Chapter 26

Capacitance

and

Dielectrics

Capacitors

- Capacitors are devices that store electric charge
- Examples of where capacitors are used include:
  - radio receivers
  - filters in power supplies
  - energy-storing devices in electronic flashes

Definition of Capacitance

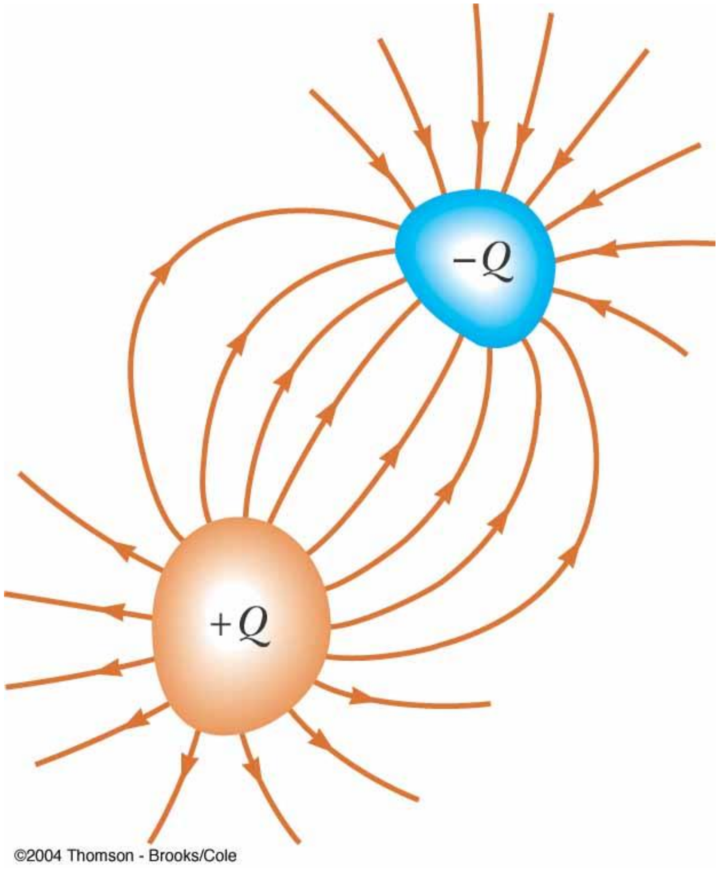
- The **capacitance**, *C*, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors

$$C = \frac{Q}{\Delta V}$$

- The SI unit of capacitance is the **farad** (F)

Makeup of a Capacitor

- A capacitor consists of two conductors
  - These conductors are called *plates*
  - When the conductor is charged, the plates carry charges of equal magnitude and opposite directions
- A potential difference exists between the plates due to the charge

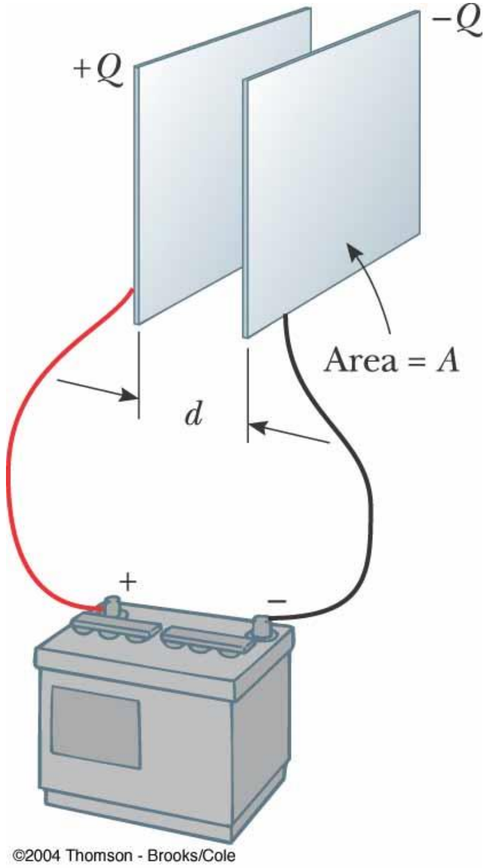


More About Capacitance

- Capacitance will always be a positive quantity
- The capacitance of a given capacitor is constant
- The capacitance is a measure of the capacitor's ability to store charge
- The farad is a large unit, typically you will see microfarads (μF) and picofarads (pF)

Parallel Plate Capacitor

- Each plate is connected to a terminal of the battery
- If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires



Capacitance – Isolated Sphere

- Assume a spherical charged conductor
- Assume $V = 0$ at infinity

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / R} = \frac{R}{k_e} = 4\pi\varepsilon_o R$$

- Note, this is independent of the charge and the potential difference

Capacitance – Parallel Plates

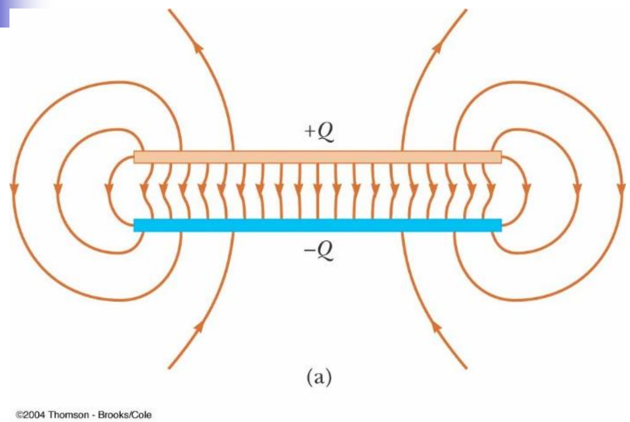
- The charge density on the plates is $\sigma = Q/A$
  - A is the area of each plate, which are equal
  - Q is the charge on each plate, equal with opposite signs
- The electric field is uniform between the plates and zero elsewhere

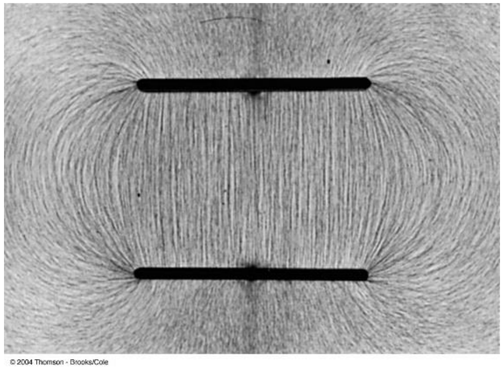
Capacitance – Parallel Plates, cont.

- The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{Qd/\varepsilon_o A} = \frac{\varepsilon_o A}{d}$$

Parallel Plate Assumptions



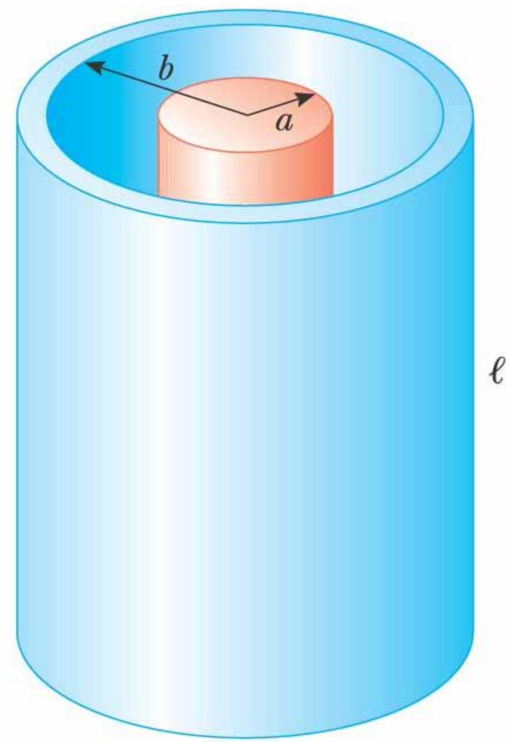


- The assumption that the electric field is uniform is valid in the central region, but not at the ends of the plates
- If the separation between the plates is small compared with the length of the plates, the effect of the non-uniform field can be ignored

Capacitance of a Cylindrical Capacitor

- From Gauss's Law, the field between the cylinders is $E = 2k_e\lambda \,/\, r$
- $\Delta V = -2k_e\lambda \ \ln\,(b/a)$
- The capacitance becomes

$$C = \frac{Q}{\Delta V} = \frac{\ell}{2k_e \ln(b/a)}$$



(a)

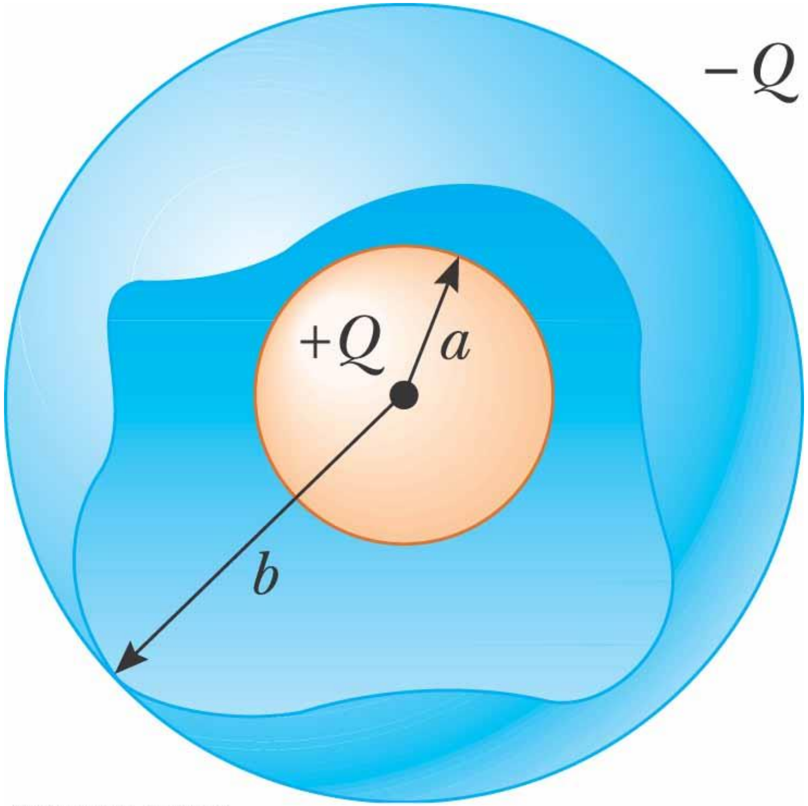
Capacitance of a Spherical Capacitor

- The potential difference will be

$$\Delta V = k_e Q\left(\frac{1}{b} - \frac{1}{a}\right)$$

- The capacitance will be

$$C = \frac{Q}{\Delta V} = \frac{ab}{k_e(b-a)}$$



Question 26.1

(a) How much charge is on each plate of a 4.00-μF capacitor when it is connected to a 12.0-V battery? (b) If this same capacitor is connected to a 1.50-V battery, what charge is stored?

Question 26.7

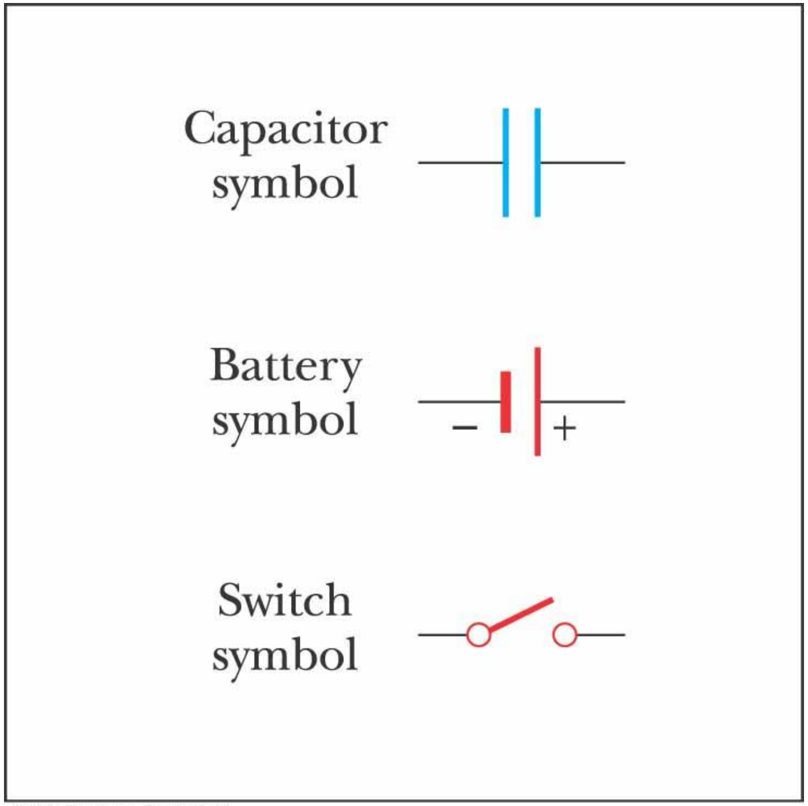
An air-filled capacitor consists of two parallel plates, each with an area of 7.60 cm^2, separated by a distance of 1.80 mm. A 20.0-V potential difference is applied to these plates. Calculate (a) the electric field between the plates, (b) the surface charge density, (c) the capacitance, and (d) the charge on each plate.

Question 26.9

When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of 30.0 nC/cm^2. What is the spacing between the plates?

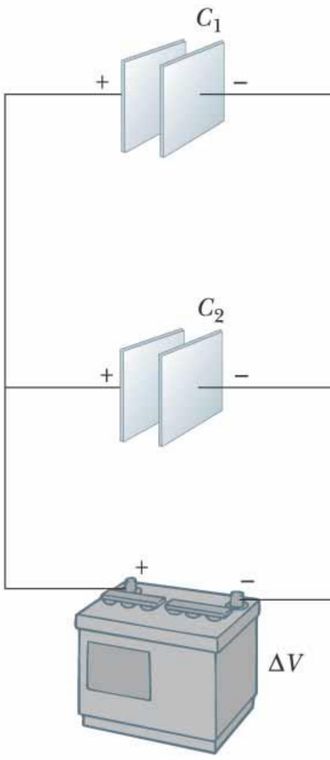
Circuit Symbols

- A circuit diagram is a simplified representation of an actual circuit
- Circuit symbols are used to represent the various elements
- Lines are used to represent wires
- The battery's positive terminal is indicated by the longer line



Capacitors in Parallel

- When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged



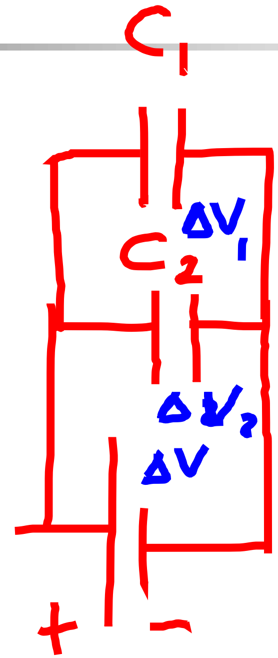
(a)

Capacitors in parallel

Parallel

$$C = \frac{Q}{\Delta V}$$

$$\Delta V = \Delta V_1 = \Delta V_2$$

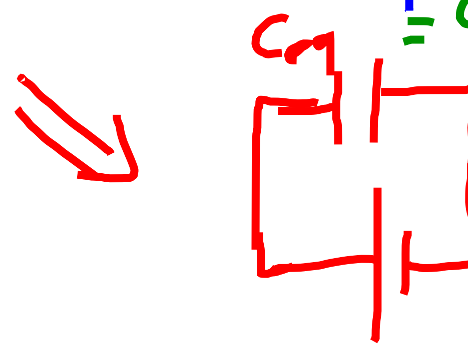


$$Q_1 = C_1 \Delta V$$

$$Q_2 = C_2 \Delta V$$

$$Q = C_{eq} * \Delta V$$

$$C_{eq} = C_1 + C_2$$



$$Q = Q_1 + Q_2$$

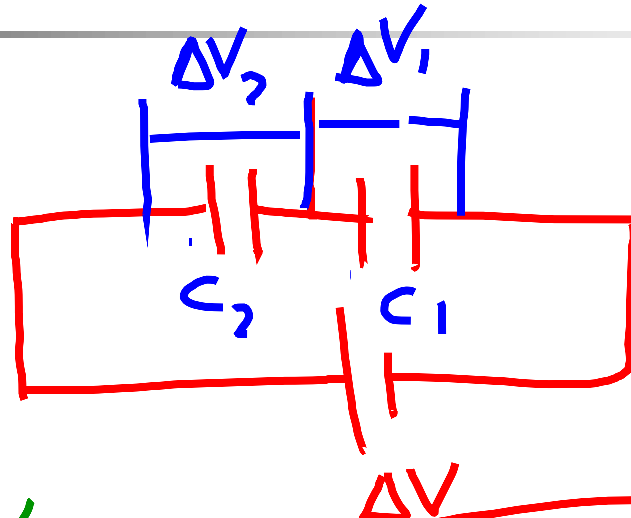
$$C_{eq} \Delta V = C_1 \Delta V + C_2 \Delta V$$

$$\boxed{C_{eq} = C_1 + C_2}$$

Capacitors in series

$$\Delta V = \frac{Q}{C}$$

$$\Delta V = \Delta V_1 + \Delta V_2$$



$$Q = Q_1 = Q_2$$

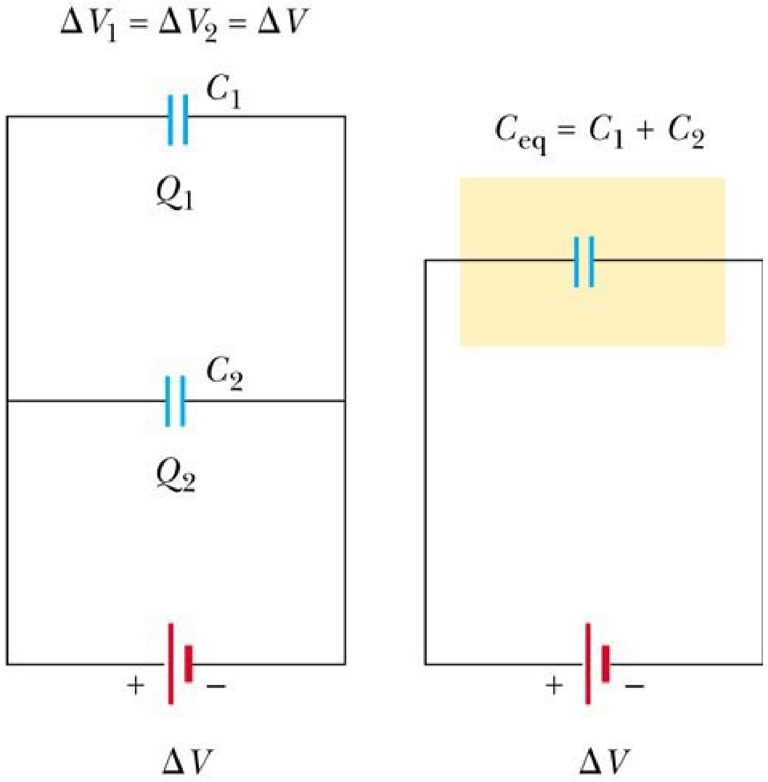
$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2} \Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Capacitors in Parallel, 2

- The total charge is equal to the sum of the charges on the capacitors
  - $Q_{total} = Q_1 + Q_2$
- The potential difference across the capacitors is the same = voltage of the battery

Capacitors in Parallel, 3

- The capacitors can be replaced with one capacitor with a capacitance of C_{eq}

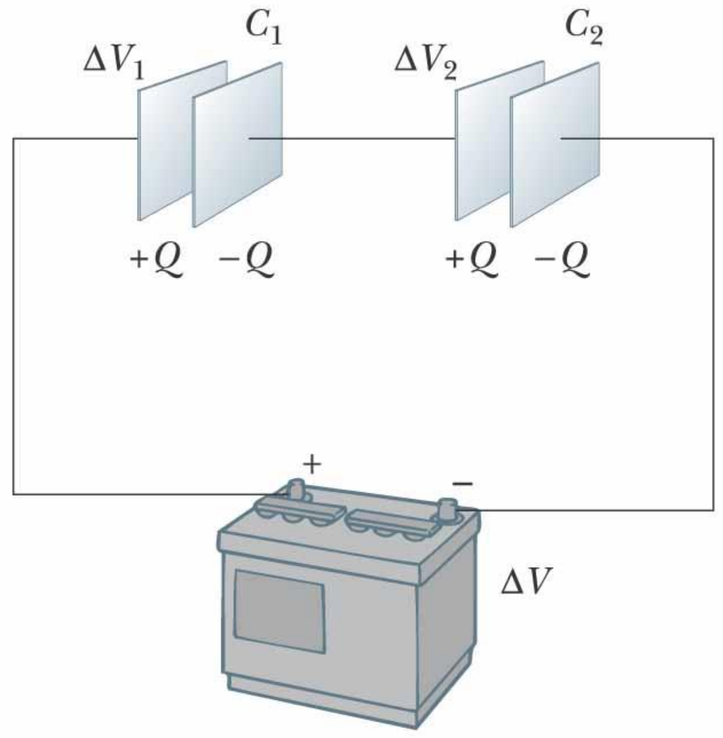


Capacitors in Parallel, final

- $C_{eq} = C_1 + C_2 + \ldots$
- The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors

Capacitors in Series

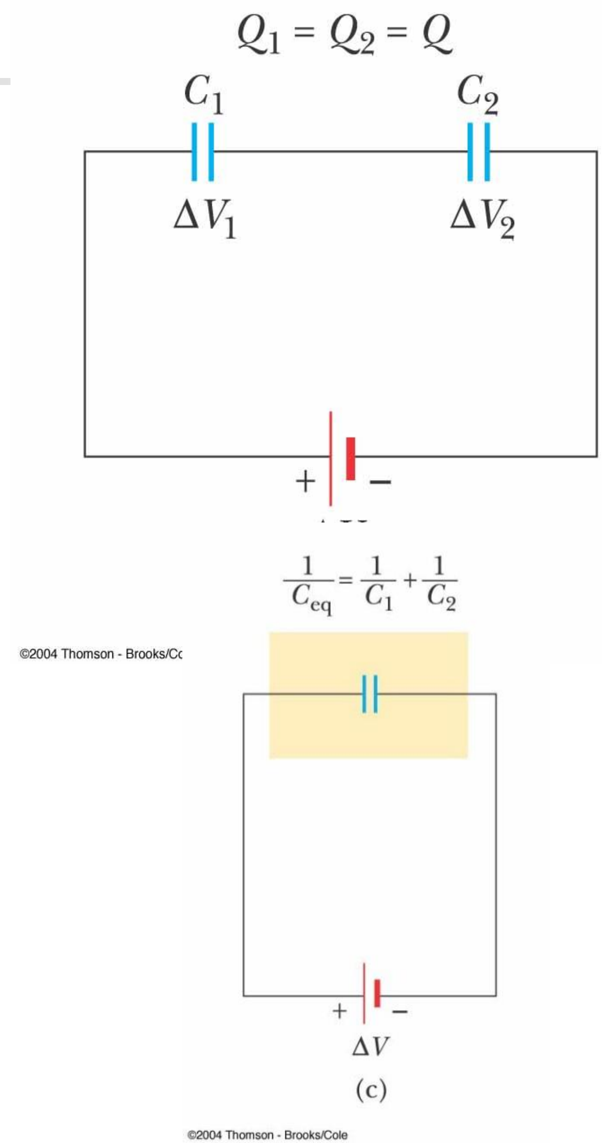
- When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery



(a)

Capacitors in Series, 2

- An equivalent capacitor can be found that performs the same function as the series combination
- The potential differences add up to the battery voltage



Capacitors in Series, final

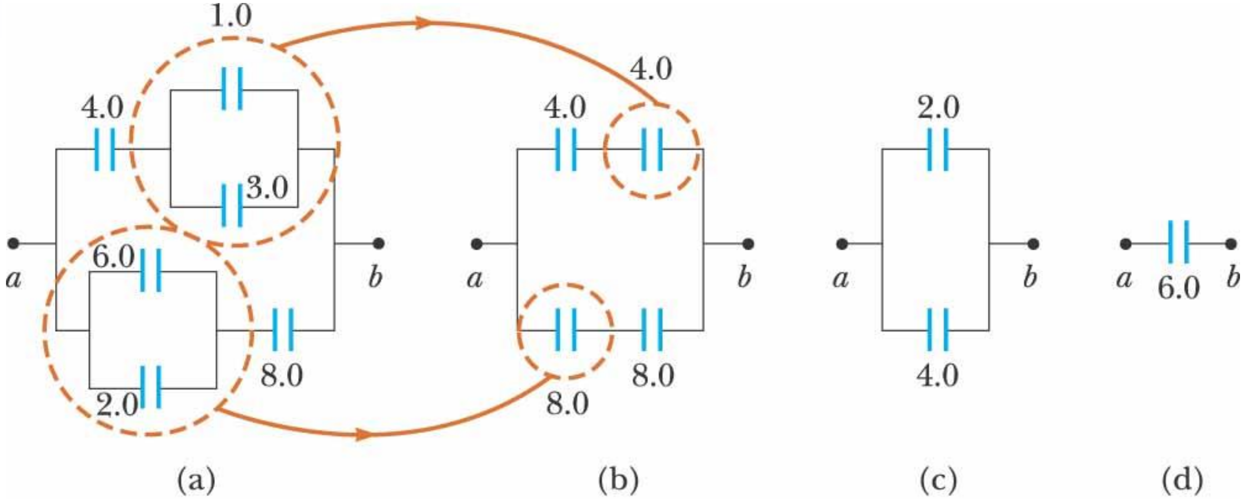
$$Q = Q_1 + Q_2 + \ldots$$

$$\Delta V = V_1 + V_2 + \ldots$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$$

- The equivalent capacitance of a series combination is always less than any individual capacitor in the combination

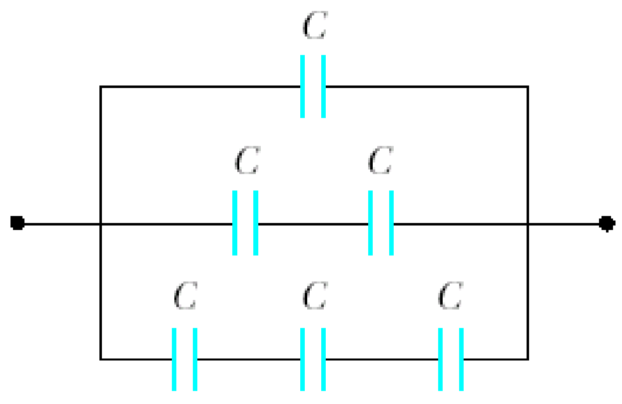
Example 26.4



- The 1.0-μF and 3.0-μF capacitors are in parallel as are the 6.0-μF and 2.0-μF capacitors
- These parallel combinations are in series with the capacitors next to them
- The series combinations are in parallel and the final equivalent capacitance can be found

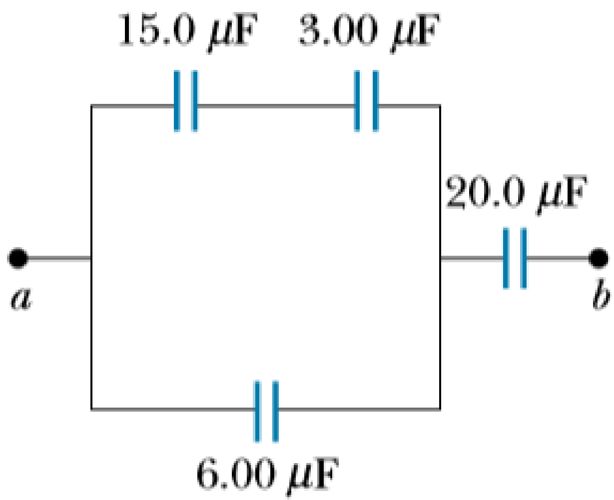
Question 26.18

Evaluate the equivalent capacitance of the configuration shown in Figure P26.18. All the capacitors are identical, and each has capacitance *C*.



Question 26.21

21. Four capacitors are connected as shown in Figure P26.21. (a) Find the equivalent capacitance between points *a* and *b*. (b) Calculate the charge on each capacitor if $\Delta V_{ab} = 15.0$ V.



$Q = C_{eq}\,\Delta V$

$= 6\,\mu F \times 15$

$= 90\,\mu C$

$C_s = \frac{15 \times 3}{15+3} = 2.5\,\mu F$

$C_6 = 2.5 + 6 = 8.5\,\mu F$

$C_{eq} = \frac{8.5 \times 20}{8.5 + 20} \simeq 6\,\mu F$

Quick Quiz 26.4

You charge a parallel-plate capacitor, remove it from the battery, and prevent the wires connected to the plates from touching each other. When you pull the plates apart, do the following quantities increase, decrease, or stay the same? (a) C; (b) Q; (c) E between the plates; (d) ΔV; (e) energy stored in the capacitor.

Energy Stored in a Capacitor

- Assume the capacitor is being charged and, at some point, has a charge *q* on it
- The work needed to transfer a charge from one plate to the other is

$$dW = \Delta V dq = \frac{q}{C} dq$$

- The total work required is

$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}$$

Energy, cont

- The work done in charging the capacitor appears as electric potential energy *U*:

$$U = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

- This applies to a capacitor of any geometry
- The energy stored increases as the charge increases and as the potential difference increases
- In practice, there is a maximum voltage before discharge occurs between the plates

Energy, final

- The energy can be considered to be stored in the electric field
- For a parallel-plate capacitor, the energy can be expressed in terms of the field as $U = \frac{1}{2} (\varepsilon_o A d) E^2$
- It can also be expressed in terms of the energy density (energy per unit volume)

  $u_E = \frac{1}{2} \varepsilon_o E^2$

Some Uses of Capacitors

- Defibrillators
  - When fibrillation occurs, the heart produces a rapid, irregular pattern of beats
  - A fast discharge of electrical energy through the heart can return the organ to its normal beat pattern
- In general, capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse

Capacitors with Dielectrics

- A *dielectric* is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance
  - Dielectrics include rubber, plastic, and waxed paper
- For a parallel-plate capacitor, $C = \kappa C_o = \kappa \varepsilon_o (A/d)$
  - The capacitance is multiplied by the factor κ when the dielectric completely fills the region between the plates

Dielectrics, cont

- In theory, *d* could be made very small to create a very large capacitance
- In practice, there is a limit to *d*
  - *d* is limited by the electric discharge that could occur though the dielectric medium separating the plates
- For a given *d*, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** of the material

Dielectrics, final

- Dielectrics provide the following advantages:
  - Increase in capacitance
  - Increase the maximum operating voltage
  - Possible mechanical support between the plates
    - This allows the plates to be close together without touching
    - This decreases *d* and increases *C*

Table 26.1

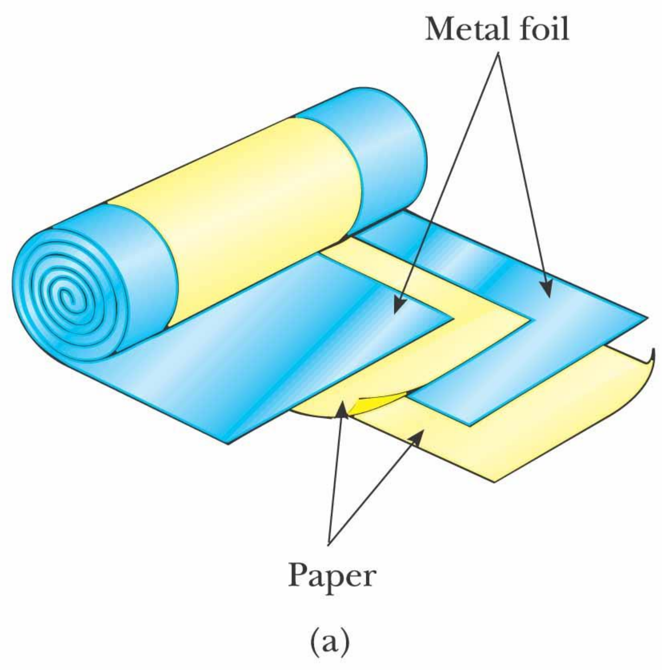
Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

[a] The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.

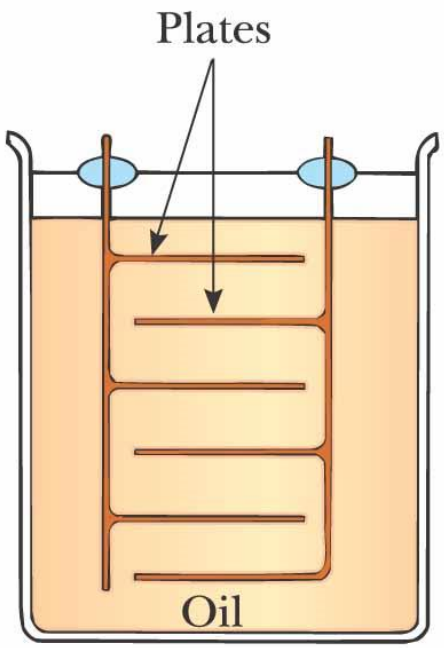
Types of Capacitors – Tubular

- Metallic foil may be interlaced with thin sheets of paper or Mylar
- The layers are rolled into a cylinder to form a small package for the capacitor



Types of Capacitors – Oil Filled

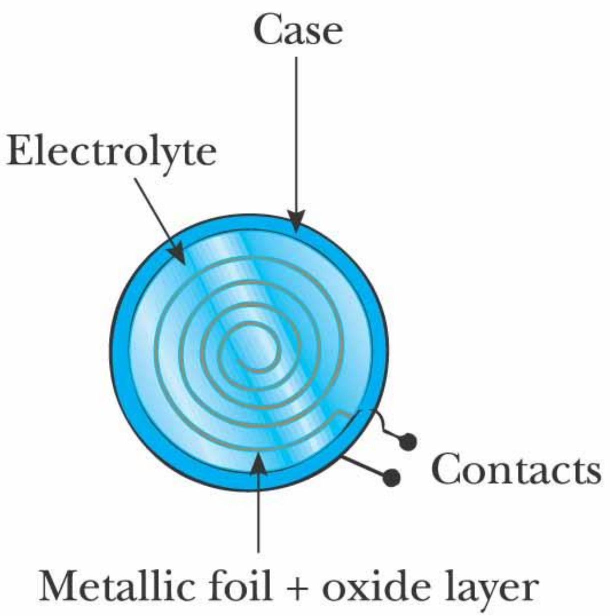
- Common for high-voltage capacitors
- A number of interwoven metallic plates are immersed in silicon oil



(b)

Types of Capacitors – Electrolytic

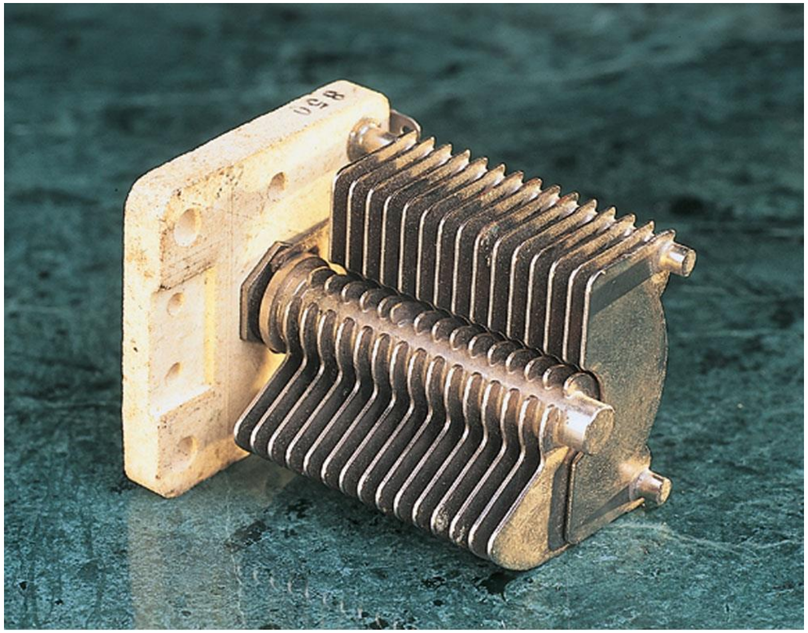
- Used to store large amounts of charge at relatively low voltages
- The electrolyte is a solution that conducts electricity by virtue of motion of ions contained in the solution



(c)

Types of Capacitors – Variable

- Variable capacitors consist of two interwoven sets of metallic plates
- One plate is fixed and the other is movable
- These capacitors generally vary between 10 and 500 pF
- Used in radio tuning circuits



Example 26.6

EXAMPLE 26.6 **A Paper-Filled Capacitor**

A parallel-plate capacitor has plates of dimensions 2.0 cm by 3.0 cm separated by a 1.0-mm thickness of paper. (a) Find its capacitance.

Solution Because $\kappa = 3.7$ for paper (see Table 26.1), we have

$$C = \kappa \frac{\epsilon_0 A}{d} = 3.7(8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2})\left(\frac{6.0 \times 10^{-4}\ \mathrm{m^2}}{1.0 \times 10^{-3}\ \mathrm{m}}\right)$$

$$= 20 \times 10^{-12}\ \mathrm{F} = \boxed{20\ \mathrm{pF}}$$

Example 26.6

(b) What is the maximum charge that can be placed on the capacitor?

Solution From Table 26.1 we see that the dielectric strength of paper is 16×10^6 V/m. Because the thickness of the paper is 1.0 mm, the maximum voltage that can be applied before breakdown is

$$\Delta V_{\text{max}} = E_{\text{max}} d = (16 \times 10^6 \text{ V/m})(1.0 \times 10^{-3} \text{ m})$$
$$= 16 \times 10^3 \text{ V}$$

Hence, the maximum charge is

$$Q_{\text{max}} = C \Delta V_{\text{max}} = (20 \times 10^{-12} \text{ F})(16 \times 10^3 \text{ V}) = \boxed{0.32\ \mu\text{C}}$$

Example 26.6

Exercise What is the maximum energy that can be stored in the capacitor?

Answer 2.6×10^{-3} J.

Example 26.7

EXAMPLE 26.7 **Energy Stored Before and After**

A parallel-plate capacitor is charged with a battery to a charge Q_0, as shown in Figure 26.17a. The battery is then removed, and a slab of material that has a dielectric constant κ is inserted between the plates, as shown in Figure 26.17b. Find the energy stored in the capacitor before and after the dielectric is inserted.

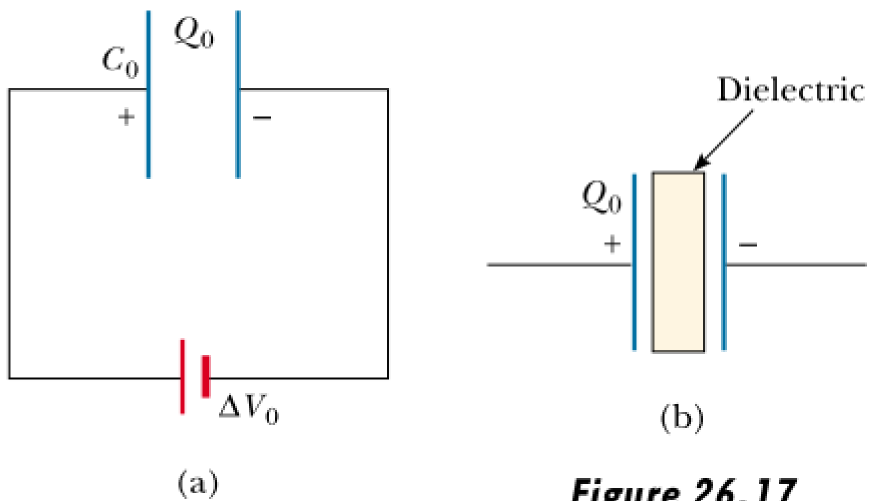


Figure 26.17

Example 26.7

Solution The energy stored in the absence of the dielectric is (see Eq. 26.11):

$$U_0 = \frac{{Q_0}^2}{2C_0}$$

After the battery is removed and the dielectric inserted, the *charge on the capacitor remains the same*. Hence, the energy stored in the presence of the dielectric is

$$U = \frac{{Q_0}^2}{2C}$$

But the capacitance in the presence of the dielectric is $C = \kappa C_0$, so U becomes

$$U = \frac{{Q_0}^2}{2\kappa C_0} = \frac{U_0}{\kappa}$$

Example 26.7

Exercise Suppose that the capacitance in the absence of a dielectric is 8.50 pF and that the capacitor is charged to a potential difference of 12.0 V. If the battery is disconnected and a slab of polystyrene is inserted between the plates, what is $U_0 - U$?

Answer 373 pJ.

Geometry of Some Capacitors

Table 26.2

Capacitance and Geometry

Geometry	Capacitance	Equation
Isolated sphere of radius R (second spherical conductor assumed to have infinite radius)	$C = 4\pi\epsilon_0 R$	26.2
Parallel-plate capacitor of plate area A and plate separation d	$C = \epsilon_0 \frac{A}{d}$	26.3
Cylindrical capacitor of length ℓ and inner and outer radii a and b, respectively	$C = \frac{\ell}{2k_e \ln(b/a)}$	26.4
Spherical capacitor with inner and outer radii a and b, respectively	$C = \frac{ab}{k_e(b-a)}$	26.6