The Law of Gravitation  Any particle in the universe attracts any other particle with a gravitational force whose magnitude is

\[ F = G \frac{m_1 m_2}{r^2} \]  

(Newton’s law of gravitation),

(13-1)

where \( m_1 \) and \( m_2 \) are the masses of the particles, \( r \) is their separation, and \( G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \) is the gravitational constant.

Gravitational Behavior of Uniform Spherical Shells The gravitational force between extended bodies is found by adding (integrating) the individual forces on individual particles within the bodies. However, if either of the bodies is a uniform spherical shell or a spherically symmetric solid, the net gravitational force it exerts on an external object may be computed as if all the mass of the shell or body were located at its center.

Superposition  Gravitational forces obey the principle of superposition; that is, if \( n \) particles interact, the net force \( F_{\text{net}} \) on a particle labeled particle 1 is the sum of the forces on it from all the other particles taken one at a time:

\[ F_{\text{net}} = \sum_{i=2}^{n} F_i \]  

(13-5)

in which the sum is a vector sum of the forces \( F_i \) on particle 1 from particles 2, 3, \ldots, \( n \). The gravitational force \( F_i \) on a particle from an extended body is found by dividing the body into units of differential mass \( dm \), each of which produces a differential force \( dF \) on the particle, and then integrating to find the sum of those forces:

\[ F_i = \int dF. \]  

(13-6)

Gravitational Acceleration  The gravitational acceleration \( a \) of a particle (of mass \( m \)) is due solely to the gravitational force acting on it. When the particle is at distance \( r \) from the center of a uniform spherical body of mass \( M \), the magnitude \( F \) of the gravitational force on the particle is given by Eq. 13-1; thus, by Newton’s second law,

\[ F = ma, \]  

(13-10)

which gives

\[ a = \frac{GM}{r^2}. \]  

(13-11)

Free-Fall Acceleration and Weight  Because Earth’s mass is not distributed uniformly, because the planet is not perfectly spherical, and because it rotates, the actual free-fall acceleration \( g \) of a particle near Earth differs slightly from the gravitational acceleration \( a \), and the particle’s weight (equal to \( mg \)) differs from the magnitude of the gravitational force on it (Eq. 13-1).

Gravitation Within a Spherical Shell  A uniform shell of matter exerts no net gravitational force on a particle located inside it. This means that if a particle is located inside a uniform solid sphere at distance \( r \) from its center, the gravitational force exerted on the particle is due only to the mass \( M_{\text{in}} \) that lies inside a sphere of radius \( r \). This mass is given by

\[ M_{\text{in}} = \rho \frac{4}{3} m^3, \]  

(13-18)

where \( \rho \) is the density of the sphere.

Gravitational Potential Energy  The gravitational potential energy \( U(r) \) of a system of two particles, with masses \( M \) and \( m \) and separated by a distance \( r \), is the negative of the work that would be done by the gravitational force of either particle acting on the other if the separation between the particles were changed from infinite (very large) to \( r \). This energy is

\[ U = -\frac{G M m}{r} \]  

(gravitational potential energy).

(13-21)

Potential Energy of a System  If a system contains more than two particles, its total gravitational potential energy \( U \) is the sum of terms representing the potential energies of all the pairs. As an example, for three particles, of masses \( m_1 \), \( m_2 \), and \( m_3 \),

\[ U = -\frac{G m_1 m_2}{r_{12}} - \frac{G m_1 m_3}{r_{13}} - \frac{G m_2 m_3}{r_{23}}. \]  

(13-22)

Escape Speed  An object will escape the gravitational pull of an astronomical body of mass \( M \) and radius \( R \) (that is, it will reach an infinite distance) if the object’s speed near the body’s surface is at least equal to the escape speed, given by

\[ v = \sqrt{\frac{2 G M}{R}}. \]  

(13-28)

Kepler’s Laws  The motion of satellites, both natural and artificial, is governed by these laws:

1. **The law of orbits.** All planets move in elliptical orbits with the Sun at one focus.

2. **The law of areas.** A line joining any planet to the Sun sweeps out equal areas in equal time intervals. (This statement is equivalent to conservation of angular momentum.)

3. **The law of periods.** The square of the period \( T \) of any planet is proportional to the cube of the semimajor axis \( a \) of its orbit. For circular orbits with radius \( r \),

\[ T^2 = \frac{4 \pi^2}{GM} a^3 \]  

(law of periods).

(13-34)

where \( M \) is the mass of the attracting body— the Sun in the case of the solar system. For elliptical planetary orbits, the semimajor axis \( a \) is substituted for \( r \).

Energy in Planetary Motion  When a planet or satellite with mass \( m \) moves in a circular orbit with radius \( r \), its potential energy \( U \) and kinetic energy \( K \) are given by

\[ U = -\frac{G M m}{r} \quad \text{and} \quad K = \frac{G M m}{2 r}. \]  

(13-21, 13-38)

The mechanical energy \( E = K + U \) is then

\[ E = -\frac{G M m}{2 r}. \]  

(13-40)

For an elliptical orbit of semimajor axis \( a \),

\[ E = -\frac{G M m}{2 a}. \]  

(13-42)

Einstein’s View of Gravitation  Einstein pointed out that gravitation and acceleration are equivalent. This principle of equivalence led him to a theory of gravitation (the general theory of relativity) that explains gravitational effects in terms of a curvature of space.
1. In Fig. 13-20, a central particle of mass $M$ is surrounded by a square array of other particles, separated by either distance $d$ or distance $d/2$ along the perimeter of the square. What are the magnitude and direction of the net gravitational force on the central particle due to the other particles?

2. Figure 13-21 shows three arrangements of the same identical particles, with three of them placed on a circle of radius 0.20 m and the fourth one placed at the center of the circle. (a) Rank the arrangements according to the magnitude of the net gravitational force on the central particle due to the other three particles, greatest first. (b) Rank them according to the magnitude of the gravitational force on particle $P$ due to the shell, greatest first.

3. In Fig. 13-22, a central particle is surrounded by two circular rings of particles, at radii $r$ and $R$, with $R > r$. All the particles have mass $m$. What are the magnitude and direction of the net gravitational force on the central particle due to the particles in the rings?

4. In Fig. 13-23, two particles, of masses $m$ and $2m$, are fixed in place on an axis. (a) Where on the axis can a third particle of mass $3m$ be placed (other than at infinity) so that the net gravitational force on it from the first two particles is zero? (b) Does the answer change if the third particle has, instead, a mass of $16m$? (c) Is there a point off the axis (other than infinity) at which the net force on the third particle would be zero?

5. Figure 13-24 shows three situations involving a point particle $P$ with mass $m$ and a spherical shell with a uniformly distributed mass $M$. The radii of the shells are given. Rank the situations according to the magnitude of the gravitational force on particle $P$ due to the shell, greatest first.

6. In Fig. 13-25, three particles are fixed in place. The mass of $B$ is greater than the mass of $C$. Can a fourth particle (particle $D$) be placed somewhere so that the net gravitational force on particle $A$ from particles $B$, $C$, and $D$ is zero? If so, in which quadrant should it be placed and which axis should it be near?

7. Rank the four systems of equal-mass particles shown in Checkpoint 2 according to the absolute value of the gravitational potential energy of the system, greatest first.

8. Figure 13-26 gives the gravitational acceleration $a_g$ for four planets as a function of the radial distance $r$ from the center of the planet, starting at the surface of the planet (at radius $R_1$). (a) Rank the planets according to the percent change in the gravitational potential energy of the rocket–moon system, greatest first. (b) The net work done on a rocket by the gravitational force from the moon, greatest first.

9. Figure 13-27 shows three particles initially fixed in place, with $B$ and $C$ identical and positioned symmetrically about the $y$ axis, at distance $d$ from $A$. (a) In which direction is the net gravitational force $\vec{F}_{\text{net}}$ on $A$? (b) If we move $C$ directly away from the origin, does $\vec{F}_{\text{net}}$ change in direction? If so, how and what is the limit of the change?

10. Figure 13-28 shows six paths by which a rocket orbiting a moon might move from point $a$ to point $b$. Rank the paths according to (a) the corresponding change in the gravitational potential energy of the rocket–moon system and (b) the net work done on the rocket by the gravitational force from the moon, greatest first.

11. Figure 13-29 shows three uniform spherical planets that are identical in size and mass. The periods of rotation $T$ for the planets are given, and six lettered points
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are indicated—three points are on the equators of the planets and three points are on the north poles. Rank the points according to the value of the free-fall acceleration \( g \) at them, greatest first.

![Fig. 13-29 Question 11.](image1)

![Fig. 13-30 Question 12.](image2)

**sec. 13-2 Newton's Law of Gravitation**

1. **1 ILW** A mass \( M \) is split into two parts, \( m \) and \( M - m \), which are then separated by a certain distance. What ratio \( m/M \) maximizes the magnitude of the gravitational force between the parts?

2. **Moon effect.** Some people believe that the Moon controls their activities. If the Moon moves from being directly on the opposite side of Earth from you to being directly overhead, by what percent does (a) the Moon’s gravitational pull on you increase and (b) your weight (as measured on a scale) decrease? Assume that the Earth–Moon (center-to-center) distance is \( 3.82 \times 10^8 \) m and Earth’s radius is \( 6.37 \times 10^6 \) m.

3. **SSM** What must the separation be between a 5.2 kg particle and a 2.4 kg particle for their gravitational attraction to have a magnitude of \( 2.3 \times 10^{-11} \) N?

4. **4** The Sun and Earth each exert a gravitational force on the Moon. What is the ratio \( F_{\text{sun}}/F_{\text{Earth}} \) of these two forces? (The average Sun–Moon distance is equal to the Sun–Earth distance.)

**sec. 13-3 Gravitation and the Principle of Superposition**

5. **Miniature black holes.** Left over from the big-bang beginning of the universe, tiny black holes might still wander through the universe. If one with a mass of \( 1 \times 10^{11} \) kg (and a radius of only \( 1 \times 10^{-16} \) m) reached Earth, at what distance from your head would its gravitational pull on you match that of Earth’s?

6. **In Fig. 13-31, a square of edge length 20.0 cm is formed by four spheres of masses \( m_1 = 5.00 \, \text{g}, m_2 = 3.00 \, \text{g}, m_3 = 1.00 \, \text{g}, \) and \( m_4 = 5.00 \, \text{g}. \) In unit-vector notation, what is the net gravitational force on one of the spheres with mass \( m_4 = 5.00 \, \text{g}? \) (This is not shown) Fig. 13-31 Problem 6.

7. **One dimension.** In Fig. 13-32, two point particles are fixed on an \( x \) axis separated by distance \( d \). Particle \( A \) has mass \( m_A \) and particle \( B \) has mass \( m_B \). What is the magnitude and (b) direction (relative to the positive direction of the \( x \) axis) of the net gravitational force on sphere \( B \) due to spheres \( A \) and \( C \)?

8. **In Fig. 13-33, three 5.00 kg spheres are located at distances \( d_1 = 0.300 \) m and \( d_2 = 0.400 \) m. What are the (a) magnitude and (b) direction (relative to the positive direction of the \( x \) axis) of the net gravitational force on sphere \( B \) due to spheres \( A \) and \( C \)?

9. **SSM** We want to position a space probe along a line that extends directly toward the Sun in order to monitor solar flares. How far from Earth’s center is the point on the line where the Sun’s gravitational pull on the probe balances Earth’s pull?

10. **Two dimensions.** In Fig. 13-34, three point particles are fixed in place in an \( xy \) plane. Particle \( A \) has mass \( m_A \), particle \( B \) has mass \( 2.00m_A \), and particle \( C \) has mass \( 3.00m_A \). A fourth particle \( D \), with mass \( 4.00m_A \), is to be placed near the other three particles. In terms of distance \( d \), at what (a) \( x \) coordinate and (b) \( y \) coordinate

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should particle $D$ be placed so that the net gravitational force on particle $A$ from particles $B$, $C$, and $D$ is zero?

- **11** As seen in Fig. 13-35, two spheres of mass $m$ and a third sphere of mass $M$ form an equilateral triangle, and a fourth sphere of mass $m_4$ is at the center of the triangle. The net gravitational force on that central sphere from the other spheres is zero. (a) What is $M$ in terms of $m$? (b) If we double the value of $m_4$, what then is the magnitude of the net gravitational force on the central sphere?

- **12** In Fig. 13-36a, particle $A$ is fixed in place at $x = -0.20$ m on the $x$ axis and particle $B$, with a mass of 1.0 kg, is fixed in place at the origin. Particle $C$ (not shown) can be moved along the $x$ axis, between particle $B$ and $x = \infty$. Figure 13-36b shows the $x$ component $F_{netx}$ of the net gravitational force on particle $B$ due to particles $A$ and $C$, as a function of position $x$ of particle $C$. The plot actually extends to the right, approaching an asymptote of $-4.17 \times 10^{-10}$ N as $x \to \infty$. What are the masses of (a) particle $A$ and (b) particle $C$?

![Figure 13-36 Problem 12.](image1)

- **13** Figure 13-37 shows a spherical hollow inside a lead sphere of radius $R = 4.00$ cm; the surface of the hollow passes through the center of the sphere and "touche(s)" the right side of the sphere. The mass of the sphere before hollowing was $M = 2.95$ kg. With what gravitational force does the hollowed-out lead sphere attract a small sphere of mass $m = 0.431$ kg that lies at a distance $d = 9.00$ cm from the center of the lead sphere, on the straight line connecting the centers of the spheres and of the hollow?

![Figure 13-37 Problem 13.](image2)

- **14** Three point particles are fixed in position in an $xyz$ coordinate system. Particle $A$, at the origin, has mass $m_A$. Particle $B$, at $xyz$ coordinates $(2.00d, 1.00d, -2.00d)$, has mass $2.00m_A$, and particle $C$, at coordinates $(-1.00d, 2.00d, -3.00d)$, has mass $3.00m_A$. A fourth particle $D$, with mass $4.00m_A$, is to be placed near the other particles. In terms of distance $d$, at what (a) $x$, (b) $y$, and (c) $z$ coordinate should $D$ be placed so that the net gravitational force on $A$ from $B$, $C$, and $D$ is zero?

- **15** Three dimensions. Three point particles are fixed in place in an $xyz$ coordinate system. Particle $A$, at the origin, has mass $m_A$. Particle $B$, at $xyz$ coordinates $(2.00d, 1.00d, -2.00d)$, has mass $2.00m_A$, and particle $C$, at coordinates $(-1.00d, 2.00d, -3.00d)$, has mass $3.00m_A$. A fourth particle $D$, with mass $4.00m_A$, is to be placed near the other particles. In terms of distance $d$, at what (a) $x$, (b) $y$, and (c) $z$ coordinate should $D$ be placed so that the net gravitational force on $A$ from $B$, $C$, and $D$ is zero?

- **16** In Fig. 13-39, a particle of mass $m_1 = 0.67$ kg is a distance $d = 23$ cm from one end of a uniform rod with length $L = 3.0$ m and mass $M = 5.0$ kg. What is the magnitude of the gravitational force $F$ on the particle from the rod?

![Figure 13-39 Problem 16.](image3)

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- **17** (a) What will an object weigh on the Moon’s surface if it weighs 100 N on Earth’s surface? (b) How many Earth radii must this same object be from the center of Earth if it is to weigh the same as it does on the Moon?

- **18** Mountain pull. A large mountain can slightly affect the direction of “down” as determined by a plumb line. Assume that we can model a mountain as a sphere of radius $R = 2.00$ km and density (mass per unit volume) $2.6 \times 10^3$ kg/m$^3$. Assume also that we hang a 0.50 m plumb line at a distance of 3$R$ from the sphere’s center and such that the sphere pulls horizontally on the lower end. How far would the lower end move toward the sphere?

- **19** SSM At what altitude above Earth’s surface would the gravitational acceleration be 4.9 m/s$^2$?

- **20** Mile-high building. In 1956, Frank Lloyd Wright proposed the construction of a mile-high building in Chicago. Suppose the building had been constructed. Ignoring Earth’s rotation, find the change in your weight if you were to ride an elevator from the street level, where you weigh 600 N, to the top of the building.

- **21** LW Certain neutron stars (extremely dense stars) are believed to be rotating at about 1 rev/s. If such a star has a radius of 20 km, what must be its minimum mass so that material on its surface remains in place during the rapid rotation?

- **22** The radius $R_h$ and mass $M_h$ of a black hole are related by $R_h = 2GM_h/c^2$, where $c$ is the speed of light. Assume that the gravitational acceleration $g_h$ at a point at a distance $r_h = 1.00R_h$ from the center of a black hole is given by Eq. 13-11 (it is, for large black holes). (a) In terms of $M_h$, find $g_h$ at $r_h$. (b) Does $g_h$ at $r_h$ increase or decrease as $M_h$ increases? (c) What is $g_h$ at $r_h$ for a very large black hole whose mass is $1.55 \times 10^{22}$ times the solar mass of 1.99 $\times 10^{30}$ kg? (d) If an astronaut of height 1.70 m is at $r_h$, with her feet down, what is the difference in gravitational acceleration between her head and feet? (e) Is the tendency to stretch the astronaut severe?

- **23** One model for a certain planet has a core of radius $R$ and mass $M$ surrounded by an outer shell of inner radius $R$, outer radius $2R$, and mass $4M$. If $M = 4.1 \times 10^{24}$ kg and $R = 6.0 \times 10^6$ m, what is the gravitational acceleration of a particle at points (a) $R$ and (b) $3R$ from the center of the planet?
**View All Solutions Here**

sec. 13-5 Gravitation Inside Earth

- 24 Two concentric spherical Earth shells with uniformly distributed masses $M_1$ and $M_2$ are situated as shown in Fig. 13-40. Find the magnitude of the net gravitational force on a particle of mass $m$, due to the shells, when the particle is located at radial distance (a) $a$, (b) $b$, and (c) $c$.

- 25 A solid uniform sphere has a mass of $1.0 \times 10^5$ kg and a radius of 1.0 m. What is the magnitude of the gravitational force due to the sphere on a particle of mass $m$ located at a distance of (a) 1.5 m and (b) 0.50 m from the center of the sphere? (c) Write a general expression for the magnitude of the gravitational force on the particle at a distance $r \leq 1.0$ m from the center of the sphere.

- 26 Consider a pulsar, a collapsed star of extremely high density, with a mass $M$ equal to that of the Sun ($1.98 \times 10^{30}$ kg), a radius $R$ of only 12 km, and a rotational period $T$ of 0.041 s. By what percentage does the free-fall acceleration $g$ differ from the gravitational acceleration $a_g$ at the equator of this spherical star?

- 27 Figure 13-41 shows, not to scale, a cross section through the interior of Earth. Rather than being uniform throughout, Earth is divided into three zones: an outer crust, a mantle, and an inner core. The dimensions of these zones and the masses contained within them are shown on the figure. Earth has a total mass of $5.98 \times 10^{24}$ kg and a radius of 6370 km. Ignore rotation and assume that Earth is spherical. (a) Calculate $a_g$ at the surface. (b) Suppose that a bore hole (the Mohole) is driven to the crust–mantle interface at a depth of 25.0 km; what would be the value of $a_g$ at the bottom of the hole? (c) Suppose that Earth were a uniform sphere with the same total mass and size. What would be the value of $a_g$ at a depth of 25.0 km? (Precise measurements of $a_g$ are sensitive probes of the interior structure of Earth, although results can be clouded by local variations in mass distribution.)

- 28 Assume a planet is a uniform sphere of radius $R$ that (somehow) has a narrow radial tunnel through its center (Fig. 13-7). Also assume we can position an apple anywhere along the tunnel or outside the sphere. Let $F_R$ be the magnitude of the gravitational force on the apple when it is located at the planet’s surface. How far from the surface is there a point where the magnitude is $\frac{1}{2}F_R$ if we move the apple (a) away from the planet and (b) into the tunnel?

sec. 13-6 Gravitational Potential Energy

- 29 Figure 13-42 gives the potential energy function $U(r)$ of a projectile, plotted outward from the surface of a planet of radius $R_s$. What least kinetic energy is required of a projectile launched at the surface if the projectile is to “escape” the planet?

- 30 In Problem 1, what ratio $m/M$ gives the least gravitational potential energy for the system?

- 31 SSM The mean diameters of Mars and Earth are $6.9 \times 10^7$ km and $1.3 \times 10^7$ km, respectively. The mass of Mars is 0.11 times Earth’s mass. (a) What is the ratio of the mean density (mass per unit volume) of Mars to that of Earth? (b) What is the value of the gravitational acceleration on Mars? (c) What is the escape speed on Mars?

- 32 (a) What is the gravitational potential energy of the two-particle system in Problem 3? If you triple the separation between the particles, how much work is done (b) by the gravitational force between the particles and (c) by you?

- 33 What multiple of the energy needed to escape from Earth gives the energy needed to escape from (a) the Moon and (b) Jupiter?

- 34 Figure 13-42 gives the potential energy function $U(r)$ of a projectile, plotted outward from the surface of a planet of radius $R_s$. If the projectile is launched radially outward from the surface with a mechanical energy of $-2.0 \times 10^8$ J, what are (a) its kinetic energy at radius $r = 1.25R_s$, and (b) its turning point (see Section 8-6) in terms of $R_s$?

- 35 Figure 13-43 shows four particles, each of mass $20.0$ g, that form a square with an edge length of $d = 0.600$ m. If $d$ is reduced to 0.200 m, what is the change in the gravitational potential energy of the four-particle system?

- 36 Zero, a hypothetical planet, has a mass of $5.0 \times 10^{24}$ kg, radius of $3.0 \times 10^6$ m, and no atmosphere. A 10 kg space probe is to be launched vertically from its surface. (a) If the probe is launched with an initial energy of $5.0 \times 10^7$ J, what will be its kinetic energy when it is $4.0 \times 10^6$ m from the center of Zero? (b) If the probe is to achieve a maximum distance of $8.0 \times 10^6$ m from the center of Zero, with what initial kinetic energy must it be launched from the surface of Zero?
** View All Solutions Here **

**37** The three spheres in Fig. 13-44, with masses \( m_A = 80 \, \text{g} \), \( m_B = 10 \, \text{g} \), and \( m_C = 20 \, \text{g} \), have their centers on a common line, with \( L = 12 \, \text{cm} \) and \( d = 4.0 \, \text{cm} \). You move sphere \( B \) along the line until its center-to-center separation from \( C \) is \( d = 4.0 \, \text{cm} \). How much work is done on sphere \( B \) (a) by you and (b) by the net gravitational force on \( B \) due to spheres \( A \) and \( C \)?

![Fig. 13-44](image)

**38** In deep space, sphere \( A \) of mass 20 kg is located at the origin of an \( x \)-axis and sphere \( B \) of mass 10 kg is located on the \( x \)-axis at \( x = 0.80 \, \text{m} \). Sphere \( B \) is released from rest while sphere \( A \) is held at the origin. (a) What is the gravitational potential energy of the two-sphere system just as \( B \) is released? (b) What is the kinetic energy of \( B \) when it has moved 0.20 m toward \( A \)?

**39** (a) What is the escape speed on a spherical asteroid whose radius is 500 km and whose gravitational acceleration at the surface is 3.0 \( \text{m/s}^2 \)? (b) How far from the surface will a particle go if it leaves the asteroid’s surface with a radial speed of 1000 m/s? (c) With what speed will an object hit the asteroid if it is dropped from 1000 km above the surface?

**40** A projectile is shot directly away from Earth’s surface. Neglect the rotation of Earth. What multiple of Earth’s radius \( R_E \) gives the radial distance a projectile reaches if (a) its initial speed is 0.500 of the escape speed from Earth and (b) its initial kinetic energy is 0.500 of the kinetic energy required to escape Earth? (c) What is the least initial mechanical energy required at launch if the projectile is to escape Earth?

**41** Two neutron stars are separated by a distance of \( 1.0 \times 10^{10} \, \text{m} \). They each have a mass of \( 1.0 \times 10^{30} \, \text{kg} \) and a radius of \( 1.0 \times 10^6 \, \text{m} \). They are initially at rest with respect to each other. As measured from that rest frame, how fast are they moving when (a) their separation has decreased to one-half its initial value and (b) they are about to collide?

**42** Figure 13-45a shows a particle \( A \) that can be moved along a \( y \)-axis from an infinite distance to the origin. That origin lies at the midpoint between particles \( B \) and \( C \), which have identical masses, and the \( y \)-axis is a perpendicular bisector between them. Distance \( D \) is 3.0587 m. Figure 13-45b shows the potential energy \( U \) of the three-particle system as a function of the position of particle \( A \) along the \( y \)-axis. The curve actually extends rightward and approaches an asymptote of \(-2.7 \times 10^{-11} \, \text{J} \) as \( y \to \infty \). What are the masses of (a) particles \( B \) and \( C \) and (b) particle \( A \)?

**sec. 13-7 Planets and Satellites: Kepler’s Laws**

**43** (a) What linear speed must an Earth satellite have to be in a circular orbit at an altitude of 160 km above Earth’s surface? (b) What is the period of revolution?

**44** A satellite is put in a circular orbit about Earth with a radius equal to one-half the radius of the Moon’s orbit. What is its period of revolution in lunar months? (A lunar month is the period of revolution of the Moon.)

**45** The Martian satellite Phobos travels in an approximately circular orbit of radius 9.4 \( \times 10^6 \, \text{m} \) with a period of 7 h 39 min. Calculate the mass of Mars from this information.

**46** The first known collision between space debris and a functioning satellite occurred in 1996: At an altitude of 700 km, a year-old French spy satellite was hit by a piece of an Ariane rocket. A stabilizing boom on the satellite was demolished, and the satellite was sent spinning out of control. Just before the collision and in kilometers per hour, what was the speed of the rocket piece relative to the satellite if both were in circular orbits and the collision was (a) head-on and (b) along perpendicular paths?

**47** The Sun, which is \( 2.2 \times 10^{19} \, \text{m} \) from the center of the Milky Way galaxy, revolves around that center once every \( 2.5 \times 10^7 \, \text{years} \). Assuming each star in the Galaxy has a mass equal to the Sun’s mass of \( 2.0 \times 10^{30} \, \text{kg} \), the stars are distributed uniformly in a sphere about the galactic center, and the Sun is at the edge of that sphere, estimate the number of stars in the Galaxy.

**48** The mean distance of Mars from the Sun is 1.52 times that of Earth from the Sun. From Kepler’s law of periods, calculate the number of years required for Mars to make one revolution around the Sun; compare your answer with the value given in Appendix C.

**49** A comet that was seen in April 574 by Chinese astronomers on a day known by them as the Woo Woo day was spotted again in May 1994. Assume the time between observations is the period of the Woo Woo day comet and take its eccentricity as 0.11. What are (a) the semimajor axis of the comet’s orbit and (b) its greatest distance from the Sun in terms of the mean orbital radius \( R_{\odot} \) of Pluto?

**50** An orbiting satellite stays over a certain spot on the equator of (rotating) Earth. What is the altitude of the orbit (called a geosynchronous orbit)?

**51** A satellite, moving in an elliptical orbit, is 360 km above Earth’s surface at its farthest point and 180 km above at its closest point. Calculate (a) the semimajor axis and (b) the eccentricity of the orbit.

**52** The Sun’s center is at one focus of Earth’s orbit. How far from this focus is the other focus, (a) in meters and (b) in terms of the solar radius, \( 6.96 \times 10^8 \, \text{m} \)? The eccentricity is 0.0167, and the semimajor axis is \( 1.50 \times 10^11 \, \text{m} \).

**53** A 20 kg satellite has a circular orbit with a period of 2.4 h and a radius of \( 8.0 \times 10^6 \, \text{m} \) around a planet of unknown mass. If the magnitude of the gravitational acceleration on the surface of the planet is \( 8.0 \, \text{m/s}^2 \), what is the radius of the planet?
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**54** Hunting a black hole. Observations of the light from a certain star indicate that it is part of a binary (two-star) system. This visible star has orbital speed \( v = 270 \text{ km/s} \), orbital period \( T = 1.70 \text{ days} \), and approximate mass \( m_1 = 6m_* \), where \( m_* \) is the Sun’s mass, 1.99 \( \times 10^{30} \text{ kg} \). Assume that the visible star and its companion star, which is dark and unseen, are both in circular orbits (Fig. 13-46). What multiple of \( m_* \) gives the approximate mass \( m_2 \) of the dark star?

**55** In 1610, Galileo used his telescope to discover four prominent moons around Jupiter. Their mean orbital radii \( a \) and periods \( T \) are as follows:

<table>
<thead>
<tr>
<th>Name</th>
<th>( a ) (10^9 m)</th>
<th>( T ) (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Io</td>
<td>4.22</td>
<td>1.77</td>
</tr>
<tr>
<td>Europa</td>
<td>6.71</td>
<td>3.55</td>
</tr>
<tr>
<td>Ganymede</td>
<td>10.7</td>
<td>7.16</td>
</tr>
<tr>
<td>Callisto</td>
<td>18.8</td>
<td>16.7</td>
</tr>
</tbody>
</table>

(a) Plot \( \log a \) (y axis) against \( \log T \) (x axis) and show that you get a straight line. (b) Measure the slope of the line and compare it with the value that you expect from Kepler’s third law. (c) Find the mass of Jupiter from the intercept of this line with the \( y \) axis.

**56** In 1993 the spacecraft Galileo sent home an image (Fig. 13-47) of asteroid 243 Ida and a tiny orbiting moon (now known as Dactyl), the first confirmed example of an asteroid–moon system. In the image, the moon, which is 1.5 km wide, is 100 km from the center of the asteroid, which is 55 km long. The shape of the moon’s orbit is not well known; assume it is circular with a period of 27 h. (a) What is the mass of the asteroid? (b) The volume of the asteroid, measured from the Galileo images, is 14 100 km\(^3\). What is the density (mass per unit volume) of the asteroid?

**57** In a certain binary-star system, each star has the same mass as our Sun, and they revolve about their center of mass. The distance between them is the same as the distance between Earth and the Sun. What is their period of revolution in years?

**58** The presence of an unseen planet orbiting a distant star can sometimes be inferred from the motion of the star as we see it. As the star and planet orbit the center of mass of the star–planet system, the star moves toward and away from us with what is called the line of sight velocity, a motion that can be detected. Figure 13-48 shows a graph of the line of sight velocity versus time for the star 14 Herculis. The star’s mass is believed to be 0.90 of the mass of our Sun. Assume that only one planet orbits the star and that our view is along the plane of the orbit. Then approximate (a) the planet’s mass in terms of Jupiter’s mass \( m_J \) and (b) the planet’s orbital radius in terms of Earth’s orbital radius \( r_E \).

**59** Three identical stars of mass \( M \) form an equilateral triangle that rotates around the triangle’s center as the stars move in a common circle about that center. The triangle has edge length \( L \). What is the speed of the stars?

**60** In Fig. 13-49, two satellites, \( A \) and \( B \), both of mass \( m = 125 \text{ kg} \), move in the same circular orbit of radius \( r = 7.87 \times 10^6 \text{ m} \) around Earth but in opposite senses of rotation and therefore on a collision course. (a) Find the total mechanical energy \( E_A + E_B \) of the two satellites + Earth system before the collision. (b) If the collision is completely inelastic so that the wreckage remains as one piece of tangled material (mass \( = 2m \)), find the total mechanical energy immediately after the collision. (c) Just after the collision, is the wreckage falling directly toward Earth’s center or orbiting around Earth?

**61** (a) At what height above Earth’s surface is the energy required to lift a satellite to that height equal to the kinetic energy required for the satellite to be in orbit at that height? (b) For greater heights, which is greater, the energy for lifting or the kinetic energy for orbiting?

**62** Two Earth satellites, \( A \) and \( B \), each of mass \( m \), are to be launched into circular orbits about Earth’s center. Satellite \( A \) is to
** View All Solutions Here **

-orbit at an altitude of 6370 km. Satellite $B$ is to orbit at an altitude of 19 110 km. The radius of Earth $R_E$ is 6370 km. (a) What is the ratio of the potential energy of satellite $B$ to that of satellite $A$, in orbit? (b) What is the ratio of the kinetic energy of satellite $B$ to that of satellite $A$, in orbit? (c) Which satellite has the greater total energy if each has a mass of 14.6 kg? (d) By how much much earlier than Igor will Picard return to Earth? Suppose the satellite loses mechanical energy at the average rate of 1.4 \times 10^5 J per orbital revolution. Adopting the reason-

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74 The mysterious visitor that appears in the enchanting story *The Little Prince* was said to come from a planet that “was scarcely any larger than a house!” Assume that the mass per unit volume of the planet is about that of Earth and that the planet does not appreciably spin. Approximate (a) the free-fall acceleration on the planet’s surface and (b) the escape speed from the planet.

75 UU The masses and coordinates of three spheres are as follows: 20 kg, \(x = 0.50 \text{ m}, y = 1.0 \text{ m}; 40 \text{ kg}, x = -1.0 \text{ m}, y = -1.0 \text{ m}; 60 \text{ kg}, x = 0 \text{ m}, y = -0.50 \text{ m}. What is the magnitude of the gravitational force on a 20 kg sphere located at the origin due to these three spheres?

76 SSM A very early, simple satellite consisted of an inflated spherical aluminum balloon 30 m in diameter and of mass 20 kg. Suppose a meteor having a mass of 7.0 kg passes within 3.0 m of the surface of the satellite. What is the magnitude of the gravitational force on the meteor from the satellite at the closest approach?

77 UU Four uniform spheres, with masses \(m_A = 40 \text{ kg}, m_B = 35 \text{ kg}, m_C = 200 \text{ kg}, m_D = 50 \text{ kg} \), have \((x, y)\) coordinates of \((0, 50 \text{ cm}), (0, 0), (-80 \text{ cm}, 0), (40 \text{ cm}, 0)\), respectively. In unit-vector notation, what is the net gravitational force on sphere \(B\) due to the other spheres?

78 (a) In Problem 77, remove sphere \(A\) and calculate the gravitational potential energy of the remaining three-particle system. (b) If \(A\) is then put back in place, is the potential energy of the four-particle system more or less than that of the system in (a)? (c) In (a), is the work done by you to remove \(A\) positive or negative? (d) In (b), is the work done by you to replace \(A\) positive or negative?

79 SSM A certain triple-star system consists of two stars, each of mass \(m\), revolving in the same circular orbit of radius \(r\) around a central star of mass \(M\) (Fig. 13-53). The two orbiting stars are always at opposite ends of a diameter of the orbit. Derive an expression for the period of revolution of the stars.

80 The fastest possible rate of rotation of a planet is that for which the gravitational force on material at the equator just barely provides the centripetal force needed for the rotation. (Why?) (a) Show that the corresponding shortest period of rotation is

\[
T = \sqrt{\frac{3\pi}{G\rho}}
\]

where \(\rho\) is the uniform density (mass per unit volume) of the spherical planet. (b) Calculate the rotation period assuming a density of 3.0 g/cm\(^3\), typical of many planets, satellites, and asteroids. No astronomical object has ever been found to be spinning with a period shorter than that determined by this analysis.

81 SSM In a double-star system, two stars of mass \(3.0 \times 10^{30}\) kg each rotate about the system’s center of mass at radius \(1.0 \times 10^{13}\) m. (a) What is their common angular speed? (b) If a meteoroid passes through the system’s center of mass perpendicular to their orbital plane, what minimum speed must it have at the center of mass if it is to escape to “infinity” from the two-star system?

82 A satellite is in elliptical orbit with a period of \(8.00 \times 10^4\) s about a planet of mass \(7.00 \times 10^{22}\) kg. At aphelion, at radius \(4.5 \times 10^{10}\) m, the satellite’s angular speed is \(7.158 \times 10^{-2}\) rad/s. What is its angular speed at perihelion?

83 SSM In a shuttle craft of mass \(m = 3000\) kg, Captain Janeway orbits a planet of mass \(M = 9.50 \times 10^{23}\) kg, in a circular orbit of radius \(r = 4.20 \times 10^{10}\) m. What are (a) the period of the orbit and (b) the speed of the shuttle craft? Janeway briefly fires a forward-pointing thruster, reducing her speed by 2.00%. Just then, what are (c) the speed, (d) the kinetic energy, (e) the gravitational potential energy, and (f) the mechanical energy of the shuttle craft? (g) What is the semimajor axis of the elliptical orbit now taken by the craft? (h) What is the difference between the period of the original circular orbit and that of the new elliptical orbit? (i) Which orbit has the smaller period?

84 A uniform solid sphere of radius \(R\) produces a gravitational acceleration of \(g\) on its surface. At what distance from the sphere’s center are there points (a) inside and (b) outside the sphere where the gravitational acceleration is \(g/3\)?

85 UU A projectile is fired vertically from Earth’s surface with an initial speed of 10 km/s. Neglecting air drag, how far above the surface of Earth will it go?

86 An object lying on Earth’s equator is accelerated (a) toward the center of Earth because Earth rotates, (b) toward the Sun because Earth revolves around the Sun in an almost circular orbit, and (c) toward the center of our galaxy because the Sun moves around the galactic center. For the latter, the period is \(2.5 \times 10^6\) y and the radius is \(2.2 \times 10^{20}\) m. Calculate these three accelerations as multiples of \(g = 9.8 \text{ m/s}^2\).

87 (a) If the legendary apple of Newton could be released from rest at a height of 2 m from the surface of a neutron star with a mass 1.5 times that of our Sun and a radius of 20 km, what would be the apple’s speed when it reached the surface of the star? (b) If the apple could rest on the surface of the star, what would be the approximate difference between the gravitational acceleration at the top and at the bottom of the apple? (Choose a reasonable size for an apple; the answer indicates that an apple would never survive near a neutron star.)

88 With what speed would mail pass through the center of Earth if falling in a tunnel through the center?

89 SSM The orbit of Earth around the Sun is almost circular: The closest and farthest distances are \(1.47 \times 10^{10}\) km and \(1.52 \times 10^{10}\) km respectively. Determine the corresponding variations in (a) total energy, (b) gravitational potential energy, (c) kinetic energy, and (d) orbital speed. (\textit{Hint: Use conservation of energy and conservation of angular momentum.})

90 A 50 kg satellite circles planet Cruton every 6.0 h. The magnitude of the gravitational force exerted on the satellite by Cruton is 80 N. (a) What is the radius of the orbit? (b) What is the kinetic energy of the satellite? (c) What is the mass of planet Cruton?

91 We watch two identical astronomical bodies \(A\) and \(B\), each of mass \(m\), fall toward each other from rest because of the gravitational force on each from the other. Their initial center-to-center separation is \(R\). Assume that we are in an inertial reference frame that is stationary with respect to the center of mass of this two-body system. Use the principle of conservation of mechanical
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energy \((K_f + U_f = K_i + U_i)\) to find the following when the center-to-center separation is 0.5 \(R_i\): (a) the total kinetic energy of the system, (b) the kinetic energy of each body, (c) the speed of each body relative to us, and (d) the speed of body \(B\) relative to body \(A\).

Next assume that we are in a reference frame attached to body \(A\) (we ride on the body). Now we see body \(B\) fall from rest toward us. From this reference frame, again use \(K_f + U_f = K_i + U_i\) to find the following when the center-to-center separation is 0.5 \(R_i\): (e) the kinetic energy of body \(B\) and (f) the speed of body \(B\) relative to body \(A\). (g) Why are the answers to (d) and (f) different? Which answer is correct?

92  A 150.0 kg rocket moving radially outward from Earth has a speed of 3.70 km/s when its engine shuts off 200 km above Earth’s surface. (a) Assuming negligible air drag, find the rocket’s kinetic energy when the rocket is 1000 km above Earth’s surface. (b) What maximum height above the surface is reached by the rocket?

93  Planet Roton, with a mass of \(7.0 \times 10^{24}\) kg and a radius of 1600 km, gravitationally attracts a meteorite that is initially at rest relative to the planet, at a distance great enough to take as infinite. The meteorite falls toward the planet. Assuming the planet is airless, find the speed of the meteorite when it reaches the planet’s surface.

94  Two 20 kg spheres are fixed in place on a \(y\) axis, one at \(y = 0.40\) m and the other at \(y = -0.40\) m. A 10 kg ball is then released from rest at a point on the \(x\) axis that is at a great distance (effectively infinite) from the spheres. If the only forces acting on the ball are the gravitational forces from the spheres, then when the ball reaches the \((x, y)\) point \((0.30\) m, 0), what are (a) its kinetic energy and (b) the net force on it from the spheres, in unit-vector notation?

95  Sphere \(A\) with mass 80 kg is located at the origin of an \(xy\) coordinate system; sphere \(B\) with mass 60 kg is located at coordinates \((0.25\) m, 0); sphere \(C\) with mass 0.20 kg is located in the first quadrant 0.20 m from \(A\) and 0.15 m from \(B\). In unit-vector notation, what is the gravitational force on \(C\) due to \(A\) and \(B\)?

96  In his 1865 science fiction novel *From the Earth to the Moon*, Jules Verne described how three astronauts are shot to the Moon by means of a huge gun. According to Verne, the aluminum capsule containing the astronauts is accelerated by ignition of nitrocellulose to a speed of 11 km/s along the gun barrel’s length of 220 m. (a) In \(g\) units, what is the average acceleration of the capsule and astronauts in the gun barrel? (b) Is that acceleration tolerable or deadly to the astronauts?

A modern version of such gun-launched spacecraft (although without passengers) has been proposed. In this modern version, called the SHARP (Super High Altitude Research Project) gun, ignition of methane and air shoves a piston along the gun’s tube, compressing hydrogen gas that then launches a rocket. During this launch, the rocket moves 3.5 km and reaches a speed of 7.0 km/s. (c) In \(g\) units, what would be the average acceleration of the rocket within the launcher? (d) How much additional speed is needed (via the rocket engine) if the rocket is to orbit Earth at an altitude of 700 km?

97  An object of mass \(m\) is initially held in place at radial distance \(r = 3R_E\) from the center of Earth, where \(R_E\) is the radius of Earth. Let \(M_E\) be the mass of Earth. A force is applied to the object to move it to a radial distance \(r = 4R_E\), where it again is held in place. Calculate the work done by the applied force during the move by integrating the force magnitude.

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