Chapter 29

Sources of the Magnetic Field

Magnetic Fields

The origin of the magnetic field is moving charges.

The magnetic field due to various current distributions can be calculated.

Ampère’s law is useful in calculating the magnetic field of a highly symmetric configuration carrying a steady current.

Magnetic effects in matter can be explained on the basis of atomic magnetic moments.

Biot-Savart Law – Introduction

Biot and Savart conducted experiments on the force exerted by an electric current on a nearby magnet.

They arrived at a mathematical expression that gives the magnetic field at some point in space due to a current.

The magnetic field described by the Biot-Savart Law is the field *due to* a given current carrying conductor.

- Do not confuse this field with any external field applied to the conductor from some other source.

Biot-Savart Law – Observations

The vector $d\vec{\mathbf{B}}$ is perpendicular to both $d\vec{\mathbf{s}}$ and to the unit vector $\hat{\mathbf{r}}$ directed from $d\vec{\mathbf{s}}$ toward P.

The magnitude of $d\vec{\mathbf{B}}$ is inversely proportional to r^2, where r is the distance from $d\vec{\mathbf{s}}$ to P.

The magnitude of $d\vec{\mathbf{B}}$ is proportional to the current and to the magnitude ds of the length element $d\vec{\mathbf{s}}$.

The magnitude of $d\vec{\mathbf{B}}$ is proportional to $\sin\theta$, where θ is the angle between the vectors $d\vec{\mathbf{s}}$ and $\hat{\mathbf{r}}$.

Biot-Savart Law – Equation

The observations are summarized in the mathematical equation called the **Biot-Savart law**:

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

The constant μ_o is called the **permeability of free space.**

$$\mu_0 = 4\pi \times 10^{-7}\ \text{T} \cdot\ \text{m/A}$$

Total Magnetic Field

$d\vec{\mathbf{B}}$ is the field created by the current in the length segment ds.

To find the total field, sum up the contributions from all the current elements $I d\vec{\mathbf{s}}$

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

- The integral is over the entire current distribution.

The law is also valid for a current consisting of charges flowing through space.

- For example, this could apply to the beam in an accelerator.

Magnetic Field Compared to Electric Field

Distance

- The magnitude of the magnetic field varies as the inverse square of the distance from the source.
- The electric field due to a point charge also varies as the inverse square of the distance from the charge.

Direction

- The electric field created by a point charge is radial in direction.
- The magnetic field created by a current element is perpendicular to both the length element $d\vec{\mathbf{s}}$ and the unit vector $\hat{\mathbf{r}}$.

Magnetic Field Compared to Electric Field, cont.

Source

- An electric field is established by an isolated electric charge.
- The current element that produces a magnetic field must be part of an extended current distribution.
    - Therefore you must integrate over the entire current distribution.

Problem 29.01:

Calculate the magnitude of the magnetic field at a point $25.0\,\text{cm}$ from a long, thin conductor carrying a current of $2.00\,\text{A}$.

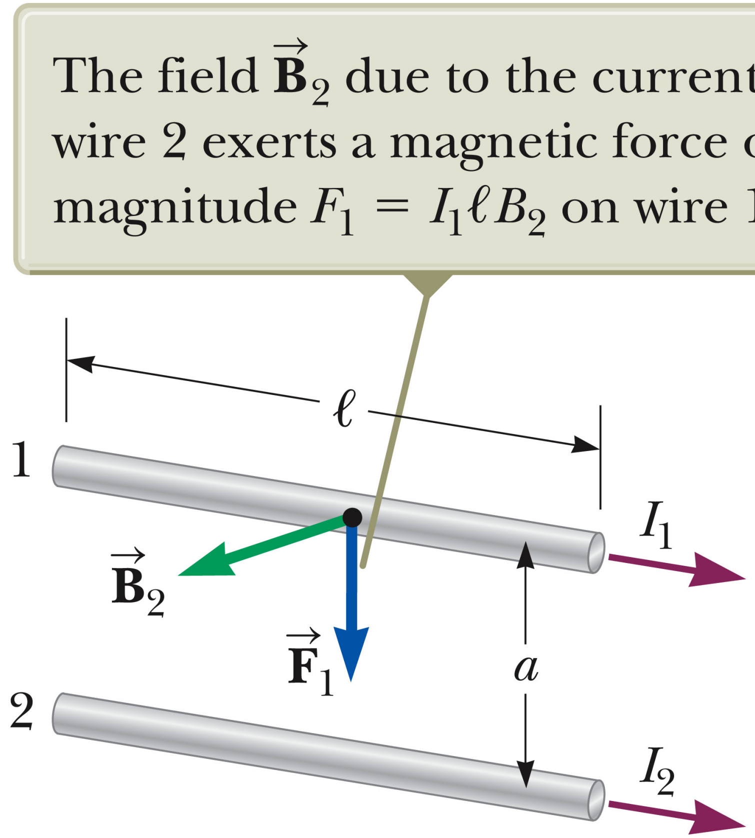
The magnetic field is given by

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\ \text{m/A}\right)(2.00\ \text{A})}{2\pi(0.250\ \text{m})} = 1.60 \times 10^{-6}\ \text{T}$$

Magnetic Force Between Two Parallel Conductors

Two parallel wires each carry a steady current.

The field $\vec{\mathbf{B}}$ due to the current in wire 2 exerts a force on wire 1 of $F_1 = I_1 \ell B_2$.



The field $\vec{\mathbf{B}}_2$ due to the current in wire 2 exerts a magnetic force of magnitude $F_1 = I_1 \ell B_2$ on wire 1.

Magnetic Force Between Two Parallel Conductors, cont.

Substituting the equation for the magnetic field (B_2) gives

$$F_1 = \frac{\mu_o I_1 I_2}{2\pi a}\ell$$

Parallel conductors carrying currents in the same direction attract each other.

- Parallel conductors carrying current in opposite directions repel each other.

Magnetic Force Between Two Parallel Conductors, final

The result is often expressed as the magnetic force between the two wires, F_B.

This can also be given as the force per unit length:

$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

The derivation assumes both wires are long compared with their separation distance.

- Only one wire needs to be long.
- The equations accurately describe the forces exerted on each other by a long wire and a straight, parallel wire of limited length, ℓ.

Problem 29.12:

Two parallel wires separated by $4.00\,\mathrm{cm}$ repel each other with a force per unit length of $2.00 \times 10^{-4}\ \mathrm{N/m}$. The current in one wire is $5.00\ \mathrm{A}$. (a) Find the current in the other wire. (b) Are the currents in the same direction or in opposite directions? (c) What would happen if the direction of one current were reversed and doubled?

(a) The force per unit length that parallel conductors exert on each other is, from, $F/\ell = \mu_0 I_1 I_2 / 2\pi d$. Thus, if $F/\ell = 2.00 \times 10^{-4}\ \mathrm{N/m}, I_1 = 5.00\ \mathrm{A}$, and $d = 4.00\ \mathrm{cm}$, the current in the second wire must be

$$I_2 = \frac{2\pi d}{\mu_0 I_1}\left(\frac{F}{\ell}\right)$$

$$= \left[\frac{2\pi\left(4.00 \times 10^{-2}\ \mathrm{m}\right)}{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot\ m/A}\right)(5.00\ \mathrm{A})}\right]\left(2.00 \times 10^{-4}\ \mathrm{N/m}\right)$$

$$= 8.00\ \mathrm{A}$$

(b) Since parallel conductors carrying currents in the same direction attract each other, the currents in these conductors which repel each other must be in opposite directions.

(c) The force is directly proportional to the product of the currents. The result of reversing the direction of either of the currents and doubling the magnitude would be that the force of interaction would be attractive and the magnitude of the force would double.

Problem 29.13:

Two parallel wires are separated by $6.00\,\mathrm{cm}$, each carrying $3.00\,\mathrm{A}$ of current in the same direction. (a) What is the magnitude of the force per unit length between the wires? (b) Is the force attractive or repulsive?

(a) The force per unit length that one wire exerts on the other is $F/\ell = \mu_0 I_1 I_2/2\pi d$, where d is the distance separating the two wires. In this case, the value of this force is

$$\frac{F}{\ell} = \frac{\left(4\pi\times 10^{-7}\ \mathrm{T\cdot\ m/A}\right)(3.00\ \mathrm{A})^2}{2\pi\left(6.00\times 10^{-2}\ \mathrm{m}\right)} = 3.00\times 10^{-5}\ \mathrm{N/m}$$

(b) The force one wire exerts on the other is an attractive force.

Andre-Marie Ampère

1775 – 1836

French physicist

Credited with the discovery of electromagnetism

- The relationship between electric current and magnetic fields

Also worked in mathematics

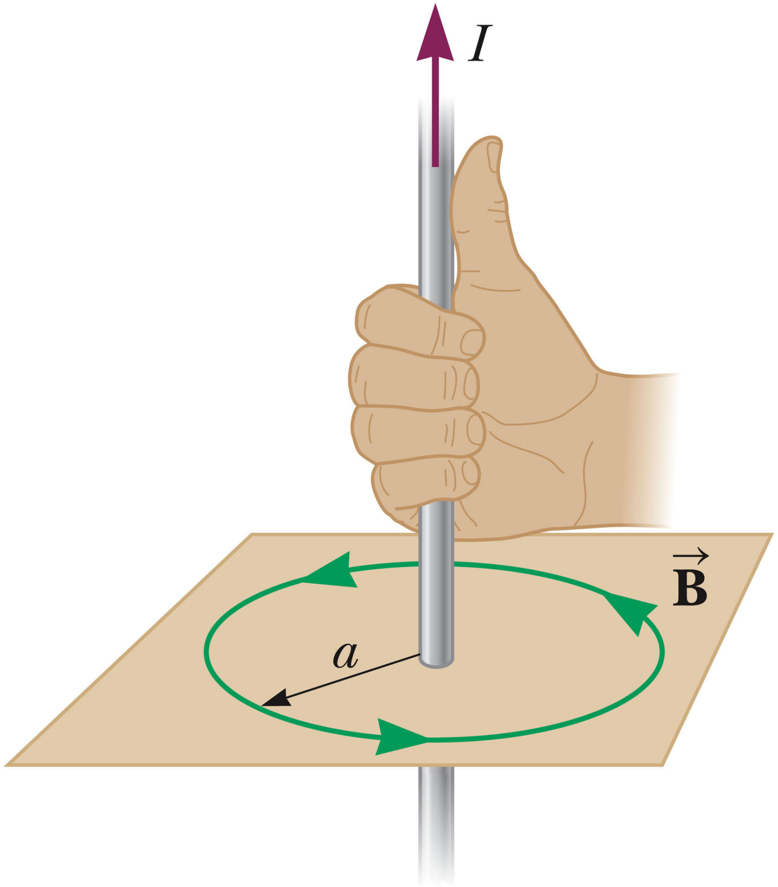
Magnetic Field for a Long, Straight Conductor: Direction

The magnetic field lines are circles concentric with the wire.

The field lines lie in planes perpendicular to the wire.

The magnitude of the field is constant on any circle of radius *a.*

The right-hand rule for determining the direction of the field is shown.



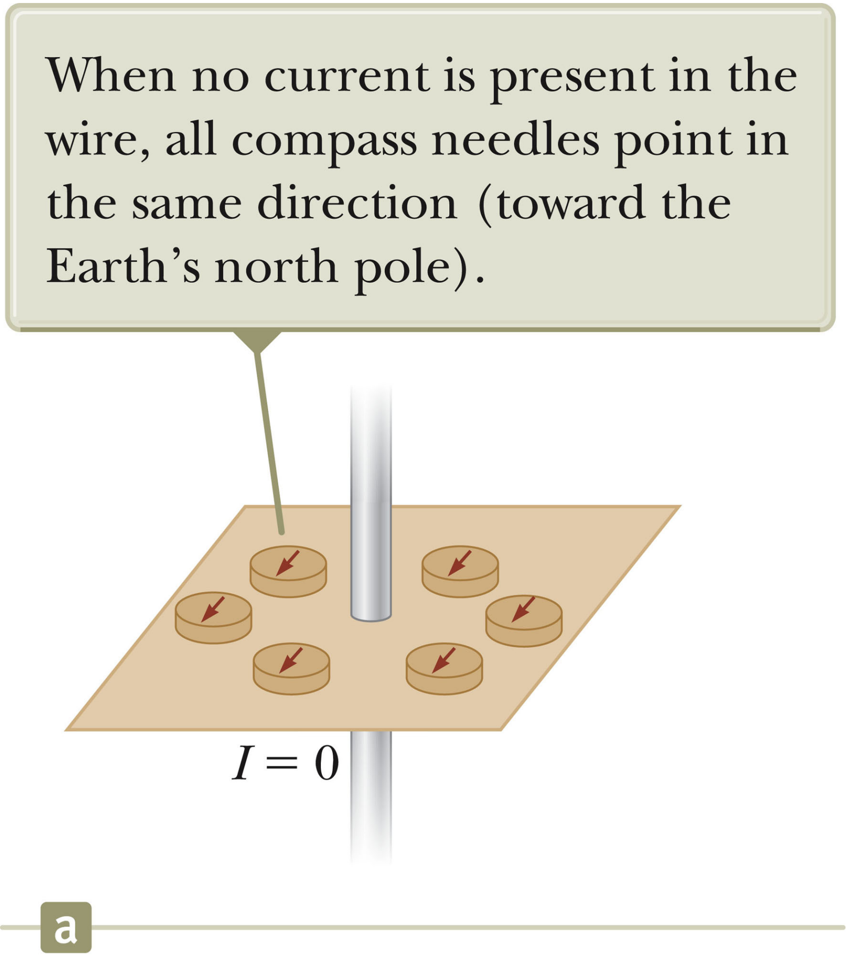
Magnetic Field of a Wire

A compass can be used to detect the magnetic field.

When there is no current in the wire, there is no field due to the current.

The compass needles all point toward the Earth's north pole.

- Due to the Earth's magnetic field


When no current is present in the wire, all compass needles point in the same direction (toward the Earth's north pole).

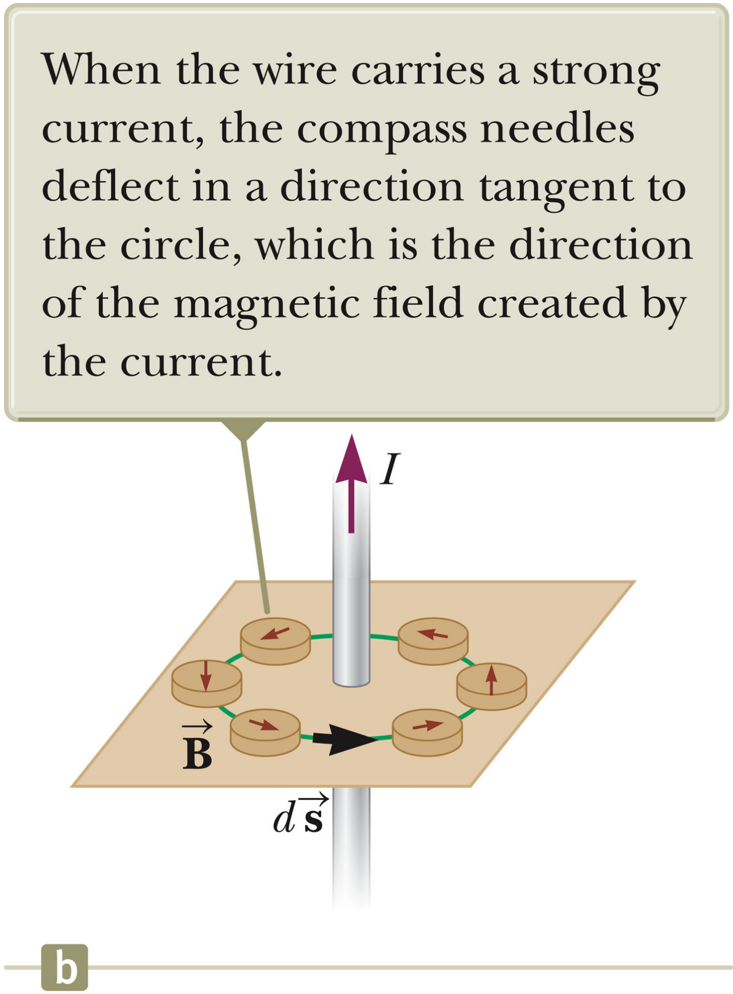
Magnetic Field of a Wire, cont.

Here the wire carries a strong current.

The compass needles deflect in a direction tangent to the circle.

This shows the direction of the magnetic field produced by the wire.

If the current is reversed, the direction of the needles also reverse.



When the wire carries a strong current, the compass needles deflect in a direction tangent to the circle, which is the direction of the magnetic field created by the current.

Magnetic Field of a Wire, final

The circular magnetic field around the wire is shown by the iron filings.

Ampere’s Law

The product of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ can be evaluated for small length elements $d\vec{\mathbf{s}}$ on the circular path defined by the compass needles for the long straight wire.

Ampere’s law states that the line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed path equals $\mu_o I$ where I is the total steady current passing through any surface bounded by the closed path:

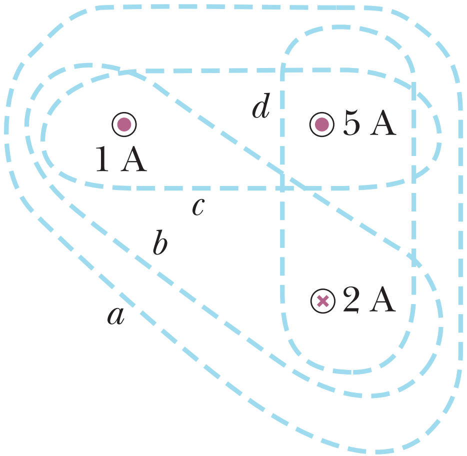
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 \boldsymbol{I}$$

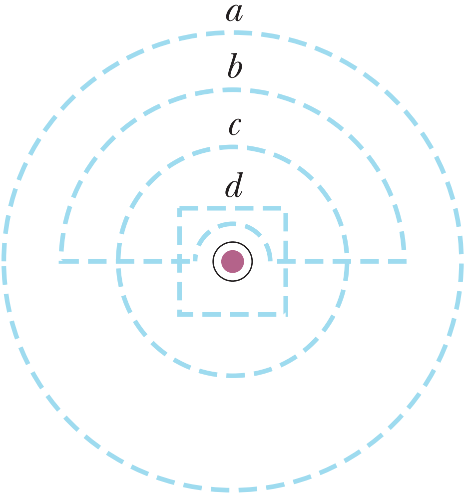
Ampere’s law describes the creation of magnetic fields by all continuous current configurations.

- Most useful for this course if the current configuration has a high degree of symmetry.

Put the thumb of your right hand in the direction of the current through the amperian loop and your fingers curl in the direction you should integrate around the loop.

Quick quiz:





29.3 Rank the magnitudes of $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for the closed paths a through d in the figure from greatest to least.

29.4 Rank the magnitudes of $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ for the closed paths a through d in the figure from greatest to least.

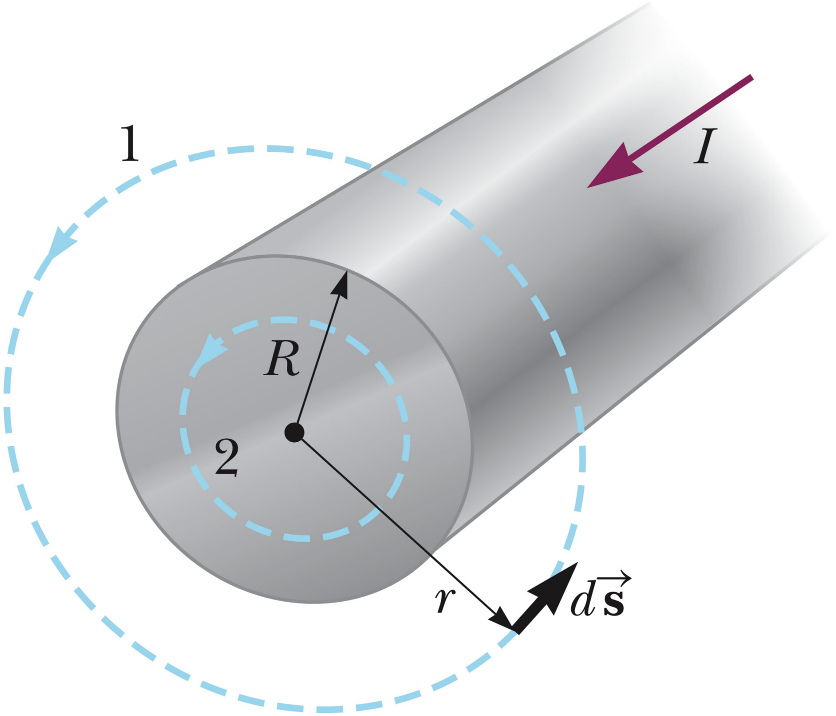
Example 29.05: The Magnetic Field Created by a Long Current-Carrying Wire

Calculate the magnetic field at a distance r from the center of a wire carrying a steady current I.

The current is uniformly distributed through the cross section of the wire.

Since the wire has a high degree of symmetry, the problem can be categorized as a Ampère's Law problem.

- For $r \geq R,$ this should be the same result as obtained from the Biot-Savart Law.



Field Due to a Long Straight Wire – Results From Ampere's Law

Outside of the wire, $r > R$

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I \rightarrow B = \frac{\mu_o I}{2\pi r}$$

Inside the wire, we need I', the current inside the amperian circle.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B(2\pi r) = \mu_0 I' \rightarrow \qquad I' = \frac{r^2}{R^2} I$$

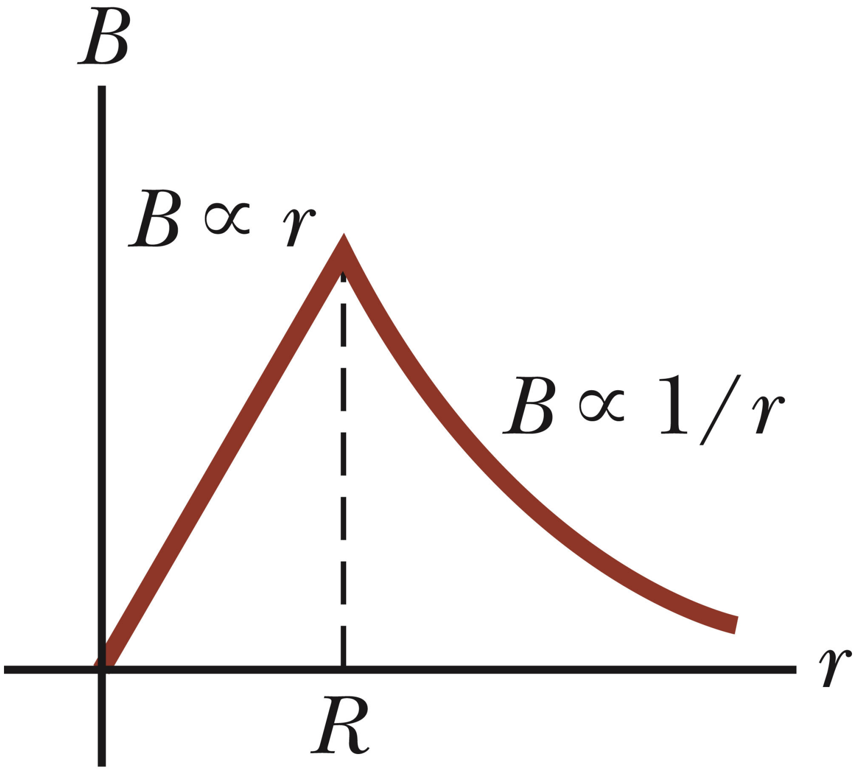
$$B = \left(\frac{\mu_0 I r}{2\pi R^2} \right)$$

Field Due to a Long Straight Wire – Results Summary

The field is proportional to r inside the wire.

The field varies as $1/r$ outside the wire.

Both equations are equal at $r = R$.

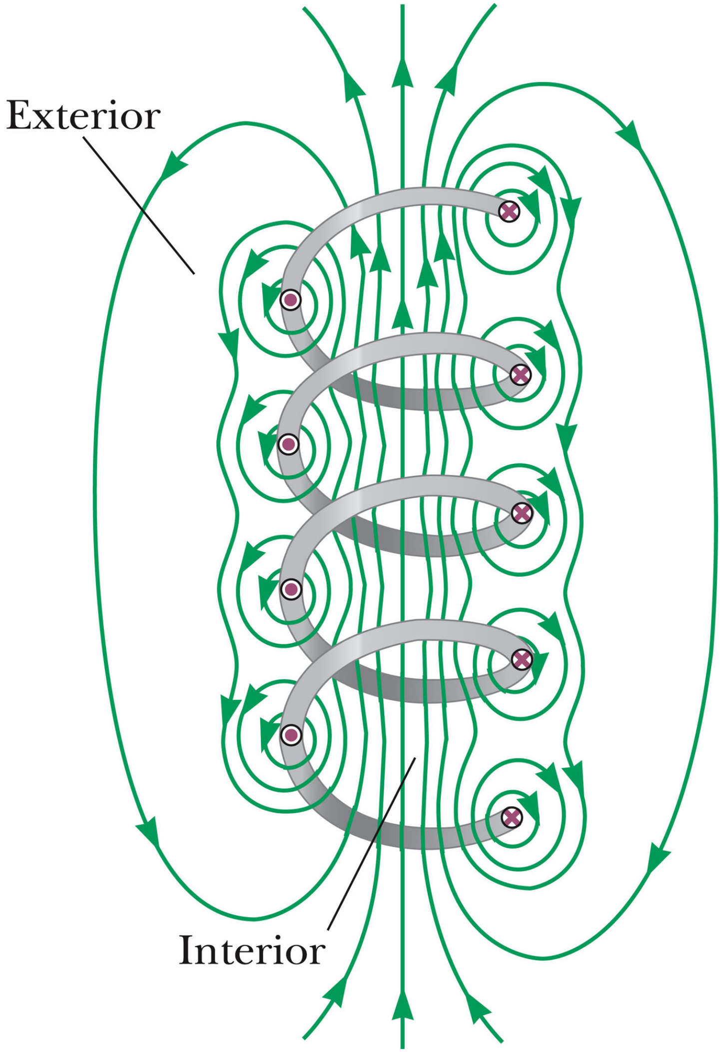


Magnetic Field of a Solenoid

A solenoid is a long wire wound in the form of a helix.

A reasonably uniform magnetic field can be produced in the space surrounded by the turns of the wire.

- The interior of the solenoid



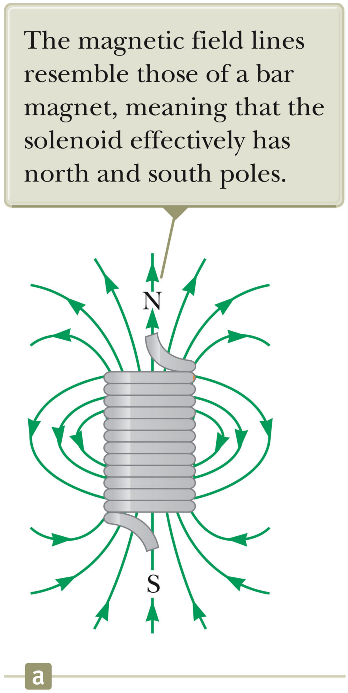
Magnetic Field of a Solenoid, Description

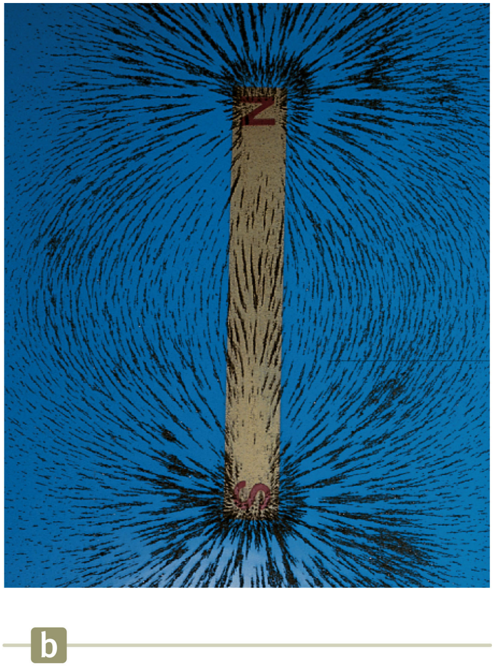
The field lines in the interior are

- Nearly parallel to each other
- Uniformly distributed
- Close together

This indicates the field is strong and almost uniform.

Magnetic Field of a Tightly Wound Solenoid


The magnetic field lines resemble those of a bar magnet, meaning that the solenoid effectively has north and south poles.



The field distribution is similar to that of a bar magnet.

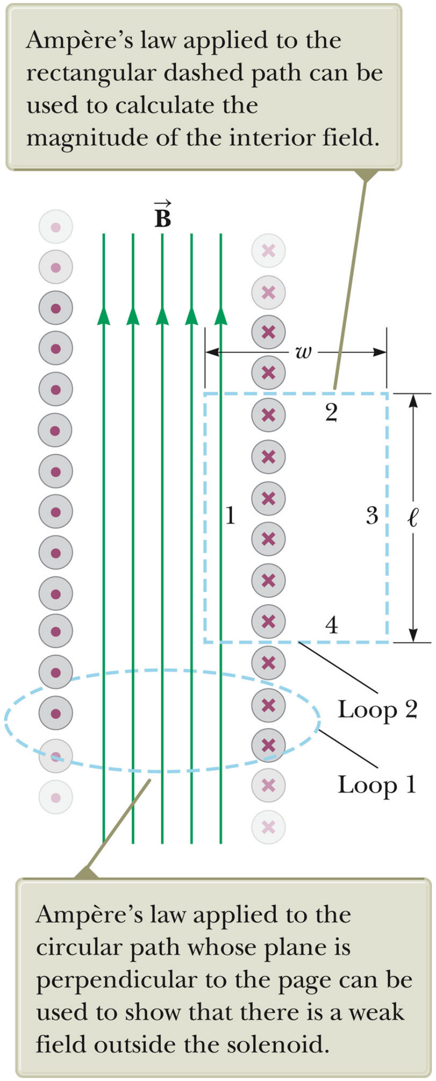
As the length of the solenoid increases,

- The interior field becomes more uniform.
- The exterior field becomes weaker.

Ideal Solenoid – Characteristics

An ideal solenoid is approached when:

- The turns are closely spaced.
- The length is much greater than the radius of the turns.



Ampère's law applied to the rectangular dashed path can be used to calculate the magnitude of the interior field.

Ampère's law applied to the circular path whose plane is perpendicular to the page can be used to show that there is a weak field outside the solenoid.

Ampere’s Law Applied to a Solenoid

Consider an amperian loop (loop 1 in the diagram) surrounding the ideal solenoid.

- The loop encloses a small current.
- There is a weak field external to the solenoid.
- A second layer of turns of wire could be used to eliminate the field.

Ampere’s law can also be used to find the interior magnetic field of the solenoid.

- Consider a rectangle with side ℓ parallel to the interior field and side w perpendicular to the field.
  - This is loop 2 in the diagram.
- The side of length ℓ inside the solenoid contributes to the field.
  - This is side 1 in the diagram.
  - Sides 2, 3, and 4 give contributions of zero to the field.

Ampere's Law Applied to a Solenoid, cont.

Applying Ampere's Law gives

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \int_{\text{path1}} \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B \int_{\text{path1}} ds = B\ell$$

The total current through the rectangular path equals the current through each turn multiplied by the number of turns.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = B\ell = \mu_0 N l$$

Solving Ampere's law for the magnetic field is

$$B = \mu_o \frac{N}{\ell} I = \mu_o n I$$

- $n = N/\ell$ is the number of turns per unit length.

This is valid only at points near the center of a very long solenoid.

Problem 29.23:

A long solenoid that has 1000 turns uniformly distributed over a length of $0.400\ \mathrm{m}$ produces a magnetic field of magnitude $1.00 \times 10^{-4}\ \mathrm{T}$ at its center. What current is required in the windings for that to occur?

The magnetic field at the center of a solenoid is $B = \mu_0 \frac{N}{\ell} I$, so

$$I = \frac{B}{\mu_0 n} = \frac{\left(1.00 \times 10^{-4}\ \mathrm{T}\right)(0.400\ \mathrm{m})}{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot\ m/A}\right)(1000)} = 31.8\ \mathrm{mA}$$

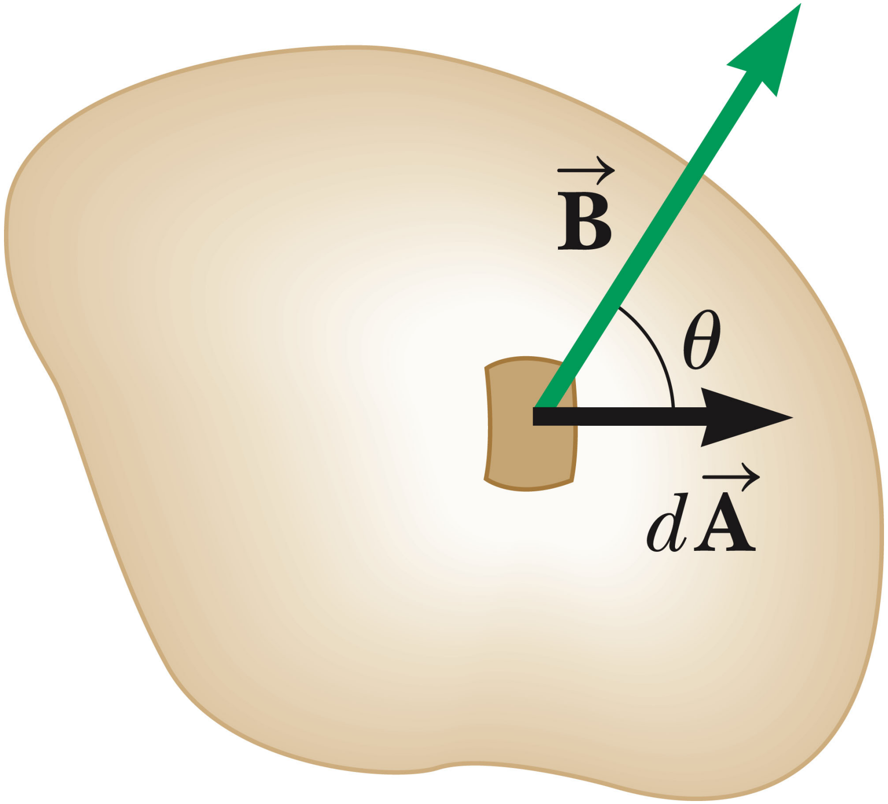
Magnetic Flux

The magnetic flux associated with a magnetic field is defined in a way similar to electric flux.

Consider an area element dA on an arbitrarily shaped surface.

The magnetic field in this element is $\vec{\mathbf{B}}$.

$d\vec{\mathbf{A}}$ is a vector that is perpendicular to the surface and has a magnitude equal to the area dA.



Magnetic Flux, cont.

The magnetic flux Φ_B is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$$

The unit of magnetic flux is $\mathrm{T} \cdot \mathrm{m}^2 = \mathrm{Wb}$

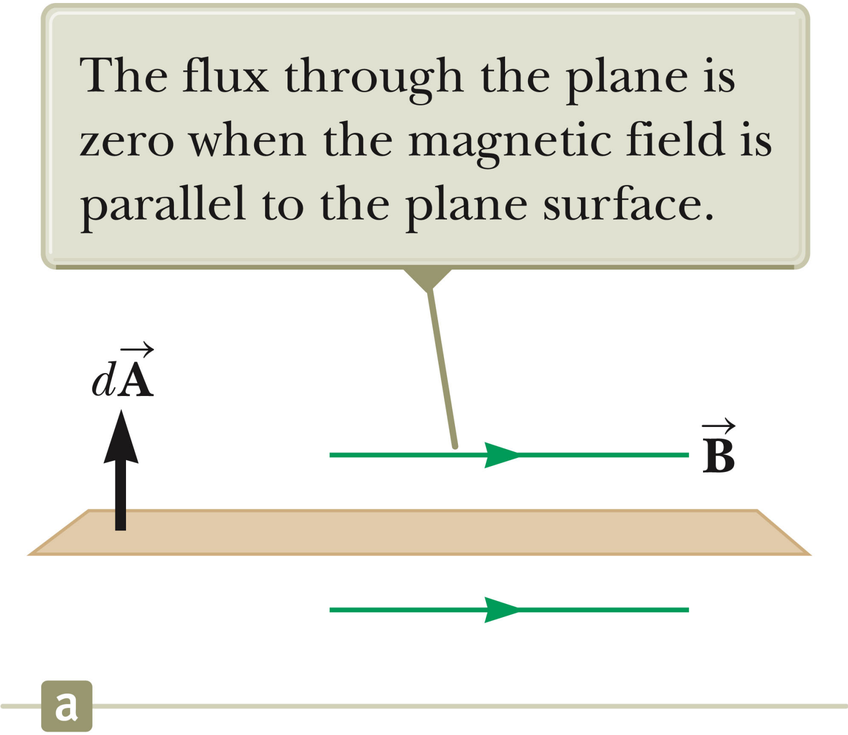
- Wb is a weber

Magnetic Flux Through a Plane, 1

A special case is when a plane of area *A* makes an angle θ with $d\vec{\mathbf{A}}$.

The magnetic flux is $\Phi_B = BA\cos\theta$.

In this case, the field is parallel to the plane and $\Phi_B = 0$.



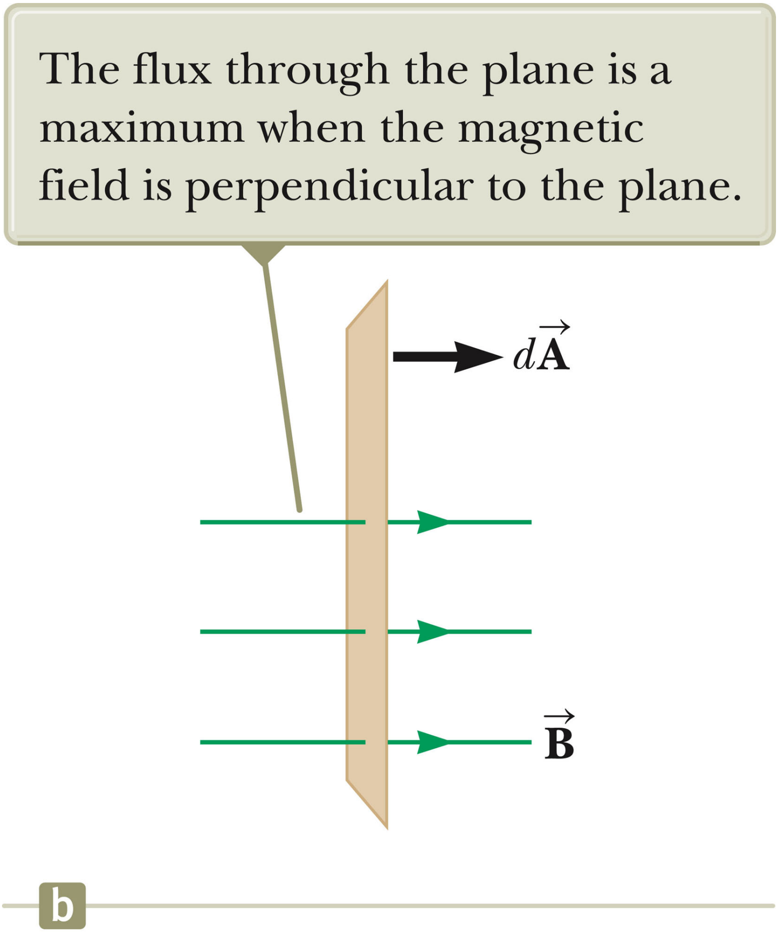
The flux through the plane is zero when the magnetic field is parallel to the plane surface.

Magnetic Flux Through A Plane, 2

The magnetic flux is $\Phi_B = BA\cos\theta$.

In this case, the field is perpendicular to the plane and $\Phi = BA$.

- This is the maximum value of the flux.



The flux through the plane is a maximum when the magnetic field is perpendicular to the plane.

Gauss' Law in Magnetism

Magnetic fields do not begin or end at any point.

- Magnetic field lines are continuous and form closed loops.
- The number of lines entering a surface equals the number of lines leaving the surface.

Gauss' law in magnetism says the magnetic flux through any closed surface is always zero:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$$

This indicates that isolated magnetic poles (monopoles) have never been detected.

- Perhaps they do not exist
- Certain theories do suggest the possible existence of magnetic monopoles.

Example 29.07: Magnetic Flux Through a Rectangular Loop

A rectangular loop of width a and length b is located near a long wire carrying a current I. The distance between the wire and the closest side of the loop is c. The wire is parallel to the long side of the loop. Find the total magnetic flux through the loop due to the current in the wire.

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA = \int \frac{\mu_0 I}{2\pi r}\,dA$$

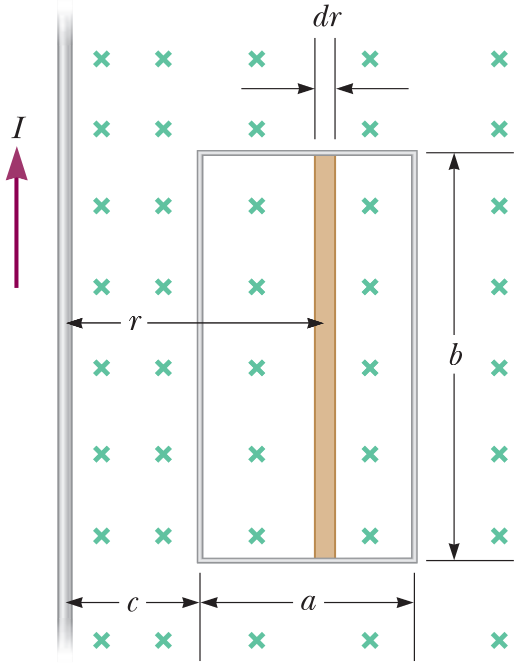
Express the area element as $dA = b\,dr$ and substitute:

$$\Phi_B = \int \frac{\mu_0 I}{2\pi r} b\,dr = \frac{\mu_0 I b}{2\pi} \int \frac{dr}{r}$$

Integrate from $r = c$ to $r = a + c$:

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{dr}{r} = \frac{\mu_0 I b}{2\pi} \ln r \bigg|_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi} \ln\left(\frac{a+c}{c}\right) = \frac{\mu_0 I b}{2\pi} \ln\left(1 + \frac{a}{c}\right)$$



Problem 29.27:

Consider the hemispherical closed surface in the figure. The hemisphere is in a uniform magnetic field that makes an angle θ with the vertical. Calculate the magnetic flux through (a) the flat surface S_1 and (b) the hemispherical surface S_2.

(a) The magnetic flux through the flat surface S_1 is

$$(\Phi_B)_{\text{flat}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = B\pi R^2 \cos(180 - \theta) = -B\pi R^2 \cos\theta$$

(b) The net flux out of the closed surface is zero:

$$(\Phi_B)_{\text{flat}} + (\Phi_B)_{\text{curved}} = 0$$

Therefore,

$$(\Phi_B)_{\text{curved}} = B\pi R^2 \cos\theta$$