



**324 Stat**  
**Lecture Notes**

**(4) Some Discrete  
Probability  
Distributions**

( Book: Chapter 5 ,pg 143)

# 4.1 Discrete Uniform Distribution:

*Discrete Uniform is not in the book, it should be studied from the notes*

- If the random variable **X** assume the values with equal probabilities, then the discrete uniform distribution is given by:

$$\begin{aligned} P(X, K) &= \frac{1}{K} , & X &= x_1, x_2, \dots, x_K \\ &= 0 & & \textit{elsewhere} \end{aligned} \quad (1)$$

# Theorem:

- The mean and variance of the discrete uniform distribution  $P(X, K)$  are:

$$\mu = \frac{\sum_{i=1}^K X_i}{K} \quad , \quad \sigma^2 = \frac{\sum_{i=1}^K (X - \mu)^2}{K} \quad (2)$$

## EX (1):

- When a die is tossed once, each element of the sample space  $S = \{1,2,3,4,5,6\}$  occurs with probability  $1/6$ . Therefore we have a uniform distribution with:

$$P(X,6) = \frac{1}{6}, X = 1,2,3,4,5,6$$

- Find:
- 1.  $P(1 \leq x < 4)$
- 2.  $P(x < 3)$
- 3.  $P(3 < x < 6)$

# Solution:

- 1.  $P(1 \leq x < 4)$

$$= P(x = 1) + P(x = 2) + P(x = 3) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6}$$

- 2.  $P(x < 3) = P(x = 1) + P(x = 2) = \frac{2}{6}$

- 3.  $P(3 < x < 6) = P(x = 4) + P(x = 5) = \frac{2}{6}$

## EX (2):

- For example (1): Find  $\mu$  and  $\sigma^2$

## Solution:

$$\mu = \frac{\sum_{i=1}^k X_i}{k} = \frac{1+2+3+4+5+6}{6} = 3.5$$

$$\sigma^2 = \frac{\sum_{i=1}^k (X_i - \mu)^2}{k}$$

$$= \frac{(1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2}{6} = \frac{35}{12}$$

# The Bernoulli Process:

- Bernoulli trials are an experiment with:
  - 1) Only two possible outcomes.
  - 2) Labeled as success (S), failure (F).
  - 3) Probability of success =  $p$ , probability of failure =  $q = 1 - p$  ( $p + q = 1$ ).

# Binomial Distribution:

- \* The **Binomial** trials must possess the following properties:
  - 1) The experiment consists of **n** repeated trials.
  - 2) Each trial results in an outcome that may be classified as a success or a failure.
  - 3) The probability of success denoted by **p** remains constant from trial to trial.
  - 4) The repeated trials are independent.
  - 5) The parameters of binomial are **n, p**.



# Binomial Distribution:

- The probability distribution of the binomial random variable **X**, the number of successes in **n** independent trials is:

$$b(x, n, p) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, 2, \dots, n \quad (5)$$

## Theorem:

- The mean and the variance of the **binomial** distribution  $b(x, n, p)$  are:

$$\mu = np \quad \text{and} \quad \sigma^2 = npq \quad (6)$$

## EX (4):

- According to a survey by the Administrative Management society,  $\frac{1}{3}$  of U.S. companies give employees four weeks of vacation after they have been with the company for **15** years. Find the probability that among **6** companies surveyed at random, the number that gives employees **4** weeks of vacation after **15** years of employment is:
  - a) anywhere from **2** to **5**;
  - b) fewer than **3**;
  - c) at most **1**;
  - d) at least **5**;
  - e) greater than **2**;
  - f) calculate  $\mu$  and  $\sigma^2$

# Solution:

$$n = 6, p = 1/3, q = 2/3$$

$$(a) P(2 \leq x \leq 5) = P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5)$$

$$= \binom{6}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4 + \binom{6}{3} \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^3 + \binom{6}{4} \left(\frac{1}{3}\right)^4 \left(\frac{2}{3}\right)^2 + \binom{6}{5} \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^1$$

$$= \frac{240}{729} + \frac{160}{729} + \frac{60}{729} + \frac{12}{729} = \frac{472}{729} = 0.647$$

$$(b) P(x < 3) = P(x = 0) + P(x = 1) + P(x = 2)$$

$$= \binom{6}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6 + \binom{6}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^5 + \binom{6}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4$$

$$= \frac{64}{729} + \frac{192}{729} + \frac{240}{729} = \frac{496}{729} = 0.68$$

$$\begin{aligned}(c) P(x \leq 1) &= P(x = 0) + P(x = 1) \\ &= \binom{6}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6 + \binom{6}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^5 \\ &= \frac{64}{729} + \frac{192}{729} = \frac{256}{729} = 0.351\end{aligned}$$

$$\begin{aligned}(d) P(x \geq 5) &= P(x = 5) + P(x = 6) \\ &= \binom{6}{5} \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right)^1 + \binom{6}{6} \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^0 \\ &= \frac{12}{729} + \frac{1}{729} = \frac{13}{729} = 0.018\end{aligned}$$

$$(e) P(x > 2) = 1 - P(x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2)$$

$$= 1 - \left\{ \binom{6}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6 + \binom{6}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^5 + \binom{6}{2} \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4 \right\}$$

$$= 1 - \left\{ \frac{64}{729} + \frac{192}{729} + \frac{240}{729} \right\} = 1 - \frac{496}{729} = \frac{233}{729} = 0.319$$

$$(f) \mu = np = (6) \left(\frac{1}{3}\right) = \frac{6}{3} = 2$$

$$\sigma^2 = npq = (6) \left(\frac{1}{3}\right) \left(\frac{2}{3}\right) = \frac{12}{9} = 1.333$$

## EX (5):

- The probability that a person suffering from headache will obtain relief with a particular drug is **0.9**. **Three** randomly selected sufferers from headache are given the drug. Find the probability that the number obtaining relief will be:
  - a) exactly zero;
  - b) at most one;
  - c) more than one;
  - d) two or fewer;
  - e) Calculate  $\mu$  and  $\sigma^2$

See Ex 5.2 pg 146

## Solution:

$$n = 3, p = 0.9, q = 0.1$$


$$(a) P(X = 0) = \binom{3}{0} (0.9)^0 (0.1)^3 = 0.001$$

$$(b) P(x \leq 1) = P(x = 0) + P(x = 1)$$

$$= \binom{3}{0} (0.9)^0 (0.1)^3 + \binom{3}{1} (0.9)^1 (0.1)^2$$

$$= 0.001 + 0.027 = 0.028$$




$$\begin{aligned}(c) P(X > 1) &= 1 - p(x \leq 1) = 1 - [P(x = 0) + P(x = 1)] \\ &= 1 - \left[ \binom{3}{0} (0.9)^0 (0.1)^3 + \binom{3}{1} (0.9)^1 (0.1)^2 \right] \\ &= 1 - (0.001 + 0.027) = 1 - 0.028 = 0.972\end{aligned}$$

$$\begin{aligned}(d) P(x \leq 2) &= P(x = 0) + P(x = 1) + P(x = 2) \\ &= \binom{3}{0} (0.9)^0 (0.1)^3 + \binom{3}{1} (0.9)^1 (0.1)^2 + \binom{3}{2} (0.9)^2 (0.1)^1 \\ &= 0.001 + 0.027 + 0.243 = 0.271\end{aligned}$$