Chapter 32

Alternating Current Circuits

Alternating Current Circuits

Electrical appliances in the house use alternating current (AC) circuits.

If an AC source applies an alternating voltage to a series circuit containing resistor, inductor, and capacitor, what are the amplitude and time characteristics of the alternating current.

Other devices will be discussed

- Transformers
- Power transmission
- Electrical filters

AC Circuits

An AC circuit consists of a combination of circuit elements and a power source.

The power source provides an alternating voltage, Δv.

Notation note:

- Lower case symbols will indicate instantaneous values.
- Capital letters will indicate fixed values.

AC Voltage

The output of an AC power source is sinusoidal and varies with time according to the following equation:

- $\Delta v = \Delta V_{\max} \sin \omega t$
  - Δv is the instantaneous voltage.
  - $\Delta V_{\max}$ is the maximum output voltage of the source.
    - Also called the **voltage amplitude**
  - ω is the angular frequency of the AC voltage.

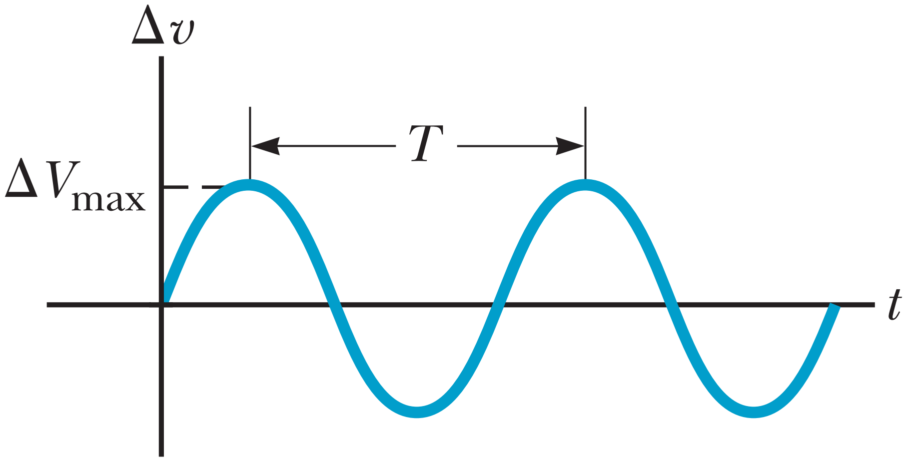
AC Voltage, cont.

The angular frequency is

$$\omega = 2\pi f = \frac{2\pi}{T}$$

- f is the frequency of the source.
- T is the period of the source.

The voltage is positive during one half of the cycle and negative during the other half.



AC Voltage, final

The current in any circuit driven by an AC source is an alternating current that varies sinusoidally with time.

Commercial electric power plants in the US use a frequency of $60\ \mathrm{Hz}$.

- This corresponds with an angular frequency of $377\ \mathrm{rad/s}$.

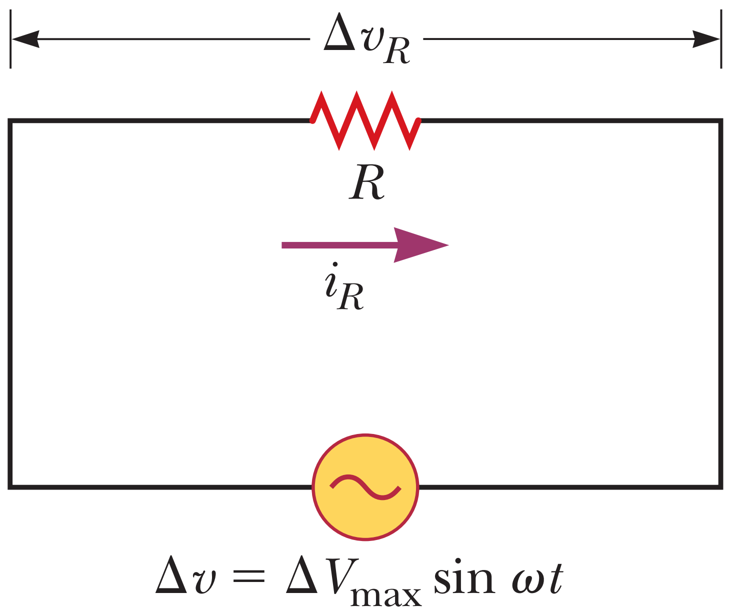
Resistors in an AC Circuit

Consider a circuit consisting of an AC source and a resistor.

The AC source is symbolized by

$$\Delta v_R = \Delta V_{max} \sin \omega t$$

Δv_R is the instantaneous voltage across the resistor.



Resistors in an AC Circuit, cont.

The instantaneous current in the resistor is

$$i_R = \frac{\Delta v}{R} = \frac{\Delta V_{\max}}{R} \sin \omega t = I_{\max} \sin \omega t$$

The instantaneous voltage across the resistor is also given as

$$\Delta v_R = i_R R = I_{\max} R \sin \omega t$$

Resistors in an AC Circuit, final

The graph shows the current through and the voltage across the resistor.

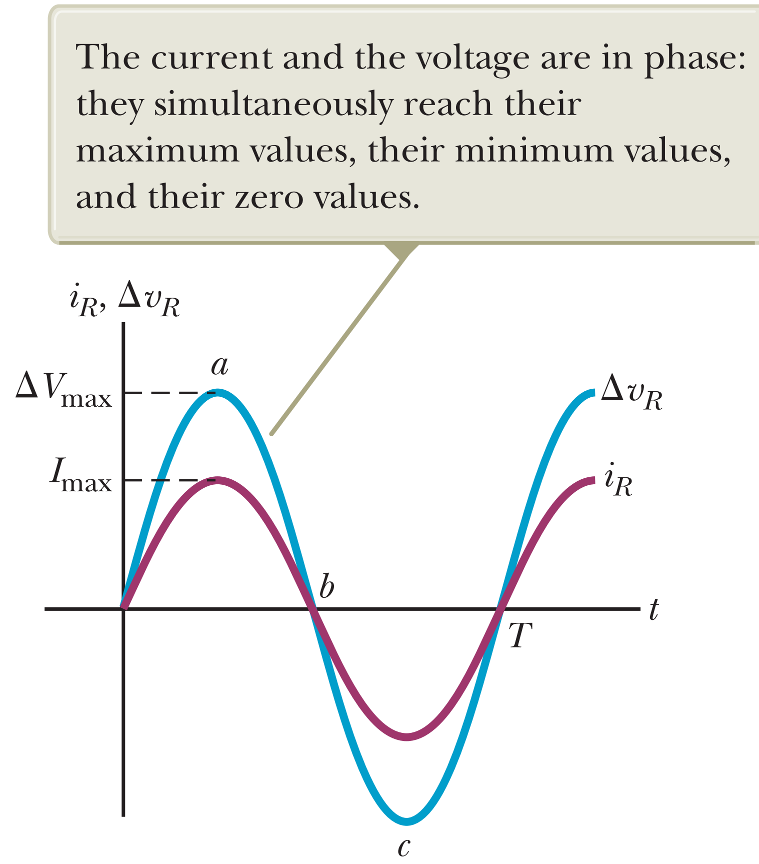
The current and the voltage reach their maximum values at the same time.

The current and the voltage are said to be *in phase.*

For a sinusoidal applied voltage, the current in a resistor is always in phase with the voltage across the resistor.

The direction of the current has no effect on the behavior of the resistor.

Resistors behave essentially the same way in both DC and AC circuits.



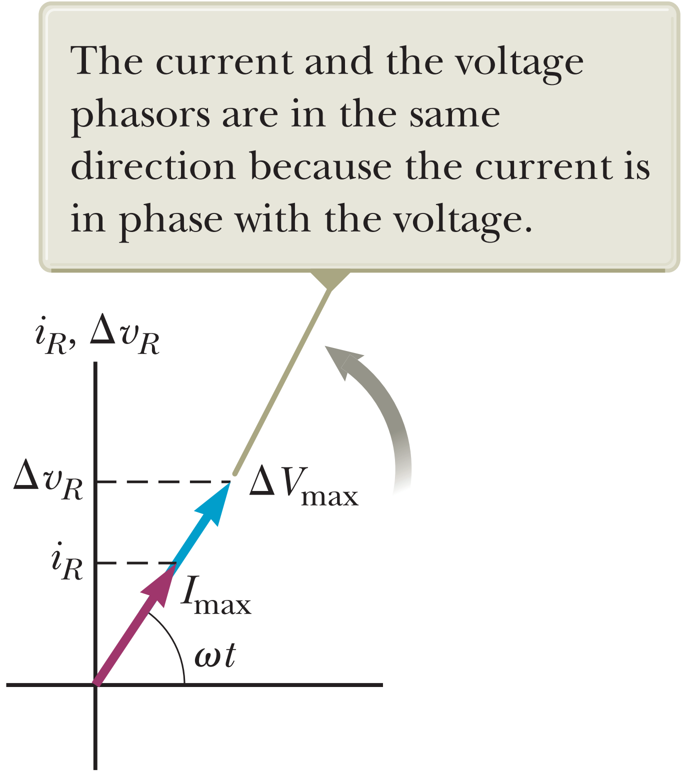
Phasor Diagram

To simplify the analysis of AC circuits, a graphical constructor called a *phasor diagram* can be used.

A **phasor** is a vector whose length is proportional to the maximum value of the variable it represents.

The vector rotates counterclockwise at an angular speed equal to the angular frequency associated with the variable.

The projection of the phasor onto the vertical axis represents the instantaneous value of the quantity it represents.



A Phasor is Like a Graph

An alternating voltage can be presented in different representations.

One graphical representation is using rectangular coordinates.

- The voltage is on the vertical axis.
- Time is on the horizontal axis.

The phase space in which the phasor is drawn is similar to polar coordinate graph paper.

- The radial coordinate represents the amplitude of the voltage.
- The angular coordinate is the phase angle.
- The vertical axis coordinate of the tip of the phasor represents the instantaneous value of the voltage.
- The horizontal coordinate does not represent anything.

Alternating currents can also be represented by phasors.

rms Current and Voltage

The average current in one cycle is zero.

Resistors experience a temperature increase which depends on the magnitude of the current, but not the direction of the current.

The power is related to the square of the current.

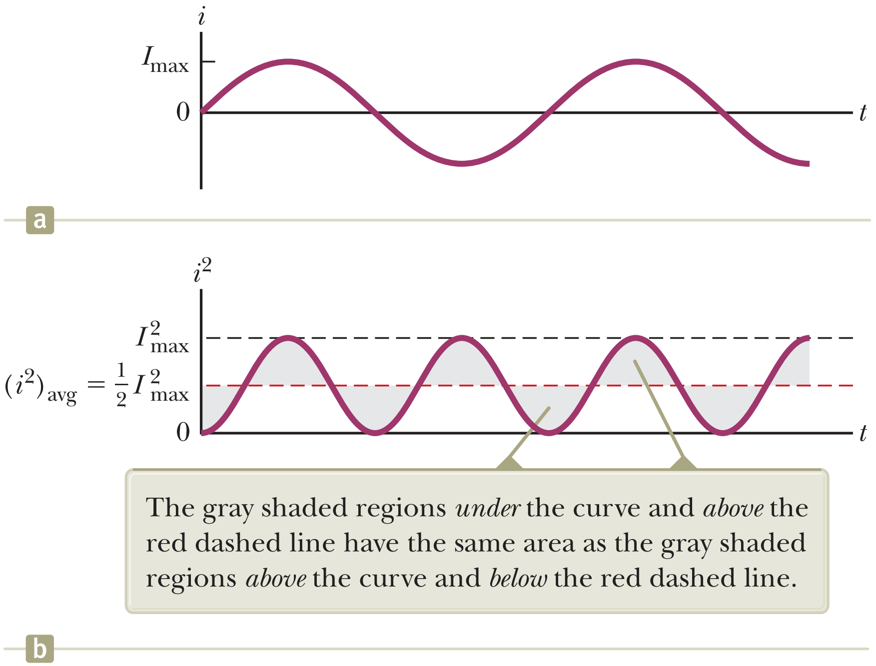
The rms current is the average of importance in an AC circuit.

- *rms* stands for root mean square

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707 I_{max}$$

Alternating voltages can also be discussed in terms of rms values.

$$\Delta V_{rms} = \frac{\Delta V_{max}}{\sqrt{2}} = 0.707 \Delta V_{max}$$



Power

The rate at which electrical energy is delivered to a resistor in the circuit is given by

- $p = i^2 R$
  - i is the *instantaneous current.*
  - The heating effect produced by an AC current with a maximum value of I_{max} is not the same as that of a DC current of the same value.
  - The maximum current occurs for a small amount of time.
- The average power delivered to a resistor that carries an alternating current is

$$P_{\text{avg}} = I_{\text{rms}}^2 R$$

Notes About rms Values

rms values are used when discussing alternating currents and voltages because

- AC ammeters and voltmeters are designed to read rms values.
- Many of the equations that will be used have the same form as their DC counterparts.

Example 32.01: What Is the rms Current?

The voltage output of an AC source is given by the expression $\Delta v = 200 \sin \omega t$, where Δv is in volts. Find the rms current in the circuit when this source is connected to a $47.0 - \Omega$ resistor.

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{\Delta V_{max}}{\sqrt{2}R}$$

$$I_{rms} = \frac{200 \text{ V}}{\sqrt{2}(47.0\Omega)} = 3.01 \text{ A}$$

Problem 32.01:

(a) What is the resistance of a lightbulb that uses an average power of $75.0\ \mathrm{W}$ when connected to a $60.0-\mathrm{Hz}$ power source having a maximum voltage of $170\ \mathrm{V}$? (b) What If? What is the resistance of a $100-\mathrm{W}$ lightbulb?

The rms voltage is

$$\Delta V_{\mathrm{rms}} = \frac{170\ \mathrm{V}}{\sqrt{2}} = 120\ \mathrm{V}$$

(a) $P = \dfrac{(\Delta V_{\mathrm{rms}})^2}{R} \rightarrow R = \dfrac{(120\ \mathrm{V})^2}{75.0\ \mathrm{W}} = 193\Omega$

(b) $R = \frac{(120\ \mathrm{V})^2}{100\ \mathrm{W}} = 144\Omega$

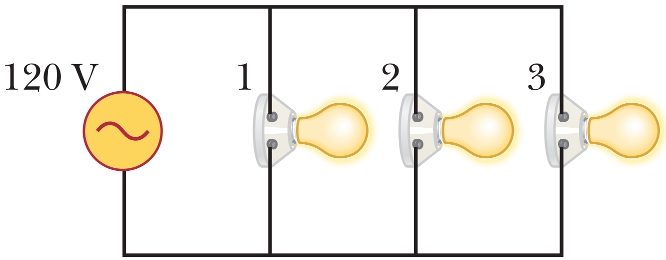
Problem 32.04:

The figure shows three lightbulbs connected to a $120-\mathrm{V}$ AC (rms) household supply voltage. Bulbs 1 and 2 have a power rating of $150\ \mathrm{W}$, and bulb 3 has a $100-\mathrm{W}$ rating. Find (a) the rms current in each bulb and (b) the resistance of each bulb. (c) What is the total resistance of the combination of the three lightbulbs?

All lamps are connected in parallel with the voltage source, so $\Delta V_{\mathrm{rms}} = 120\ \mathrm{V}$ for each lamp. Also, the current is $I_{\mathrm{rms}} = P_{\mathrm{avg}}/\Delta V_{\mathrm{rms}}$ and the resistance is $R = \Delta V_{\mathrm{rms}}/I_{\mathrm{rms}}$.

(a) For the $150-\mathrm{W}$ bulbs, $I_{\mathrm{rms}} = \dfrac{150\ \mathrm{W}}{120\ \mathrm{V}} = 1.25\ \mathrm{A}$

For the $100-\mathrm{W}$ bulb, $I_{\mathrm{rms}} = \dfrac{100\ \mathrm{W}}{120\ \mathrm{V}} = 0.833\ \mathrm{A}$



(b) The resistance in bulbs 1 and 2 is

$$R_1 = R_2 = \frac{120\ \mathrm{V}}{1.25\ \mathrm{A}} = 96.0\Omega$$

and the resistance in bulb 3 is

$$R_3 = \frac{120\ \mathrm{V}}{0.833\ \mathrm{A}} = 144\Omega$$

(c) The bulbs are in parallel, so

$$\frac{1}{R_{\mathrm{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{96.0\Omega} + \frac{1}{96.0\Omega} + \frac{1}{144\Omega}$$

$$R_{\mathrm{eq}} = 36.0\Omega$$

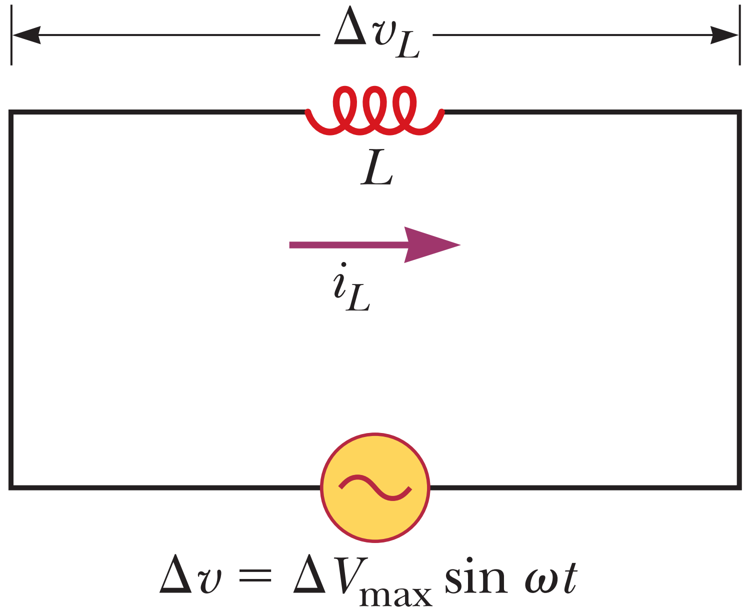
Inductors in an AC Circuit

Kirchhoff's loop rule can be applied and gives:

$\Delta v + \Delta v_L = 0$, Or

$$\Delta v - L\frac{di_L}{dt} = 0$$

$$\Delta v = L\frac{di_L}{dt} = \Delta V_{\max} \sin \omega t$$



Current in an Inductor

The equation obtained from Kirchhoff's loop rule can be solved for the current

$$i_L = \frac{\Delta V_{\max}}{L} \int \sin \omega t dt = -\frac{\Delta V_{\max}}{\omega L} \cos \omega t$$

$$i_L = \frac{\Delta V_{\max}}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right); \quad I_{\max} = \frac{\Delta V_{\max}}{\omega L}$$

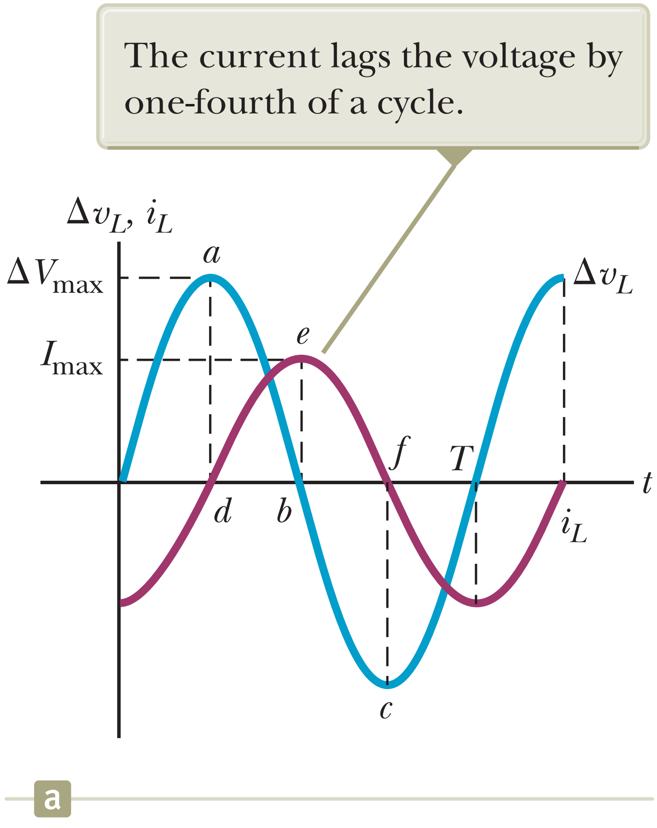
This shows that the instantaneous current i_L in the inductor and the instantaneous voltage Δv_L across the inductor are out of phase by $(\pi/2)\,\text{rad} = 90°$.

Phase Relationship of Inductors in an AC Circuit

The current is a maximum when the voltage across the inductor is zero.

- The current is momentarily not changing

For a sinusoidal applied voltage, the current in an inductor always lags behind the voltage across the inductor by $90°$ $(\pi/2)$.

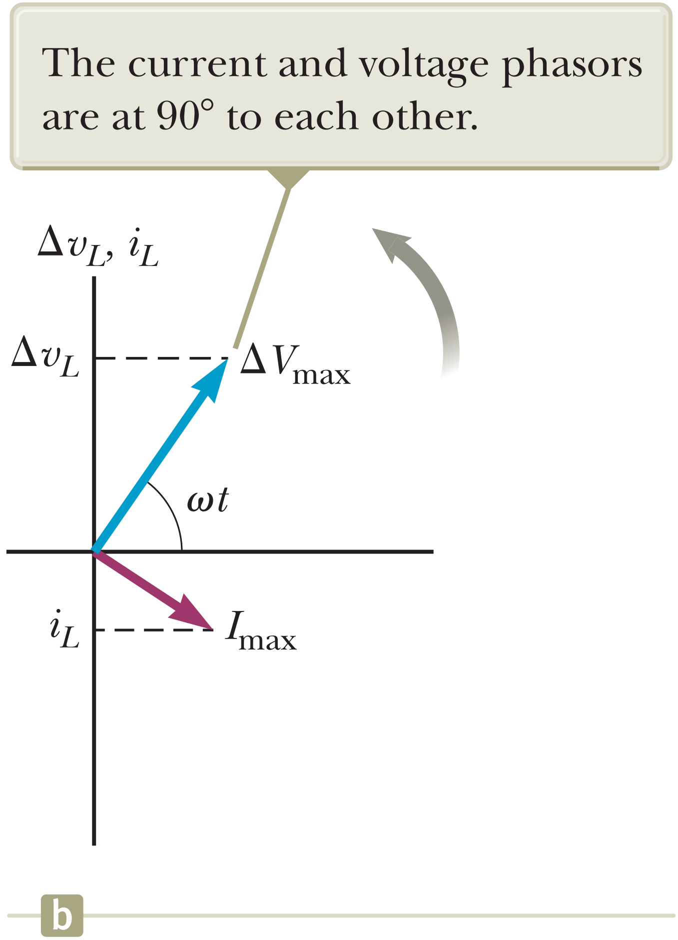


Phasor Diagram for an Inductor

The phasors are at $90°$ with respect to each other.

This represents the phase difference between the current and voltage.

Specifically, the current lags behind the voltage by $90°$.



Inductive Reactance

The factor ωL has the same units as resistance and is related to current and voltage in the same way as resistance.

Because ωL depends on the frequency, it reacts differently, in terms of offering resistance to current, for different frequencies.

The factor is the **inductive reactance** and is given by:

- $X_L = \omega L$

Inductive Reactance, cont.

Current can be expressed in terms of the inductive reactance:

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{X_L} \text{ or } I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L}$$

As the frequency increases, the inductive reactance increases

- This is consistent with Faraday's Law:
  - The larger the rate of change of the current in the inductor, the larger the back emf, giving an increase in the reactance and a decrease in the current.

Voltage Across the Inductor

The instantaneous voltage across the inductor is

$$\Delta v_L = -L\frac{di_L}{dt} = -\Delta V_{\max}\sin\omega t = -I_{\max}X_L\sin\omega t$$

Problem 32.06:

In a purely inductive AC circuit, $\Delta V_{\max} = 100\ \text{V}$. (a) The maximum current is $7.50\ \text{A}$ at $50.0\ \text{Hz}$. Calculate the inductance L. (b) What If? At what angular frequency ω is the maximum current $2.50\ \text{A}$?

(a) $X_L = \omega L = \frac{\Delta V_{\max}}{I_{\max}}$

$$L = \frac{\Delta V_{\max}}{\omega I_{\max}} = \frac{100\ \text{V}}{2\pi(50.0\ \text{Hz})(7.50\ \text{A})} = 0.0424\text{H}$$

(b) From $I_{\max} = \frac{\Delta V_{\max}}{X_L} = \frac{\Delta V_{\max}}{\omega L}$, we see that is current inversely proportional to angular frequency:

$$\frac{I_{\max}}{I'_{\max}} = \frac{\omega'}{\omega}$$

$$\omega' = \omega \frac{I_{\max}}{I'_{\max}} = [2\pi(50.0\ \text{Hz})]\frac{7.50\ \text{A}}{2.50\ \text{A}} = 942\text{rad/s}$$

Problem 32.09:

An AC source has an output rms voltage of $78.0\ \text{V}$ at a frequency of $80.0\ \text{Hz}$. If the source is connected across a 25.0-mH inductor, what are (a) the inductive reactance of the circuit, (b) the rms current in the circuit, and (c) the maximum current in the circuit?

(a) $X_L = 2\pi f L = 2\pi(80.0\ \text{Hz})\left(25.0 \times 10^{-3}\text{H}\right) = 12.6\Omega$

(b) $I_{\text{rms}} = \frac{\Delta V_{L,\text{rms}}}{X_L} = \frac{78.0\ \text{V}}{X_L} = 6.21\ \text{A}$

(c) $I_{\max} = \sqrt{2} I_{\text{rms}} = \sqrt{2}(6.21\ \text{A}) = 8.78\ \text{A}$

Capacitors in an AC Circuit

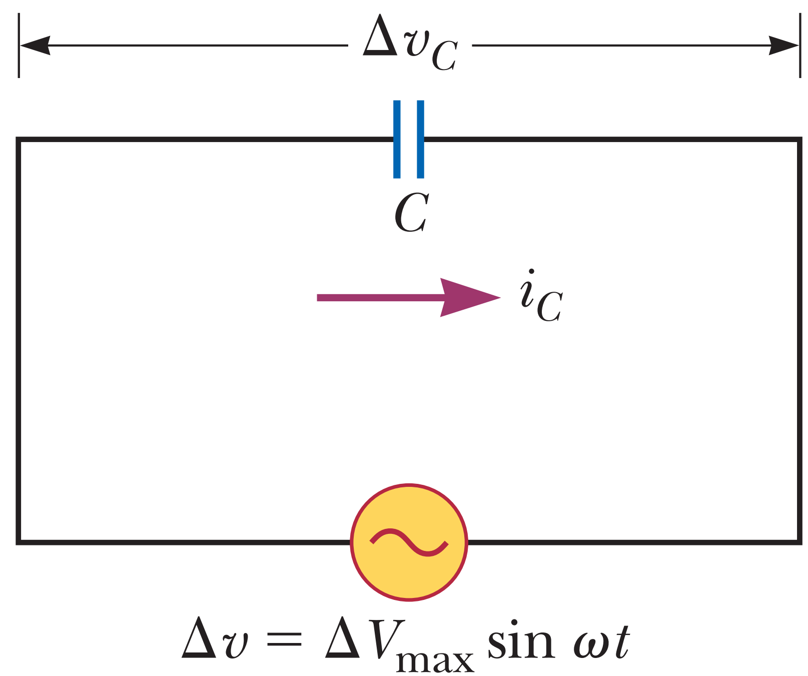
The circuit contains a capacitor and an AC source.

Kirchhoff's loop rule gives:

$\Delta v + \Delta v_C = 0$ and so

$$\Delta v - \frac{q}{C} = 0$$

- Δv_C is the instantaneous voltage across the capacitor.



Capacitors in an AC Circuit, cont.

The charge is $q = C\Delta V_{\max} \sin \omega t$

The instantaneous current is given by

$$i_C = \frac{dq}{dt} = \omega C \Delta V_{\max} \cos \omega t$$

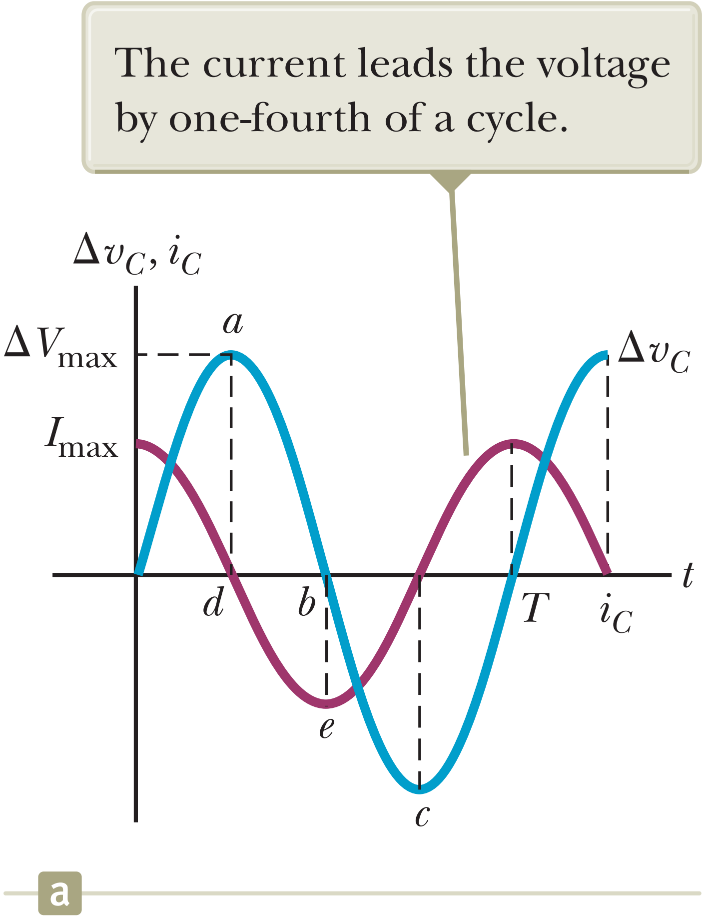
Or $i_C = \omega C \Delta V_{\max} \sin\left(\omega t + \frac{\pi}{2}\right)$

The current is $\pi/2 \text{ rad} = 90°$ out of phase with the voltage

More About Capacitors in an AC Circuit

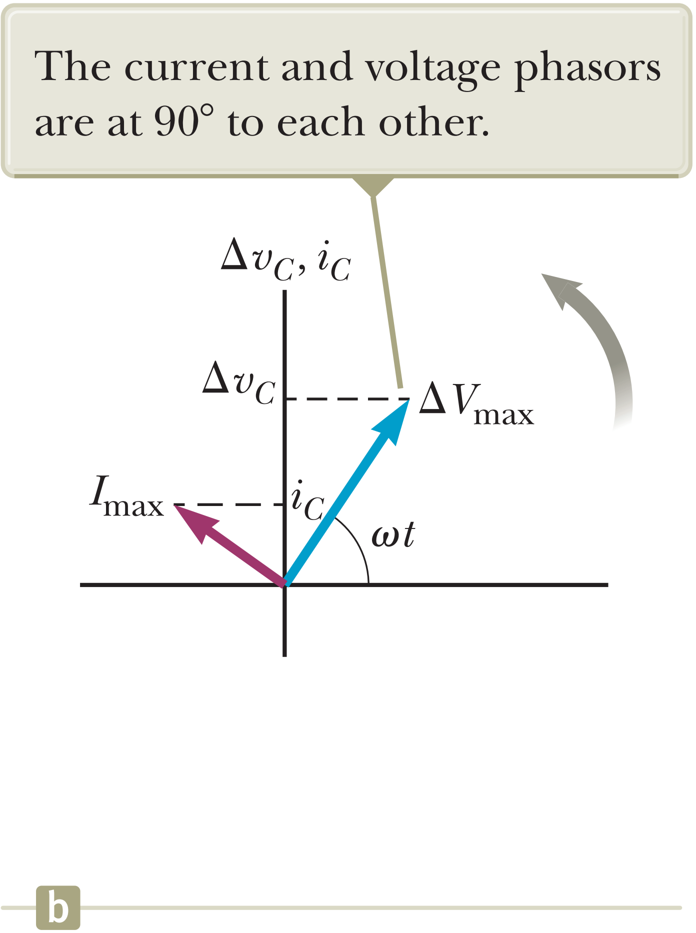
The current reaches its maximum value one quarter of a cycle sooner than the voltage reaches its maximum value.

The current leads the voltage by $90°$.



Phasor Diagram for Capacitor

The phasor diagram shows that for a sinusoidally applied voltage, the current always leads the voltage across a capacitor by $90°$.



Capacitive Reactance

The maximum current in the circuit occurs at $\cos \omega t = 1$ which gives

$$I_{\max} = \omega C \Delta V_{\max} = \frac{\Delta V_{\max}}{(1/\omega C)}$$

The impeding effect of a capacitor on the current in an AC circuit is called the **capacitive reactance** and is given by

$$X_C \equiv \frac{1}{\omega C} \text{ which gives } I_{\max} = \frac{\Delta V_{\max}}{X_C}$$

Voltage Across a Capacitor

The instantaneous voltage across the capacitor can be written as

$$\Delta v_C = \Delta V_{\max} \sin \omega t = I_{\max} X_C \sin \omega t$$

As the frequency of the voltage source increases, the capacitive reactance decreases and the maximum current increases.

As the frequency approaches zero, X_C approaches infinity and the current approaches zero.

- This would act like a DC voltage and the capacitor would act as an open circuit.

Problem 32.12:

An AC source with an output rms voltage of $36.0\ \mathrm{V}$ at a frequency of $60.0\ \mathrm{Hz}$ is connected across a $12.0\text{-}\mu\mathrm{F}$ capacitor. Find (a) the capacitive reactance, (b) the rms current, and (c) the maximum current in the circuit. (d) Does the capacitor have its maximum charge when the current has its maximum value? Explain.

(a) $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\ \mathrm{Hz})(12.0 \times 10^{-6}\ \mathrm{F})} = 221\Omega$

(b) $I_{\mathrm{rms}} = \frac{\Delta V_{C,\mathrm{rms}}}{X_C} = \frac{36.0\ \mathrm{V}}{221\Omega} = 0.163\ \mathrm{A}$

(c) $I_{\max} = \sqrt{2} I_{\mathrm{rms}} = 0.230\ \mathrm{A}$

(d) No. Current leads voltage, and thus charge, by $90°$ in a capacitor. The current reaches its maximum value one-quarter cycle before the voltage reaches its maximum value. From the definition of capacitance, the capacitor reaches its maximum charge when the voltage across it is also a maximum. Consequently, the maximum charge and the maximum current do not occur at the same time.

Problem 32.13:

What is the maximum current in a $2.20\text{-}\mu\text{F}$ capacitor when it is connected across (a) a North American electrical outlet having $\Delta V_{\text{rms}} = 120\ \text{V}$ and $f = 60.0\ \text{Hz}$ and (b) a European electrical outlet having $\Delta V_{\text{rms}} = 240\ \text{V}$ and $f = 50.0\ \text{Hz}$?

The maximum current in the capacitor is given by

$$I_{\max} = \sqrt{2} I_{\text{rms}} = \frac{\sqrt{2}\left(\Delta V_{\text{rms}}\right)}{X_C} = \sqrt{2}\left(\Delta V_{\text{rms}}\right) 2\pi f C$$

(a) For the North American electrical outlet,

$$I_{\max} = \sqrt{2}(120\ \text{V})2\pi(60.0/\text{s})\left(2.20 \times 10^{-6}\text{C/V}\right) = 141\ \text{mA}$$

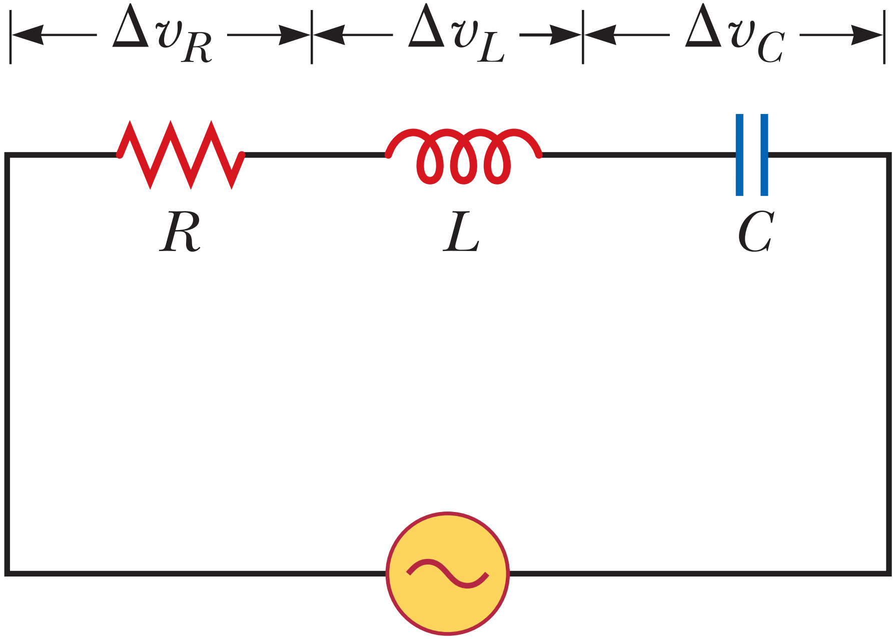
(b) For the European electrical outlet,

$$I_{\max} = \sqrt{2}(240\ \text{V})2\pi(50.0/\text{s})\left(2.20 \times 10^{-6}\ \text{F}\right) = 235\ \text{mA}$$

The *RLC* Series Circuit

The resistor, inductor, and capacitor can be combined in a circuit.

The current and the voltage in the circuit vary sinusoidally with time.



The *RLC* Series Circuit, cont.

The instantaneous voltage would be given by $\Delta v = \Delta V_{\max} \sin \omega t$.

The instantaneous current would be given by $i = I_{\max} \sin(\omega t - \phi)$.

- ϕ is the **phase angle** between the current and the applied voltage.

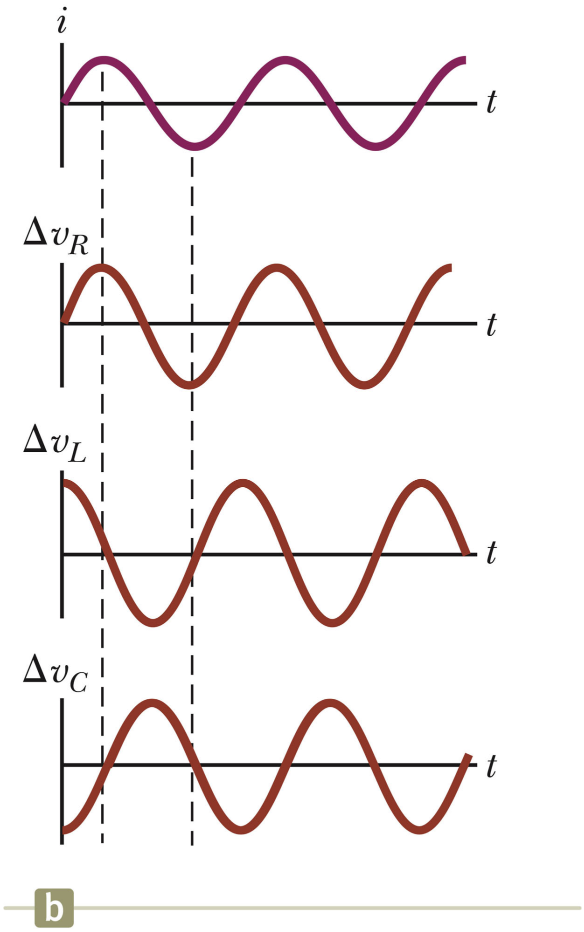
Since the elements are in series, the current at all points in the circuit has the same amplitude and phase.

i and *v* Phase Relationships – Graphical View

The instantaneous voltage across the resistor is in phase with the current.

The instantaneous voltage across the inductor leads the current by $90°$.

The instantaneous voltage across the capacitor lags the current by $90°$.



i and *v* Phase Relationships – Equations

The instantaneous voltage across each of the three circuit elements can be expressed as

$$\Delta v_R = I_{\max} R \sin \omega t = \Delta V_R \sin \omega t$$

$$\Delta v_L = I_{\max} X_L \sin \left(\omega t + \frac{\pi}{2} \right) = \Delta V_L \cos \omega t$$

$$\Delta v_C = I_{\max} X_C \sin \left(\omega t - \frac{\pi}{2} \right) = - \Delta V_C \cos \omega t$$

More About Voltage in RLC Circuits

ΔV_R is the maximum voltage across the resistor and $\Delta V_R = I_{\max} R$.

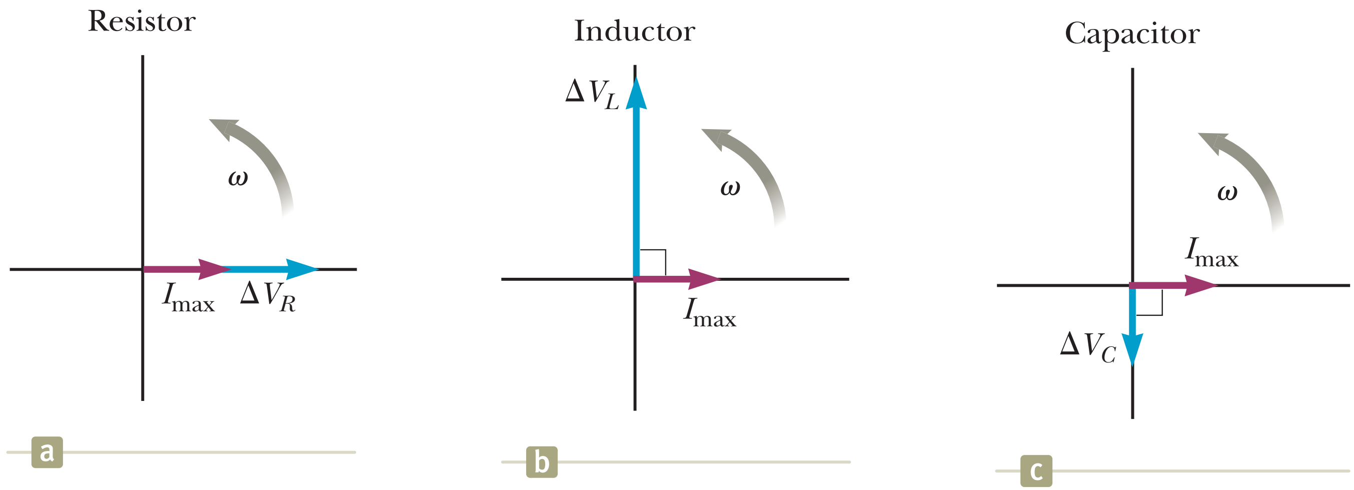
ΔV_L is the maximum voltage across the inductor and $\Delta V_L = I_{\max} X_L$.

ΔV_C is the maximum voltage across the capacitor and $\Delta V_C = I_{\max} X_C$.

The sum of these voltages must equal the voltage from the AC source.

Because of the different phase relationships with the current, they cannot be added directly.

Phasor Diagrams



To account for the different phases of the voltage drops, vector techniques are used.

Remember the phasors are rotating vectors

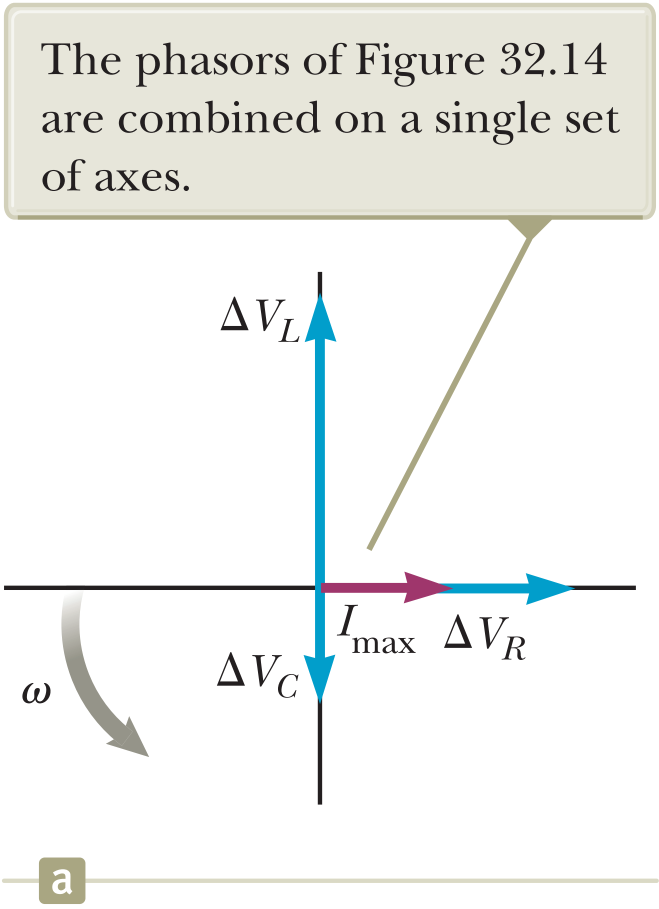
The phasors for the individual elements are shown.

Resulting Phasor Diagram

The individual phasor diagrams can be combined.

Here a single phasor I_{max} is used to represent the current in each element.

- In series, the current is the same in each element.



Vector Addition of the Phasor Diagram

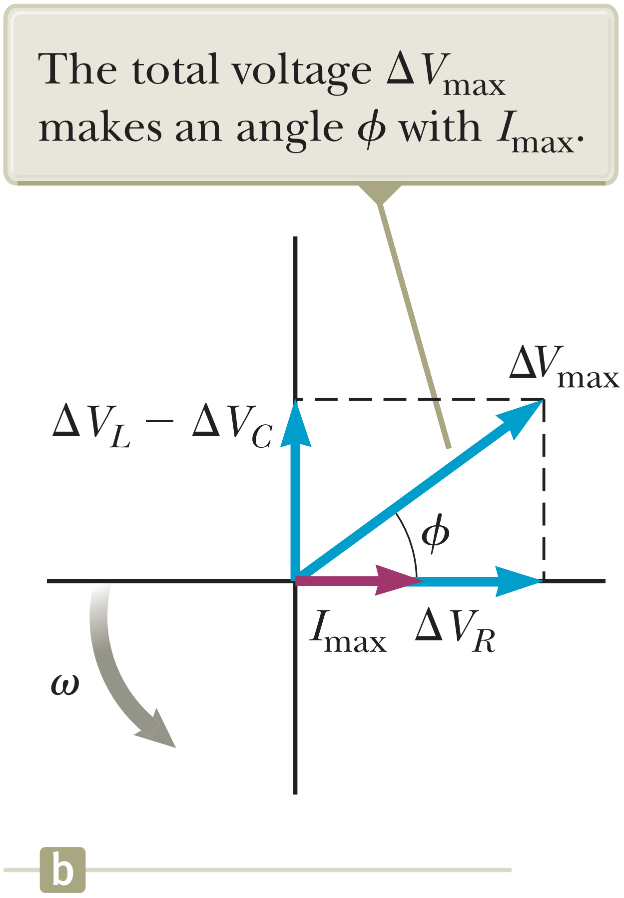
Vector addition is used to combine the voltage phasors.

ΔV_L and ΔV_C are in opposite directions, so they can be combined.

Their resultant is perpendicular to ΔV_R.

The resultant of all the individual voltages across the individual elements is ΔV_{max}.

- This resultant makes an angle of ϕ with the current phasor I_{max}.



Total Voltage in RLC Circuits

From the vector diagram, $\Delta V_{\max}$ can be calculated

$$\Delta V_{\max} = \sqrt{\Delta V_R^2 + \left(\Delta V_L - \Delta V_C\right)^2} = \sqrt{\left(I_{\max} R\right)^2 + \left(I_{\max} X_L - I_{\max} X_C\right)^2}$$

$$\Delta V_{\max} = I_{\max} \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

Impedance

The current in an *RLC* circuit is

$$I_{\max} = \frac{\Delta V_{\max}}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

Z is called the impedance of the circuit and it plays the role of resistance in the circuit, where

$$Z \equiv \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

- Impedance has units of ohms

Now the current is

$$I_{\max} = \frac{\Delta V_{\max}}{Z}$$

Phase Angle

The right triangle in the phasor diagram can be used to find the phase angle, ϕ.

$$\phi = \tan^{-1}\left(\frac{\Delta V_L - \Delta V_C}{\Delta V_R}\right) = \tan^{-1}\left(\frac{I_{\max}X_L - I_{\max}X_C}{I_{\max}R}\right)$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

The phase angle can be positive or negative and determines the nature of the circuit.

Determining the Nature of the Circuit

If ϕ is positive

- $X_L > X_C$ (which occurs at high frequencies)
- The current lags the applied voltage.
- The circuit is *more inductive than capacitive.*

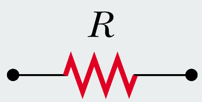
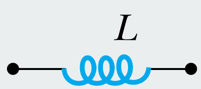
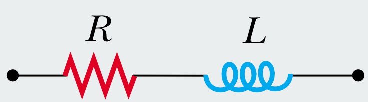
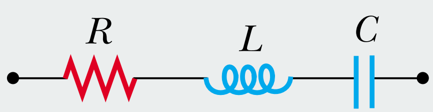
If ϕ is negative

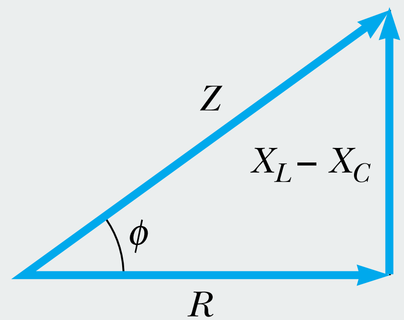
- $X_L < X_C$ (which occurs at low frequencies)
- The current leads the applied voltage.
- The circuit is *more capacitive than inductive.*

If ϕ is zero

- $X_L = X_C$
- The circuit is *purely resistive.*

Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
	R	$0°$
	X_C	$-90°$
	X_L	$+90°$
	$\sqrt{R^2 + X_C{}^2}$	Negative, between $-90°$ and $0°$
	$\sqrt{R^2 + X_L{}^2}$	Positive, between $0°$ and $90°$
	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$



[a] In each case, an AC voltage (not shown) is applied across the elements.

Example 32.04: Analyzing a Series RLC Circuit

A series RLC circuit has $R = 425\Omega$, $L = 1.25\text{H}$, and $C = 3.50\ \mu\text{F}$. It is connected to an AC source with $f = 60.0\ \text{Hz}$ and $\Delta V_{\text{max}} = 150\ \text{V}$.

(A) Determine the inductive reactance, the capacitive reactance, and the impedance of the circuit.

(B) Find the maximum current in the circuit.

(C) Find the phase angle between the current and voltage.

(D) Find the maximum voltage across each element.

(E) What replacement value of L should an engineer analyzing the circuit choose such that the current leads the applied voltage by $30.0°$ rather than $34.0°$? All other values in the circuit stay the same.

(A)

$$\omega = 2\pi f = 2\pi(60.0\ \text{Hz}) = 377\ \text{s}^{-1}$$

$$X_L = \omega L = \left(377\ \text{s}^{-1}\right)(1.25\text{H}) = 471\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(377\ \text{s}^{-1}\right)\left(3.50 \times 10^{-6}\ \text{F}\right)} = 758\Omega$$

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

$$= \sqrt{(425\Omega)^2 + (471\Omega - 758\Omega)^2} = 513\Omega$$

(B) $$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{Z} = \frac{150\ \text{V}}{513\Omega} = 0.293\ \text{A}$$

(C) $$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{471\Omega - 758\Omega}{425\Omega}\right) = -34.0°$$

(D)

$$\Delta V_R = I_{\text{max}} R = (0.293\ \text{A})(425\Omega) = 124\ \text{V}$$

$$\Delta V_L = I_{\text{max}} X_L = (0.293\ \text{A})(471\Omega) = 138\ \text{V}$$

$$\Delta V_C = I_{\text{max}} X_C = (0.293\ \text{A})(758\Omega) = 222\ \text{V}$$

(E)

$$X_L = X_C + R\tan\phi$$

$$\omega L = \frac{1}{\omega C} + R\tan\phi$$

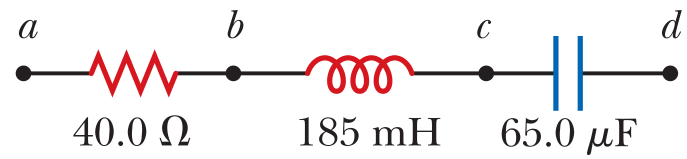
$$L = \frac{1}{\omega}\left(\frac{1}{\omega C} + R\tan\phi\right)$$

$$L = \frac{1}{\left(377\ \text{s}^{-1}\right)}\left[\frac{1}{\left(377\ \text{s}^{-1}\right)\left(3.50 \times 10^{-6}\ \text{F}\right)} + (425\Omega)\tan(-30.0°)\right]$$

$$L = 1.36\text{H}$$

Problem 32.16:

An AC source with $\Delta V_{\max} = 150\ \text{V}$ and $f = 50.0\ \text{Hz}$ is connected between points a and d in the figure. Calculate the maximum voltages between (a) points a and b, (b) points b and c, (c) points c and d, and (d) points b and d.



We first determine the reactances of the circuit. The capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(50.0)\left(65.0\times 10^{-6}\ \text{F}\right)} = 49.0\Omega$$

the inductive reactance is,

$$X_L = \omega L = 2\pi(50.0)\left(185\times 10^{-3}\text{H}\right) = 58.1\Omega$$

and the impedance Z of the circuit is

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{(40.0\Omega)^2 + (58.1\Omega - 49.0\Omega)^2}$$
$$= 41.0\Omega$$

The current in the circuit is then

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{150\ \text{V}}{41.0\Omega} = 3.66\ \text{A}$$

(a) The maximum voltage between points a and b is the potential drop across the resistor:

$$\Delta V_R = I_{\max} R = (3.66\ \text{A})(40.0\Omega) = 146\ \text{V}$$

(b) The maximum voltage between points b and c is the potential drop across the coil:

$$\Delta V_L = I_{\max} X_L = (3.66\ \text{A})(58.1\Omega) = 212.5\ \text{V} = 212\ \text{V}$$

(c) The maximum voltage between points c and d is the potential drop across the capacitor:

$$\Delta V_C = I_{\max} X_C = (3.66\ \text{A})(49.0\Omega) = 179.1\ \text{V} = 179\ \text{V}$$

(d) The potential drop between points b and d is

$$\Delta V_L - \Delta V_C = 212.5\ \text{V} - 179.1\ \text{V} = 33.4\ \text{V}$$

Problem 32.19:

An RLC circuit consists of a $150\text{-}\Omega$ resistor, a $21.0\text{-}\mu\text{F}$ capacitor, and a 460-mH inductor connected in series with a 120-V, 60.0-Hz power supply. (a) What is the phase angle between the current and the applied voltage? (b) Which reaches its maximum earlier, the current or the voltage?

The reactance of the inductor is

$$X_L = 2\pi f L = 2\pi\left(60.0\ \text{s}^{-1}\right)(0.460\text{H}) = 173\Omega$$

The reactance of the capacitor is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi\left(60.0\ \text{s}^{-1}\right)\left(21.0\times 10^{-6}\ \text{F}\right)} = 126\Omega$$

(a) $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{173\Omega - 126\Omega}{150\Omega}\right) = 17.4°$

(b) Since $X_L > X_C$, ϕ is positive, so voltage leads the current. This means that the power-supply or total voltage goes through each maximum, zero-crossing, and minimum earlier in time than the current does.

Problem 32.20:

A $60.0\text{-}\Omega$ resistor is connected in series with a $30.0\text{-}\mu\text{F}$ capacitor and a source whose maximum voltage is 120 V, operating at 60.0 Hz. Find (a) the capacitive reactance of the circuit, (b) the impedance of the circuit, and (c) the maximum current in the circuit. (d) Does the voltage lead or lag the current? (e) How will adding an inductor in series with the existing resistor and capacitor affect the current? Explain.

(a) The capacitive reactance of the circuit is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\text{ Hz})\left(30.0 \times 10^{-6}\text{ F}\right)} = 88.4\Omega$$

(b) The impedance of the circuit is

$$Z = \sqrt{R^2 + \left(0 - X_C\right)^2} = \sqrt{R^2 + X_C^2} = \sqrt{(60.0\Omega)^2 + (88.4\Omega)^2}$$
$$= 107\Omega$$

(c) $I_{\max} = \dfrac{\Delta V\text{max}}{Z} = \dfrac{1.20 \times 10^2 V}{107\Omega} = 1.12\text{ A}$

(d) The phase angle in this RC circuit is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 88.4\Omega}{60.0\Omega}\right) = -55.8^\circ$$

Since $\phi < 0$, the voltage lags behind the current by 55.8°.

(e) Adding an inductor will change the impedance, and hence the current in the circuit. The current could be larger or smaller, depending on the inductance added. The largest current would result when the inductive reactance equals the capacitive reactance, the impedance has its minimum value, equal to 60.0Ω, and the current in the circuit is

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{\Delta V_{\max}}{R} = \frac{1.20 \times 10^2\text{ V}}{60.0\Omega} = 2.00\text{ A}$$

Power in an AC Circuit

The average power delivered by the AC source is converted to internal energy in the resistor.

- $P_{\text{avg}} = \frac{1}{2} I_{\max} \Delta V_{\max} \cos\phi = I_{\text{rms}} \Delta V_{\text{rms}} \cos\phi$
- $\cos\phi$ is called the power factor of the circuit

We can also find the average power in terms of R.

- $P_{\text{avg}} = I_{\text{rms}}^2 R$

When the load is purely resistive, $\phi = 0$ and $\cos\phi = 1$

- $P_{\text{avg}} = I_{\text{rms}} \Delta V_{\text{rms}}$

Power in an AC Circuit, cont.

The average power delivered by the source is converted to internal energy in the resistor.

No power losses are associated with pure capacitors and pure inductors in an AC circuit.

- In a capacitor, during one-half of a cycle, energy is stored and during the other half the energy is returned to the circuit and no power losses occur in the capacitor.
- In an inductor, the source does work against the back emf of the inductor and energy is stored in the inductor, but when the current begins to decrease in the circuit, the energy is returned to the circuit.

The power delivered by an AC circuit depends on the phase.

Some applications include using capacitors to shift the phase to heavy motors or other inductive loads so that excessively high voltages are not needed.

Example 32.05: Average Power in an RLC Series Circuit

Calculate the average power delivered to the series RLC circuit described in Example 32.4.

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\text{max}}}{\sqrt{2}} = \frac{150\ \text{V}}{\sqrt{2}} = 106\ \text{V}$$

$$I_{\text{rms}} = \frac{I_{\text{max}}}{\sqrt{2}} = \frac{0.293\ \text{A}}{\sqrt{2}} = 0.207\ \text{A}$$

$$P_{\text{avg}} = I_{\text{rms}} V_{\text{rms}} \cos\phi = (0.207\ \text{A})(106\ \text{V})\cos(-34.0^\circ)$$
$$= 18.2\ \text{W}$$

Problem 32.21:

A series RLC circuit has a resistance of 45.0Ω and an impedance of 75.0Ω. What average power is delivered to this circuit when $\Delta V_{\mathrm{rms}} = 210\ \mathrm{V}$?

1 From the definition of impedance, $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$, we have

$$\left(X_L - X_C\right) = \sqrt{Z^2 - R^2}$$

Substituting numerical values,

$$\left(X_L - X_C\right) = \sqrt{(75.0\Omega)^2 - (45.0\Omega)^2} = 60.0\Omega$$

The phase angle of the circuit is then

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{60.0\Omega}{45.0\Omega}\right) = 53.1^\circ$$

The rms current in the circuit is

$$I_{\mathrm{rms}} = \frac{\Delta V_{\mathrm{rms}}}{Z} = \frac{210\ \mathrm{V}}{75.0\Omega} = 2.80\ \mathrm{A}$$

Therefore, the power delivered to the circuit is

$$P = \left(\Delta V_{\mathrm{rms}}\right) I_{\mathrm{rms}} \cos\phi = (210\ \mathrm{V})(2.80\ \mathrm{A})\cos\left(53.1^\circ\right) = 353\ \mathrm{W}$$

Problem 32.23:

A series RLC circuit has a resistance of 22.0Ω and an impedance of 80.0Ω. If the rms voltage applied to the circuit is $160\ \mathrm{V}$, what average power is delivered to the circuit?

The rms current in the circuit is

$$I_{\mathrm{rms}} = \frac{\Delta V_{\mathrm{rms}}}{\mathrm{Z}} = \frac{160\ \mathrm{V}}{80.0\Omega} = 2.00\ \mathrm{A}$$

and the average power delivered to the circuit is

$$P_{\mathrm{avg}} = I_{\mathrm{rms}}^2 R = (2.00\ \mathrm{A})^2(22.0\Omega) = 88.0\ \mathrm{W}$$

Problem 32.24:

An AC voltage of the form $\Delta v = 90.0 \sin 350t$, where Δv is in volts and t is in seconds, is applied to a series RLC circuit. If $R = 50.0\Omega$, $C = 25.0\mu\ \text{F}$, and $L = 0.200\text{H}$, find (a) the impedance of the circuit, (b) the rms current in the circuit, and (c) the average power delivered to the circuit.

Given $v = \Delta V_{\text{max}} \sin(\omega t) = (90.0\ \text{V})\sin(350t)$, observe that $\Delta V_{\text{max}} = 90.0\ \text{V}$and $\omega = 350\text{rad/s}$. Also, the net reactance is

$$X_L - X_C = 2\pi f L - \frac{1}{2\pi f C} = \omega L - \frac{1}{\omega C}$$

(a) To find the impedance, we first compute

$$X_L - X_C = \omega L - \frac{1}{\omega C}$$

$$= (350\text{rad/s})(0.200\text{H}) - \frac{1}{(350\text{rad/s})\left(25.0 \times 10^{-6}\ \text{F}\right)} = -44.3\Omega$$

so the impedance is

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{(50.0\Omega)^2 + (-44.3\Omega)^2} = 66.8\Omega$$

(b) The rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{max}}/\sqrt{2}}{Z} = \frac{90.0\ \text{V}}{\sqrt{2}(66.8\Omega)} = 0.953\ \text{A}$$

(c) The phase difference between the applied voltage and the current is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{-44.3\Omega}{50.0\Omega}\right) = -41.5^\circ$$

so the average power delivered to the circuit is

$$P_{\text{avg}} = I_{\text{rms}}\Delta V_{\text{rms}} \cos\phi = I_{\text{rms}}\left(\frac{\Delta V_{\text{max}}}{\sqrt{2}}\right)\cos\phi$$

$$= (0.953\ \text{A})\left(\frac{90.0\ \text{V}}{\sqrt{2}}\right)\cos\left(-41.5^\circ\right) = 45.4\ \text{W}$$

Resonance in an AC Circuit

Resonance occurs at the frequency ω_0 where the current has its maximum value.

- To achieve maximum current, the impedance must have a minimum value.
- This occurs when $X_L = X_C$
- Solving for the frequency in $\omega_0 L = 1/\omega_0 C$ gives

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The resonance frequency also corresponds to the natural frequency of oscillation of an *LC* circuit.

The rms current has a maximum value when the frequency of the applied voltage matches the natural oscillator frequency.

At the resonance frequency, the current is in phase with the applied voltage.

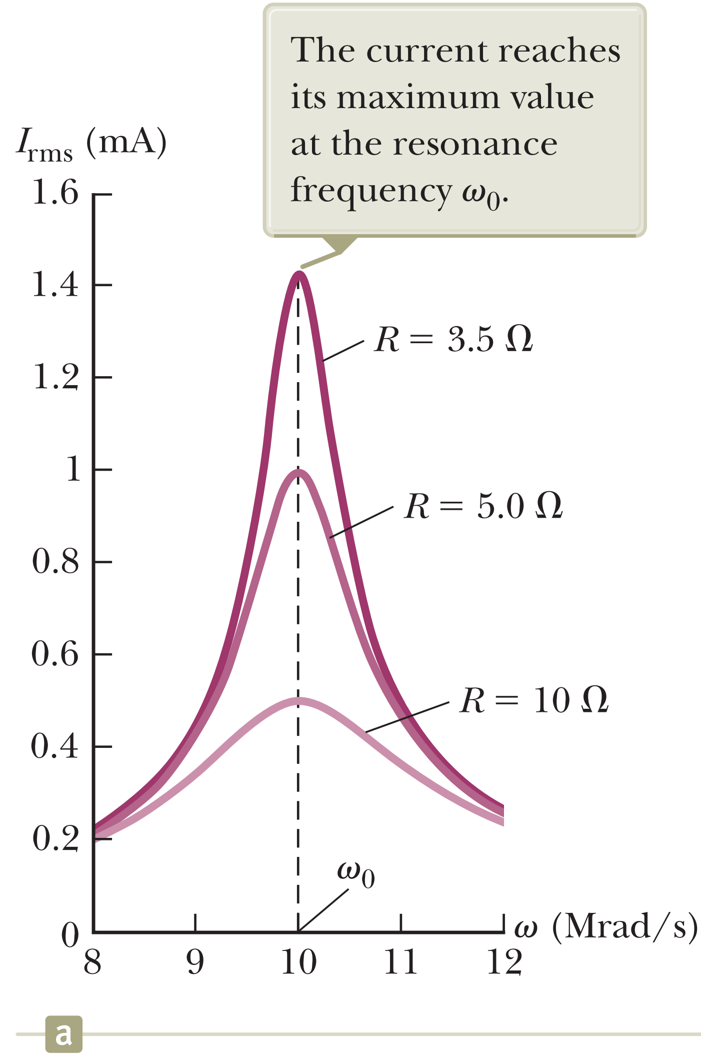
Resonance, cont.

Resonance occurs at the same frequency regardless of the value of R.

As R decreases, the curve becomes narrower and taller.

Theoretically, if $R = 0$ the current would be infinite at resonance.

- Real circuits always have some resistance.



Example 32.06: A Resonating Series RLC Circuit

Consider a series RLC circuit for which $R = 150\Omega$, $L = 20.0\text{mH}$, $\Delta V_{\text{rms}} = 20.0\ \text{V}$, and $\omega = 5000\ \text{s}^{-1}$. Determine the value of the capacitance for which the current is a maximum.

$$\omega_0 = \frac{1}{\sqrt{LC}} \rightarrow C = \frac{1}{\omega_0^2 L}$$

$$C = \frac{1}{\left(5.00 \times 10^3\ \text{s}^{-1}\right)^2 \left(20.0 \times 10^{-3}\text{H}\right)} = 2.00\mu\ \text{F}$$

Problem 32.25:

The LC circuit of a radar transmitter oscillates at $9.00\ \mathrm{GHz}$. (a) What inductance is required for the circuit to resonate at this frequency if its capacitance is $2.00\ \mathrm{pF}$? (b) What is the inductive reactance of the circuit at this frequency?

(a) The resonance frequency of a RLC circuit is $f_0 = 1/2\pi\sqrt{LC}$. Thus, the inductance is

$$L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 \left(9.00 \times 10^9\ \mathrm{Hz}\right)^2 \left(2.00 \times 10^{-12}\ \mathrm{F}\right)}$$

$$= 1.56 \times 10^{-10}\mathrm{H} = 156\mathrm{pH}$$

(b) At resonance,

$$X_L = X_C = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi \left(9.00 \times 10^9\ \mathrm{Hz}\right) \left(2.00 \times 10^{-12}\ \mathrm{F}\right)}$$

$$= 8.84\Omega$$