Chapter 30

Faraday’s Law

Induced Fields

Magnetic fields may vary in time.

Experiments conducted in 1831 showed that an emf can be induced in a circuit by a changing magnetic field.

- Experiments were done by Michael Faraday and Joseph Henry.

The results of these experiments led to *Faraday's Law of Induction.*

An *induced current* is produced by a changing magnetic field.

There is an *induced emf* associated with the induced current.

A current can be produced without a battery present in the circuit.

Faraday's law of induction describes the induced emf.

Michael Faraday

1791 – 1867

British physicist and chemist

Great experimental scientist

Contributions to early electricity include:

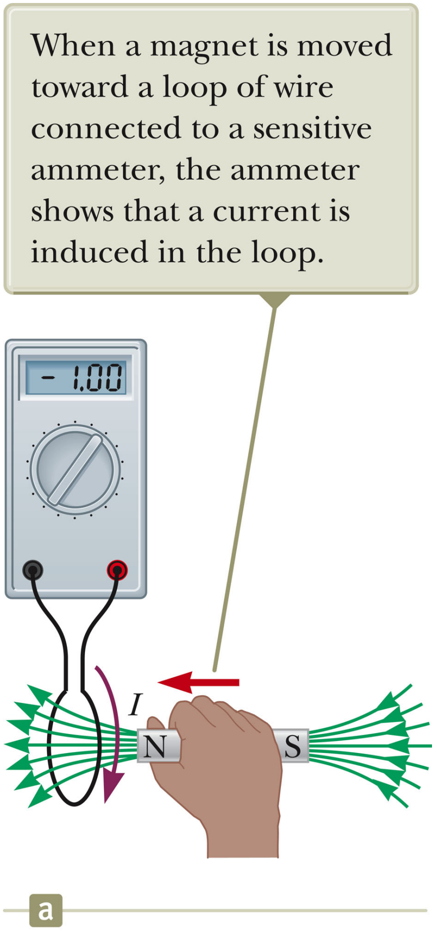
- Invention of motor, generator, and transformer
- Electromagnetic induction
- Laws of electrolysis

EMF Produced by a Changing Magnetic Field, 1

A loop of wire is connected to a sensitive ammeter.

When a magnet is moved toward the loop, the ammeter deflects.

- The direction was arbitrarily chosen to be negative.

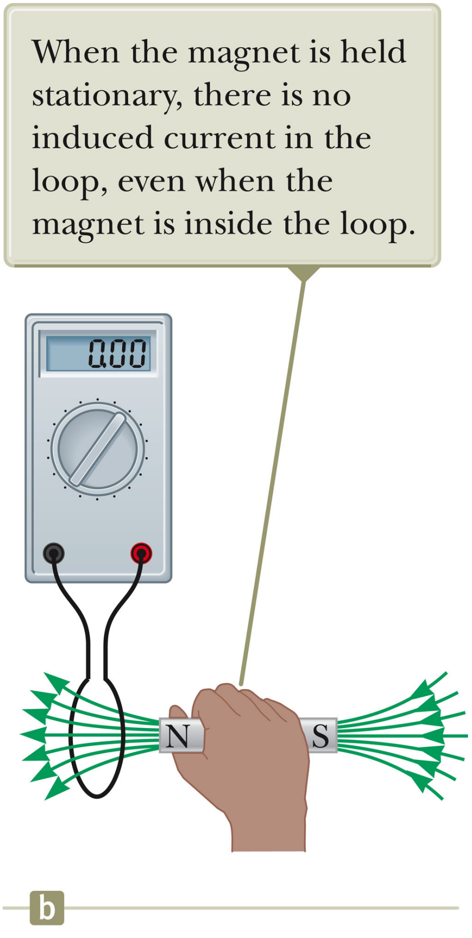


EMF Produced by a Changing Magnetic Field, 2

When the magnet is held stationary, there is no deflection of the ammeter.

Therefore, there is no induced current.

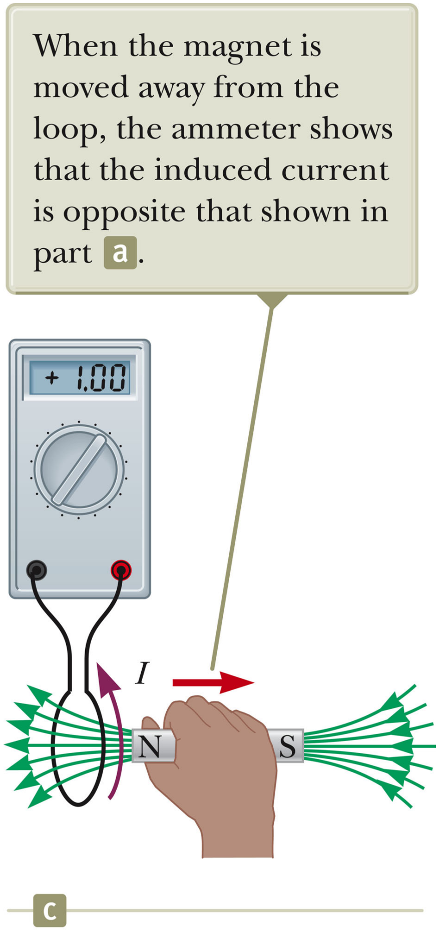
- Even though the magnet is in the loop



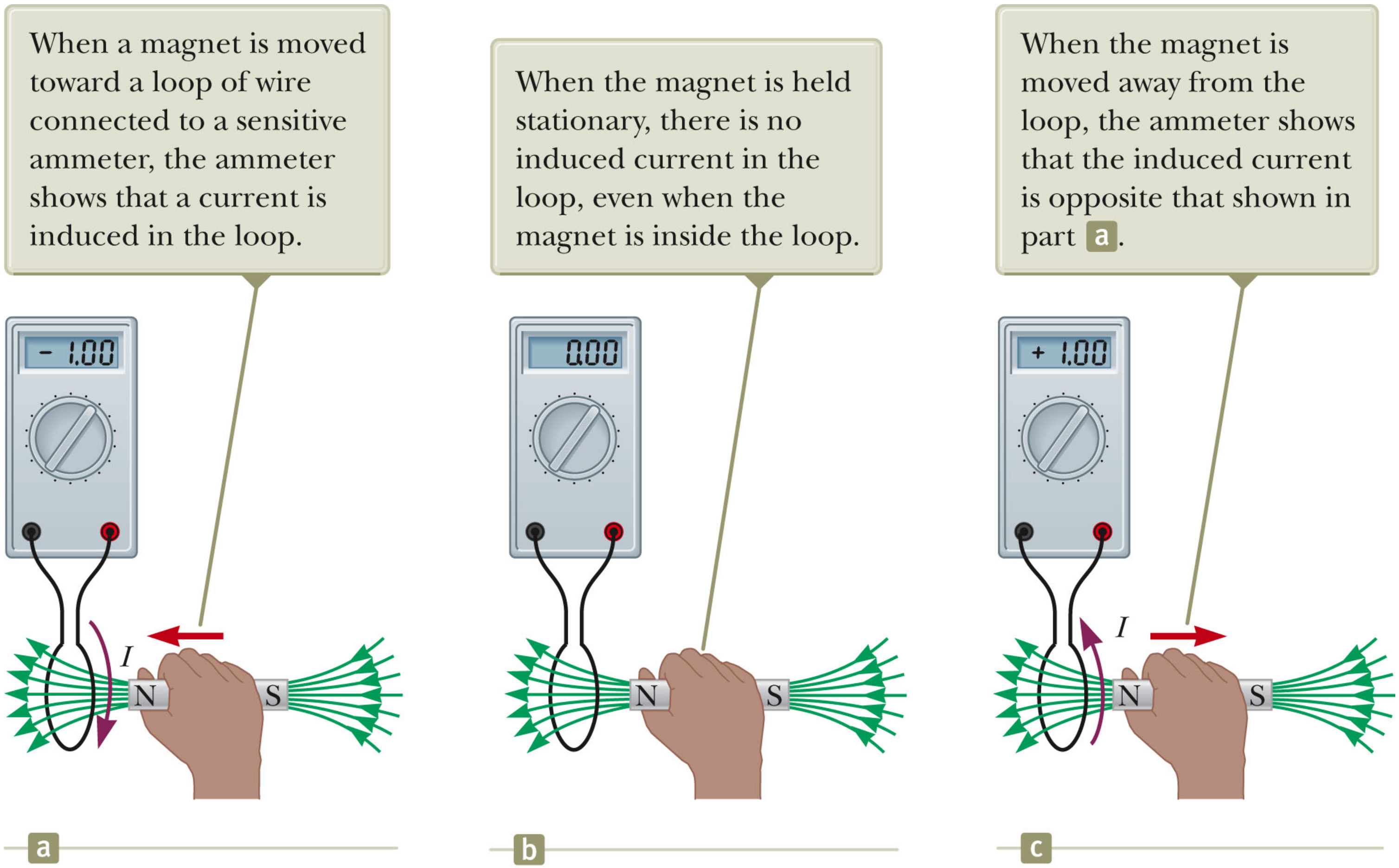
EMF Produced by a Changing Magnetic Field, 3

The magnet is moved away from the loop.

The ammeter deflects in the opposite direction.



Induced Current Experiment, Summary



EMF Produced by a Changing Magnetic Field, Summary

The ammeter deflects when the magnet is moving toward or away from the loop.

The ammeter also deflects when the loop is moved toward or away from the magnet.

Therefore, the loop detects that the magnet is moving relative to it.

- We relate this detection to a change in the magnetic field.
- This is the induced current that is produced by an induced emf.

Faraday's Experiment – Set Up

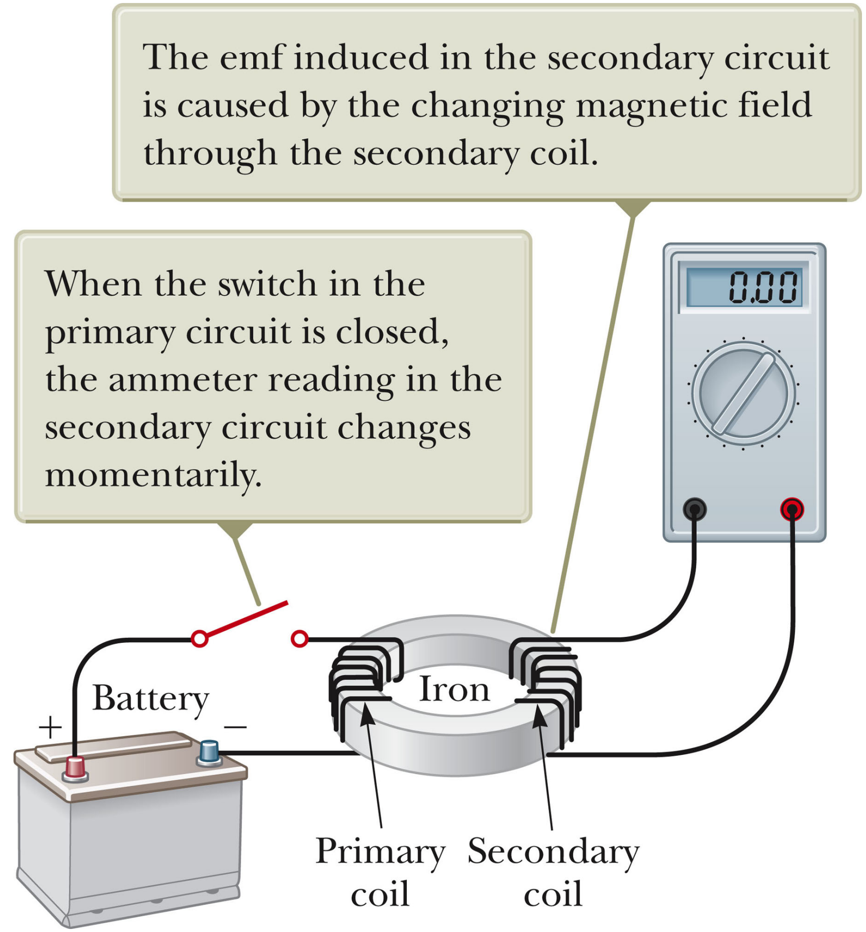
A primary coil is connected to a switch and a battery.

The wire is wrapped around an iron ring.

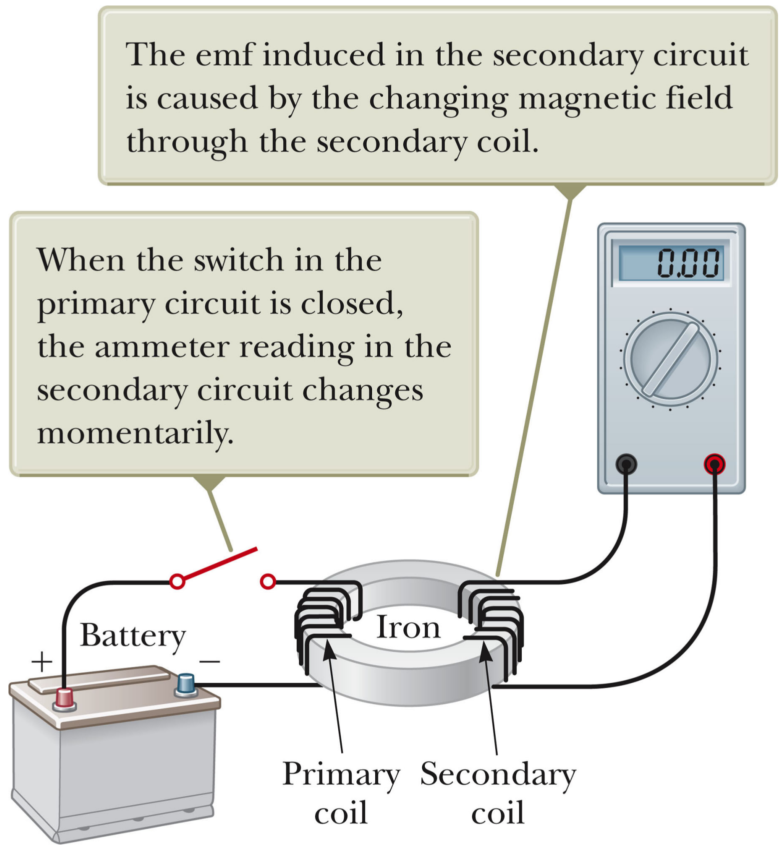
A secondary coil is also wrapped around the iron ring.

There is no battery present in the secondary coil.

The secondary coil is not directly connected to the primary coil.



Faraday's Experiment



Close the switch and observe the current readings given by the ammeter.

Faraday’s Experiment – Findings

At the instant the switch is closed, the ammeter changes from zero in one direction and then returns to zero.

When the switch is opened, the ammeter changes in the opposite direction and then returns to zero.

The ammeter reads zero when there is a steady current or when there is no current in the primary circuit.

Faraday's Experiment – Conclusions

An electric current can be induced in a loop by a changing magnetic field.

- This would be the current in the secondary circuit of this experimental set-up.

The induced current exists only while the magnetic field through the loop is changing.

This is generally expressed as: ***an induced emf is produced in the loop by the changing magnetic field.***

- The actual existence of the magnetic flux is not sufficient to produce the induced emf, the flux must be changing.

Faraday's Law of Induction – Statements

The emf induced in a circuit is directly proportional to the time rate of change of the magnetic flux through the circuit.

Mathematically,

$$\boldsymbol{\varepsilon} = -\frac{d\Phi_B}{dt}$$

Remember Φ_B is the magnetic flux through the circuit and is found by

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

If the circuit consists of N loops, all of the same area, and if Φ_B is the flux through one loop, an emf is induced in every loop and Faraday's law becomes

$$\boldsymbol{\varepsilon} = -N\frac{d\Phi_B}{dt}$$

Faraday's Law – Example

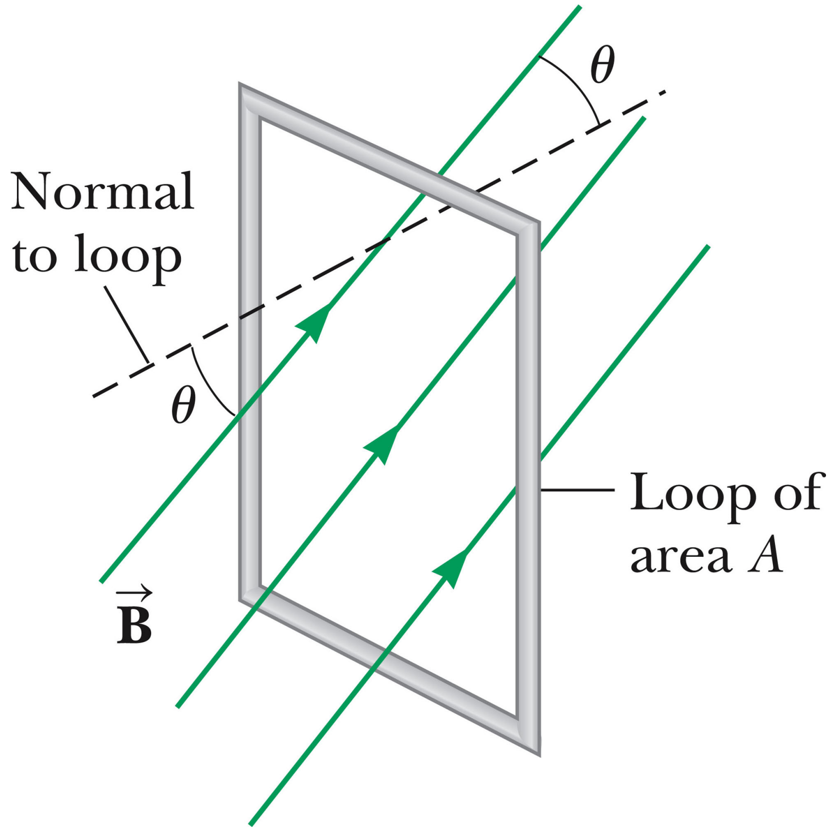
Assume a loop enclosing an area A lies in a uniform magnetic field.

The magnetic flux through the loop is

$$\Phi_B = BA\cos\theta\,.$$

The induced emf is

$$\varepsilon = -\,d/dt(BA\cos\theta).$$



Ways of Inducing an emf

The magnitude of the magnetic field can change with time.

The area enclosed by the loop can change with time.

The angle between the magnetic field and the normal to the loop can change with time.

Any combination of the above can occur.

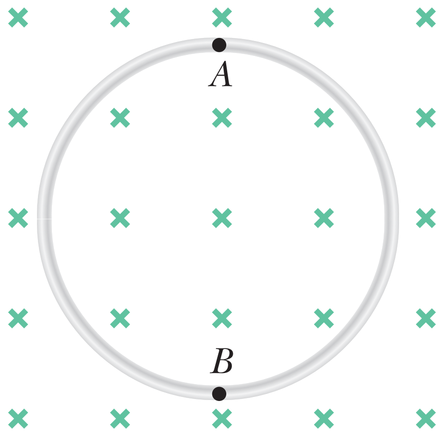
Problem 30.1:

A circular loop of wire of radius $12.0\,\text{cm}$ is placed in a magnetic field directed perpendicular to the plane of the loop as in the figure. If the field decreases at the rate of $0.0500\ \text{T/s}$ in some time interval, find the magnitude of the emf induced in the loop during this interval.

The magnitude of the induced emf in the coil is

$$|\boldsymbol{\varepsilon}| = \frac{\left|\Delta\Phi_B\right|}{\Delta t} = \left(\frac{\Delta B}{\Delta t}\right)A = (0.0500\ \text{T/s})\left[\pi(0.120\ \text{m})^2\right]$$

$$= 2.26\times10^{-3}\ \text{V} = 2.26\text{mV}$$



Example 30.1: Inducing an emf in a Coil

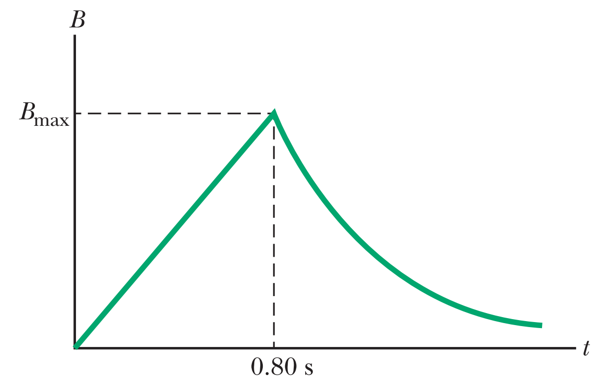
A coil consists of 200 turns of wire. Each turn is a square of side $d = 18\text{ cm}$, and a uniform magnetic field directed perpendicular to the plane of the coil is turned on. The figure shows the behavior of the magnitude of the magnetic field with time. From $t = 0$ to $t = 0.80\text{ s}$, the field changes linearly from 0 to 0.50 T . After $t = 0.80\text{ s}$, the magnitude of the field decays in time according to the expression $B = B_{\max}e^{-at}$, where a is some constant and $B_{\max} = 0.50\text{ T}$.

(A) What is the magnitude of the induced emf in the coil between $t = 0$ and $t = 0.80\text{ s}$?

(B) What is the magnitude of the induced emf in the coil after $t = 0.80\text{ s}$?

(A) $$|\boldsymbol{\varepsilon}| = N\frac{\Delta\Phi_B}{\Delta t} = N\frac{\Delta(BA)}{\Delta t} = NA\frac{\Delta B}{\Delta t} = Nd^2\frac{B_f - B_i}{\Delta t}$$

$$|\boldsymbol{\varepsilon}| = (200)(0.18\text{ m})^2\frac{(0.50\text{ T} - 0)}{0.80\text{ s}} = 4.0\text{ V}$$



(B)

$$\boldsymbol{\varepsilon} = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}\left(AB_{\max}e^{-at}\right) = -NAB_{\max}\frac{d}{dt}e^{-at} = aNd^2B_{\max}e^{-at}$$

$$\boldsymbol{\varepsilon} = a(200)(0.18\text{ m})^2(0.50\text{ T})e^{-at} = 3.2ae^{-at}$$

Problem 30.4:

A long solenoid has $n = 400$ turns per meter and carries a current given by $I = 30.0\left(1 - e^{-1.60t}\right)$, where I is in amperes and t is in seconds. Inside the solenoid and coaxial with it is a coil that has a radius of $R = 6.00$ cm and consists of a total of $N = 250$ turns of fine wire. What emf is induced in the coil by the changing current?

The solenoid creates a magnetic field

$$B = \mu_0 n I = \left(4\pi \times 10^{-7}\ \text{N/A}^2\right)(400\ \text{turns /m})(30.0\ \text{A})\left(1 - e^{-1.60t}\right)$$

$$B = \left(1.51 \times 10^{-2}\ \text{N/m}\cdot\ \text{A}\right)\left(1 - e^{-1.60t}\right)$$

The magnetic flux through one turn of the flat coil is

$$\Phi_B = B\int dA = B\left(\pi R^2\right)$$

$$= \left(1.51 \times 10^{-2}\ \text{N/m}\cdot\ \text{A}\right)\left(1 - e^{-1.60t}\right)\left[\pi(0.0600\ \text{m})^2\right]$$

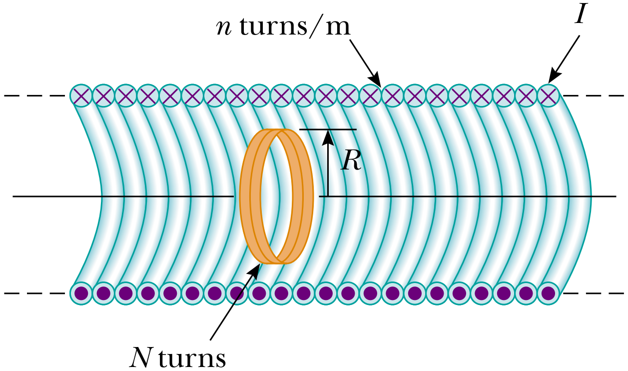
$$= \left(1.71 \times 10^{-4}\ \text{N/m}\cdot\ \text{A}\right)\left(1 - e^{-1.60t}\right)$$

The emf generated in the N-turn coil is

$$\varepsilon = -(250)\left(1.71 \times 10^{-4}\frac{\text{N}\cdot\ \text{m}}{\text{A}}\right)\frac{d\left(1 - e^{-1.60t}\right)}{dt}$$

$$= -\left(0.0426\frac{\text{N}\cdot\ \text{m}}{\text{A}}\right)\left(1.60\ \text{s}^{-1}\right)e^{-1.60t}$$

$\varepsilon = 68.2e^{-1.60t}$, where t is in seconds and ε is in mV .



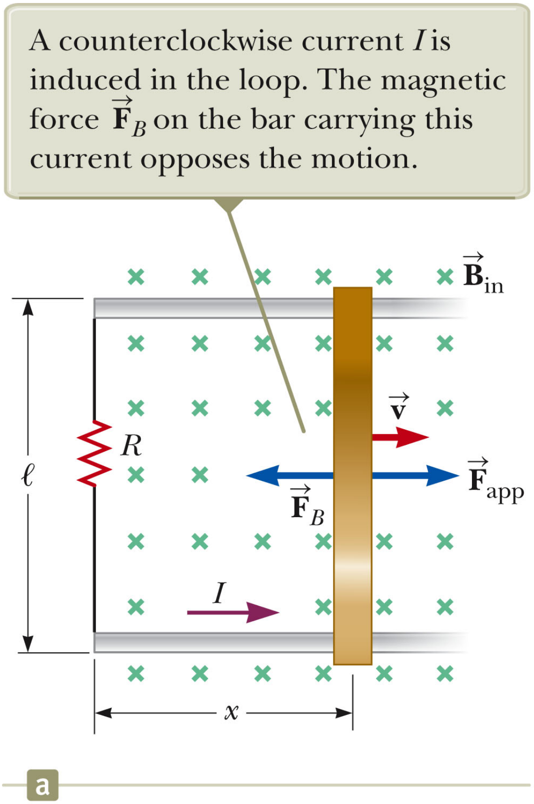
Motional emf

A motional emf is the emf induced in a conductor moving through a constant magnetic field.

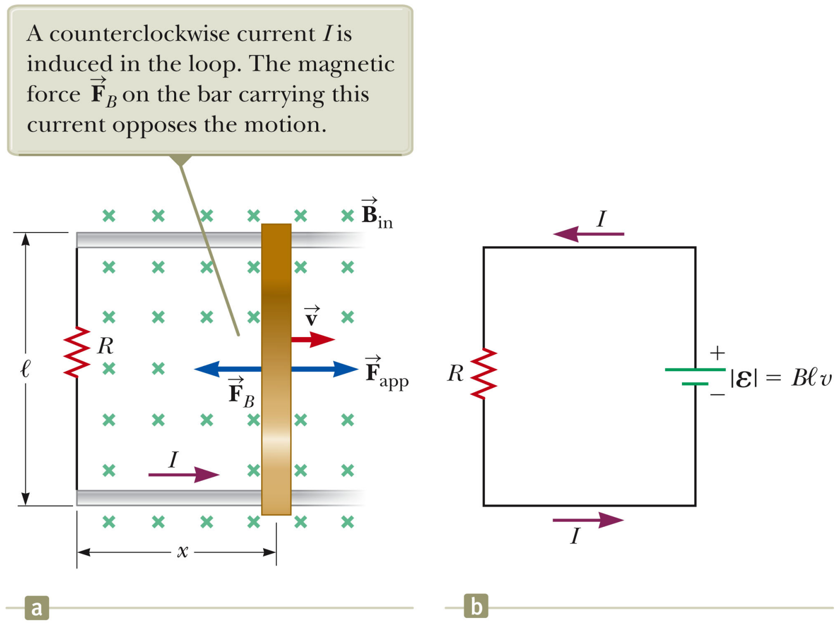
The electrons in the conductor experience a force,

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

that is directed along ℓ.



Sliding Conducting Bar



A conducting bar moving through a uniform field and the equivalent circuit diagram.

Assume the bar has zero resistance.

The stationary part of the circuit has a resistance R.

Sliding Conducting Bar, cont.

The induced emf is

$$\boldsymbol{\varepsilon} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell\frac{dx}{dt} = -B\ell v$$

Since the resistance in the circuit is R, the current is

$$I = \frac{|\boldsymbol{\varepsilon}|}{R} = \frac{B\ell v}{R}$$

Sliding Conducting Bar, Energy Considerations

The applied force does work on the conducting bar.

- Model the circuit as a nonisolated system.

This moves the charges through a magnetic field and establishes a current.

The change in energy of the system during some time interval must be equal to the transfer of energy into the system by work.

The power input is equal to the rate at which energy is delivered to the resistor.

$$P_{\text{app}} = F_{\text{app}} v = F_B v = (I\ell B)v = I(B\ell v) = I\boldsymbol{\varepsilon} = I(IR) = I^2 R = -P_{\text{elec}}$$

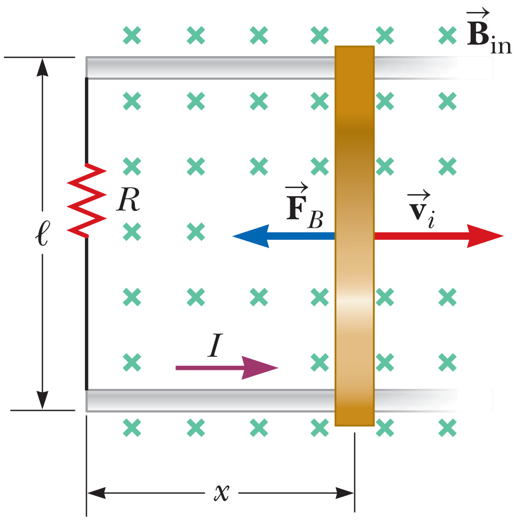
Problem 30.15:

A conducting bar of length ℓ moves to the right on two frictionless rails as shown in the figure. A uniform magnetic field directed into the page has a magnitude of $0.300\ \mathrm{T}$. Assume $R = 9.00\ \Omega$ and $\ell = 0.350$ m. (a) At what constant speed should the bar move to produce an $8.50 - \mathrm{mA}$ current in the resistor?

(b) What is the direction of the induced current?

(c) At what rate is energy delivered to the resistor?

(d) Explain the origin of the energy being delivered to the resistor.



(a) We know that $|\varepsilon| = B\ell v$ then

$$v = \frac{\varepsilon}{B\ell} = \frac{IR}{B\ell} = \frac{(8.50 \times 10^{-3}\ \mathrm{A})(9.00\Omega)}{(0.300\ \mathrm{T})(0.350\ \mathrm{m})} = 0.729\ \mathrm{m/s}$$

(b) counterclockwise.

(c)
$$P = I^2 R = \left(8.50 \times 10^{-3}\ \mathrm{A}\right)^2 (9.00\Omega)$$
$$= 6.50 \times 10^{-4}\ \mathrm{W} = 0.650\ \mathrm{mW}$$

(d) Work is being done by the external force, which is transformed into internal energy in the resistor.

Example 30.2: Magnetic Force Acting on a Sliding Bar

The conducting bar illustrated in the figure moves on two frictionless, parallel rails in the presence of a uniform magnetic field directed into the page. The bar has mass m, and its length is ℓ. The bar is given an initial velocity $\vec{\mathbf{v}}_i$ to the right and is released at $t = 0$.

(A) Using Newton's laws, find the speed of the bar as a function of time after it is released.

$$F_x = ma \rightarrow -I\ell B = m\frac{dv}{dt}$$

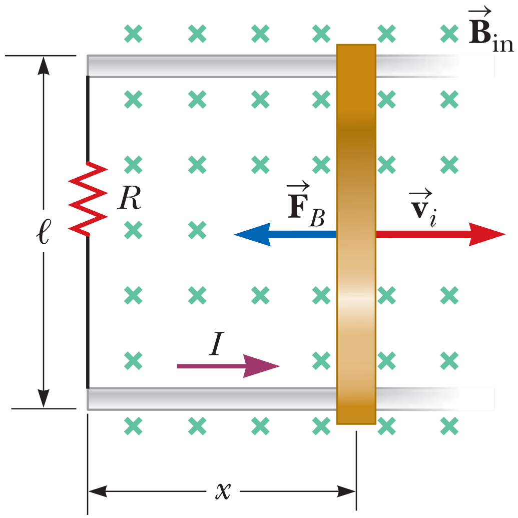
$$m\frac{dv}{dt} = -\left(\frac{B\ell v}{R}\right)\ell B = -\frac{B^2\ell^2}{R}v$$

$$\frac{dv}{v} = -\left(\frac{B^2\ell^2}{mR}\right)dt$$

$$\int_{v_i}^{v}\frac{dv}{v} = -\frac{B^2\ell^2}{mR}\int_0^t dt$$

$$\ln\left(\frac{v}{v_i}\right) = -\left(\frac{B^2\ell^2}{mR}\right)t$$

$$v = v_i e^{-t/\tau}$$



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(B) Show that the same result is found by using an energy approach.

$$\frac{dK}{dt} = \frac{dT_{\text{ET}}}{dt} = P_{\text{elec}} = -I^2R$$

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = -\left(\frac{B\ell v}{R}\right)^2 R \rightarrow mv\frac{dv}{dt} = -\frac{(B\ell v)^2}{R}$$

$$\frac{dv}{v} = -\left(\frac{B^2\ell^2}{mR}\right)dt$$