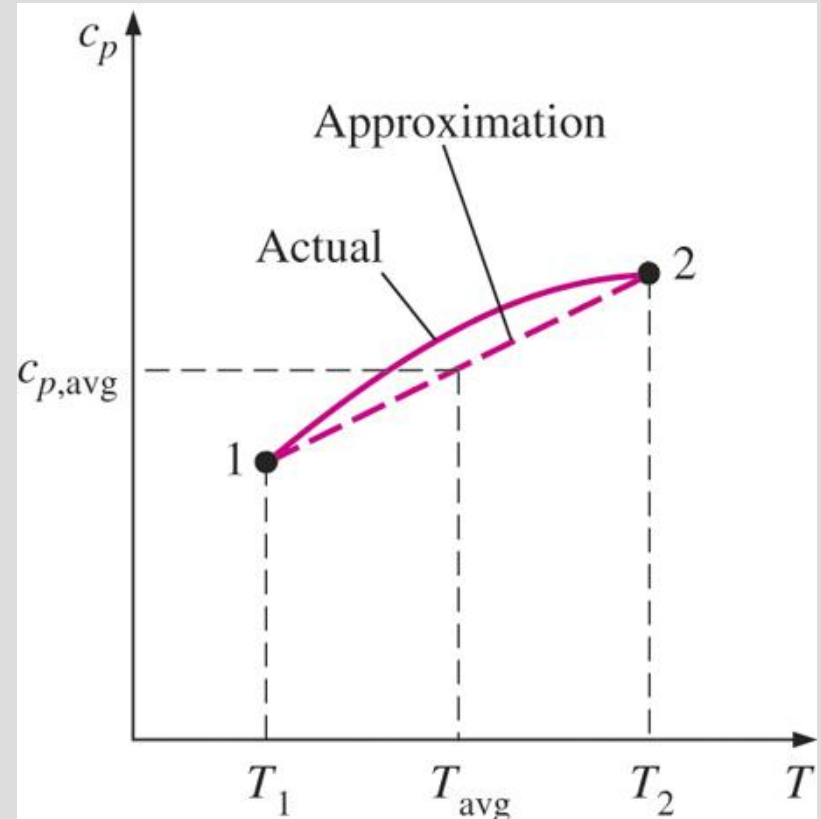
Temperature, K	c_p kJ/kg·K	c_v kJ/kg·K	k	c_p kJ/kg·K	c_v kJ/kg·K	k	c_p kJ/kg·K	c_v kJ/kg·K	k
	<i>Air</i>			<i>Carbon dioxide, CO₂</i>			<i>Carbon monoxide, CO</i>		
250	1.003	0.716	1.401	0.791	0.602	1.314	1.039	0.743	1.400
300	1.005	0.718	1.400	0.846	0.657	1.288	1.040	0.744	1.399
350	1.008	0.721	1.398	0.895	0.706	1.268	1.043	0.746	1.398
400	1.013	0.726	1.395	0.939	0.750	1.252	1.047	0.751	1.395
450	1.020	0.733	1.391	0.978	0.790	1.239	1.054	0.757	1.392
500	1.029	0.742	1.387	1.014	0.825	1.229	1.063	0.767	1.387
550	1.040	0.753	1.381	1.046	0.857	1.220	1.075	0.778	1.382
600	1.051	0.764	1.376	1.075	0.886	1.213	1.087	0.790	1.376

Finding u and h for Ideal Gases

- Yet another alternative is to use average values of C_v and C_p and consider them constant.
- For small temperature intervals, this assumption is acceptable.

$$u_2 - u_1 = c_{v,avg}(T_2 - T_1)$$

$$h_2 - h_1 = c_{p,avg}(T_2 - T_1)$$



Specific Heat Relations for Ideal Gases

- It can be easily shown that, for an ideal gas:

$$c_p = c_v + R$$

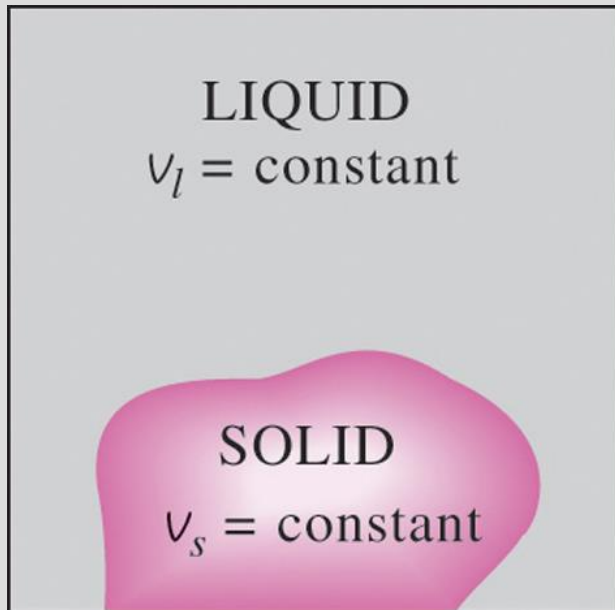
- A useful property is called the ***specific heat ratio***, and it is denoted by (k):

$$k = \frac{c_p}{c_v}$$

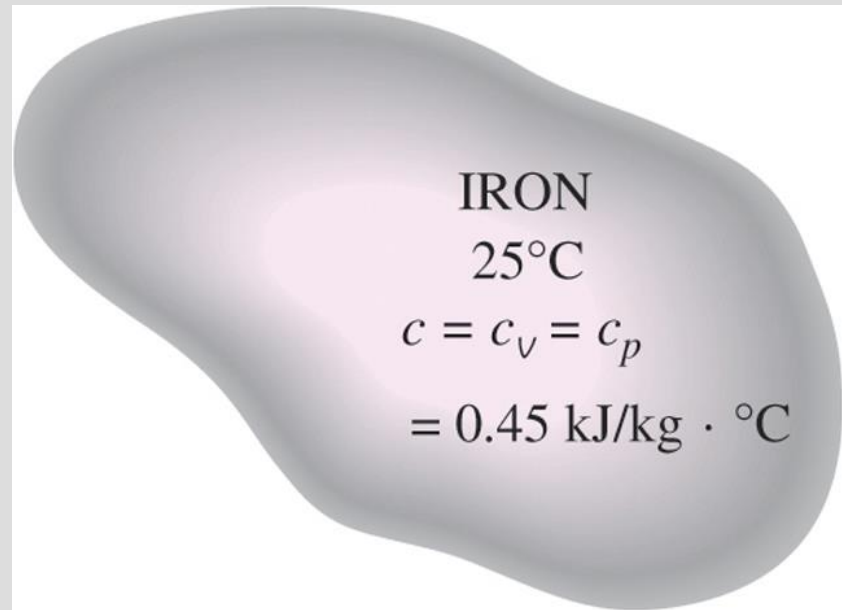
- k varies with temperature, but its variation is very small

INTERNAL ENERGY, ENTHALPY, AND SPECIFIC HEATS OF SOLIDS AND LIQUIDS

Incompressible substance: A substance whose specific volume (or density) is constant. **Solids and liquids** are incompressible substances.



The specific volumes of incompressible substances remain constant during a process.



The c_v and c_p values of incompressible substances are identical and are denoted by c .

Internal Energy Changes

$$du = c_v dT = c(T) dT$$

$$\Delta u = u_2 - u_1 = \int_1^2 c(T) dT \quad (\text{kJ/kg})$$

$$\Delta u \cong c_{\text{avg}}(T_2 - T_1) \quad (\text{kJ/kg})$$

Enthalpy Changes

$$h = u + Pv$$

$$dh = du + v dP + P dv = du + v dP$$

$$\Delta h = \Delta u + v \Delta P \cong c_{\text{avg}} \Delta T + v \Delta P \quad (\text{kJ/kg})$$

For *solids*, the term $v \Delta P$ is insignificant and thus $\Delta h = \Delta u \cong c_{\text{avg}} \Delta T$. For *liquids*, two special cases are commonly encountered:

1. *Constant-pressure processes*, as in heaters ($\Delta P = 0$): $\Delta h = \Delta u \cong c_{\text{avg}} \Delta T$
2. *Constant-temperature processes*, as in pumps ($\Delta T = 0$): $\Delta h = v \Delta P$