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Lecture Notes

(2) Random Variable and probability Distribution

(Chapter 3 of the book pg 81-94)

2.1 Definition: A Random Variable:

A random variable is a function that associates a real number with each element in the sample space.

<u>EX (3.1 pg 82):</u>

Two balls are drawn in succession without replacement from an urn containing **4** red balls and **3** black balls.

The possible outcomes and the values **y** of the random variable **Y** where is the number of red balls, are

Sample space	y
RR	2
RB	1
BR	1
BB	0

Y = number of red balls

Possible values of the random variable is y = 0,1,2

Definition: Discrete Sample Space:

If a sample space contains a finite number **n** of different values $x_1, x_2, \ldots, x_n$ or countably infinite number of different values $x_1, x_2, \ldots$ it is called a discrete sample space.

Examples of discrete random variables are;

*The number of bacteria per unit area in the study of drug control on bacterial growth.

* The number of defective television sets in a shipment of **100**.

Definition: Continuous Sample Space:

If **X** can take an infinite number of possibilities equal to the number of points on a line segment, then **X** has a continuous sample space. Examples of the continuous random variable; heights, weights, temperature, distances or life periods

`Discrete Probability Distributions:

Definition:

The set of ordered pairs **(x, f(x))** is a probability function, probability mass function or probability distribution of the discrete random variable **X** if for each possible outcome **x,**

1. $f(x) \geq 0$

2. $\sum_{\forall x} f(x) = 1$

3. $P(X = x) = f(x)$

EX (2):

A shipment of **8** similar microcomputers to a retail outlet contains **3** that are defective. If a school makes a random purchase of **2** of these computers, let **X**= # of defective in the sample. find:

1. the different values of r.v. X
2. the probability distribution of x ; f(x)
3. $P(1 \le x \le 2), P(x \le 1), P(0 < x \le 2), f(2), f(5)$

Solution:

(1)The possible values of X is: **X** =0, 1, 2

(2) $$f(0)=P(X=0)=\frac{\binom{3}{0}\binom{5}{2}}{\binom{8}{2}}=\frac{10}{28}$$

$$f(1)=P(X=1)=\frac{\binom{3}{1}\binom{5}{1}}{\binom{8}{2}}=\frac{15}{28}$$

$$f(2)=P(X=2)=\frac{\binom{3}{2}\binom{5}{0}}{\binom{8}{2}}=\frac{3}{28}$$

X	0	1	2
f(x)	10/28	15/28	3/28

(3)
$$P(1 \le x \le 2) = P(x = 1) + P(x = 2)$$
$$= 15/28 + 3/28 = 18/28$$

$$P(x \le 1) = P(x = 0) + P(x = 1)$$
$$= 10/28 + 15/28 = 25/28$$

$$P(0 < x \le 2) = P(x = 1) + P(x = 2) = 18/28$$

$$f(2) = P(x = 2) = 15/28$$

$$f(5) = P(x = 5) = 0$$

See Ex 3.8 pg 84

2.5 Definition: The Cumulative distribution Function:

The cumulative distribution function, denoted by **F(x)** of a discrete random variable **X** with probability distribution **f(x)** is given by:

$$F(x) = P(X \le x) = \sum_{X < x} f(x) \qquad for \;\; -\infty < x < \infty \qquad (1)$$

*For example $F(2) = P(x \le 2)$

$$P(a \le X \le b) = F(b) - F(a-1) \qquad (2)$$

* For example $F(3 \le x \le 7) = F(7) - F(2)$

EX (3):

For the given data find :(a) F (1) (b) $P(1 \le x \le 2)$

X	0	1	2
f(x)	10/28	15/28	3/28

Solution:

X	0	1	2
f(x)	10/28	15/28	3/28
F(x)	10/28	25/28	28/28

See Ex 3.10

a. F (1) = 25/28

b. $P(1 \le X \le 2) = F(2) - F(0) = 28/28 - 10/28 = 18/28$

2.6 Continuous Probability Distributions:

The function **f(x)** is a probability density function for the continuous random variable **X** defined over the set of real numbers **R**, if:

1. $f(x) \geq 0 \qquad for\ all\ \ x \in R$

2. $\int_{-\infty}^{\infty} f(x)\ dx = 1$

3. $P(a < X < b) = \int_{a}^{b} f(x)\,dx$

EX (3.11) pg 89:

Suppose that the error in the reaction temperature in ^{0}C for a controlled laboratory experiment is a continuous random variable **X** having the probability density function:

$$f(x) = \frac{x^2}{3} \quad , \quad -1 < x < 2$$
$$= 0 \qquad otherwise$$

a. Show that $\int_{-\infty}^{\infty} f(x)\,dx = 1$ b. Find $P(0 < X \le 1)$

c. find $P(0 < x < 3), P(x = 2), F(x), F(0.5)$

Solution:

a. $$\int_{-1}^{2} \frac{x^2}{3} dx = \frac{x^3}{9} \Big|_{-1}^{2} = \frac{1}{9}[8-(-1)^3] = \frac{8+1}{9} = 1$$

b. $$P(0 < X \leq 1) = \int_{0}^{1} \frac{x^2}{3} dx = \frac{x^3}{9} \Big|_{0}^{1} = \frac{1}{9}$$

c. $$P(0 < x < 3) = \int_{0}^{2} \frac{x^2}{3} dx = \frac{x^3}{9} \Big|_{0}^{2} = \frac{(2)^3}{9} = 8/9$$

$$P(x = 2) = 0$$

$$F(x) = \int_{-1}^{x} \frac{x^2}{3} dx = \frac{x^3}{9} \Big|_{-1}^{x} = \frac{x^3 - (-1)^3}{9} = \frac{x^3 + 1}{9}$$

$$F(0.5) = \frac{(0.5)^3 + 1}{9} = 0.139$$

2.7 Definition:

The cumulative distribution **F(x)** of a continuous random variable **X** with density function **f(x)** is given by:

$$P(x) = P(X \leq x) = \int_{-\infty}^{X} x \, dx \quad for \ -\infty < x < \infty$$

$$P(a \leq X \leq b) = P(a < X < b) = F(b) - F(a)$$

Ex 3.12 pg 90

For example (4): find $P(0 < x < 1)$

$$\because F(x) = \frac{x^3 + 1}{9}$$

$$\therefore P(0 < x < 1) = F(1) - F(0) = \frac{1+1}{9} - \frac{0+1}{9} = \frac{2}{9} - \frac{1}{9} = \frac{1}{9}$$

Therefore

$$F(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x^3+1}{9} & -1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

See Ex 3.13 pg 90