# Chapter 27

Direct Current Circuits

# Circuit Analysis

Simple electric circuits may contain batteries, resistors, and capacitors in various combinations.

For some circuits, analysis may consist of combining resistors.

In more complex complicated circuits, Kirchhoff's Rules may be used for analysis.

- These Rules are based on conservation of energy and conservation of electric charge for isolated systems.

Circuits may involve direct current or alternating current.

# Direct Current

When the current in a circuit has a constant direction, the current is called ***direct current.***

- Most of the circuits analyzed will be assumed to be in *steady state*, with constant magnitude and direction.

Because the potential difference between the terminals of a battery is constant, the battery produces direct current.

The battery is known as a source of emf.

# Electromotive Force

The electromotive force (emf), $\varepsilon$, of a battery is the maximum possible voltage that the battery can provide between its terminals.

- The emf supplies energy, it does not apply a force.

The battery will normally be the source of energy in the circuit.

The positive terminal of the battery is at a higher potential than the negative terminal.

We consider the wires to have no resistance.

# Internal Battery Resistance

If the internal resistance is zero, the terminal voltage equals the emf.

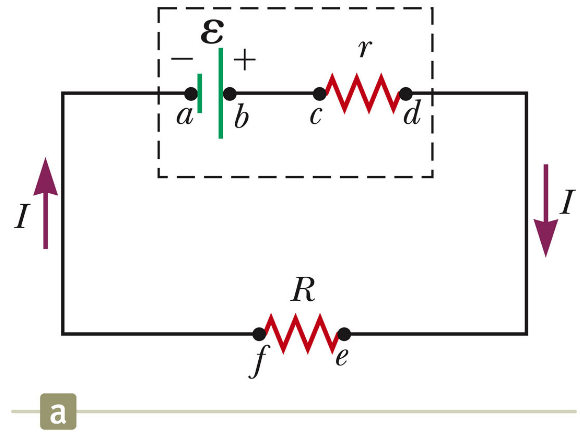
In a real battery, there is internal resistance, $r$.

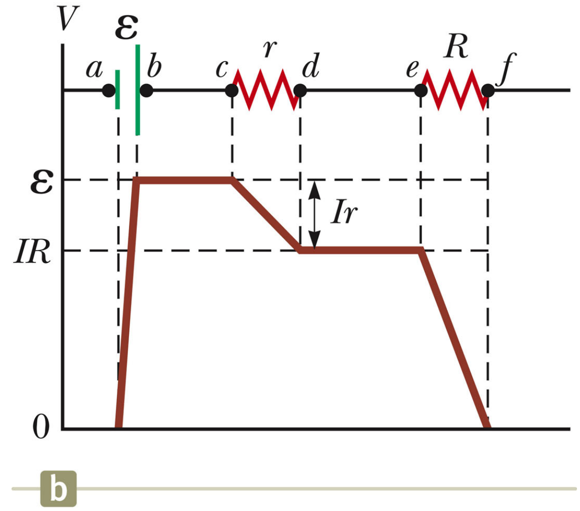
The terminal voltage, $\Delta V = \varepsilon - Ir$

The emf is equivalent to the *open-circuit* voltage.

- This is the terminal voltage when no current is in the circuit.
- This is the voltage labeled on the battery.

The actual potential difference between the terminals of the battery depends on the current in the circuit.



# Load Resistance

The terminal voltage also equals the voltage across the external resistance.

- This external resistor is called the *load resistance.*
- In the previous circuit, the load resistance is just the external resistor.
- In general, the load resistance could be any electrical device.
  - These resistances represent *loads* on the battery since it supplies the energy to operate the device containing the resistance.

# Power

The total power output of the battery is

$$P = I\Delta V = I\varepsilon$$

This power is delivered to the external resistor ($I^2R$) and to the internal resistor ($I^2r$).

$$P = I^2R + I^2r$$

The battery is a supply of constant emf.

- The battery does not supply a constant current since the current in the circuit depends on the resistance connected to the battery.
- The battery does not supply a constant terminal voltage.

# Example 27.01: Terminal Voltage of a Battery

A battery has an emf of $12.0\ \text{V}$ and an internal resistance of $0.05\ \Omega$. Its terminals are connected to a load resistance of $3.00\ \Omega$.

(A) Find the current in the circuit and the terminal voltage of the battery.

(B) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

(A)

$$I = \frac{\boldsymbol{\varepsilon}}{R+r} = \frac{12.0\ \text{V}}{3.00\Omega + 0.0500\Omega} = 3.93\ \text{A}$$

$$\Delta V = \boldsymbol{\varepsilon} - Ir = 12.0\ \text{V} - (3.93\ \text{A})(0.0500\Omega) = 11.8\ \text{V}$$

$$\Delta V = IR = (3.93\ \text{A})(3.00\Omega) = 11.8\ \text{V}$$

(B)

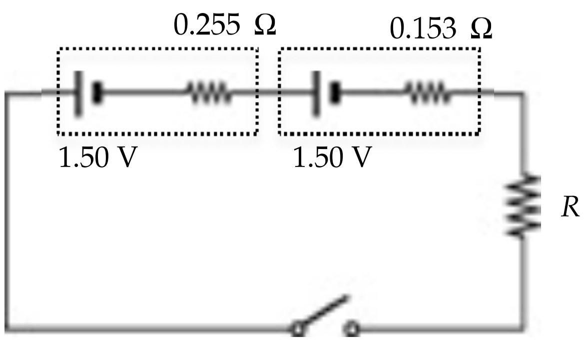
$$P_R = I^2R = (3.93\ \text{A})^2(3.00\Omega) = 46.3\ \text{W}$$

$$P_r = I^2r = (3.93\ \text{A})^2(0.0500\Omega) = 0.772\ \text{W}$$

$$P = P_R + P_r = 46.3\ \text{W} + 0.772\ \text{W} = 47.1\ \text{W}$$

# Problem 27.01:

Two $1.50 - \mathrm{V}$ batteries-with their positive terminals in the same direction-are inserted in series into a flashlight. One battery has an internal resistance of $0.255\,\Omega$, and the other has an internal resistance of $0.153\,\Omega$. When the switch is closed, the bulb carries a current of $600\ \mathrm{mA}$ . (a) What is the bulb's resistance? (b) What fraction of the chemical energy transformed appears as internal energy in the batteries?




The total resistance is $R = \dfrac{3.00\ \mathrm{V}}{0.600\ \mathrm{A}} = 5.00\,\Omega$.

(a) $R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\,\Omega - 0.408\,\Omega = 4.59\,\Omega$

(b) $\dfrac{P_{\text{batteries}}}{P_{\text{total}}} = \dfrac{(0.408\Omega)I^2}{(5.00\Omega)I^2} = 0.0816 = 8.16\,\%$

# Resistors in Series

When two or more resistors are connected end-to-end, they are said to be in series.

For a series combination of resistors, the currents are the same in all the resistors because the amount of charge that passes through one resistor must also pass through the other resistors in the same time interval.

The potential difference will divide among the resistors such that the sum of the potential differences across the resistors is equal to the total potential difference across the combination.

# Resistors in Series, cont

Currents are the same

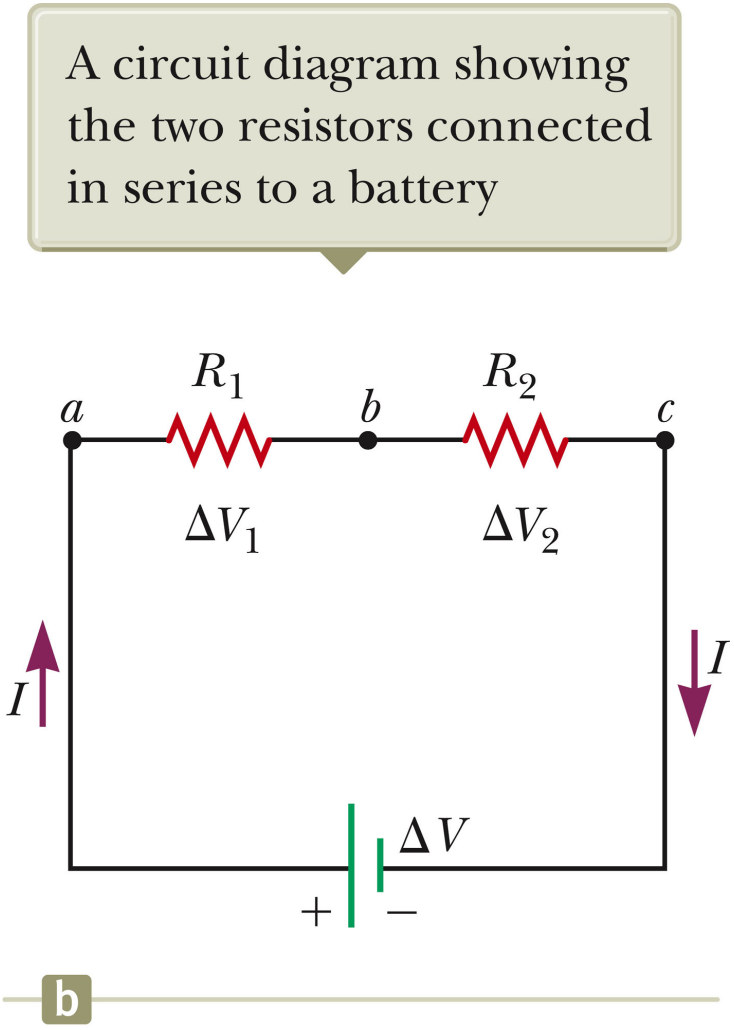
- $I = I_1 = I_2$

Potentials add

$$\Delta V = V_1 + V_2 = IR_1 + IR_2 = I(R_1 + R_2)$$

- Consequence of Conservation of Energy

The equivalent resistance has the same effect on the circuit as the original combination of resistors.



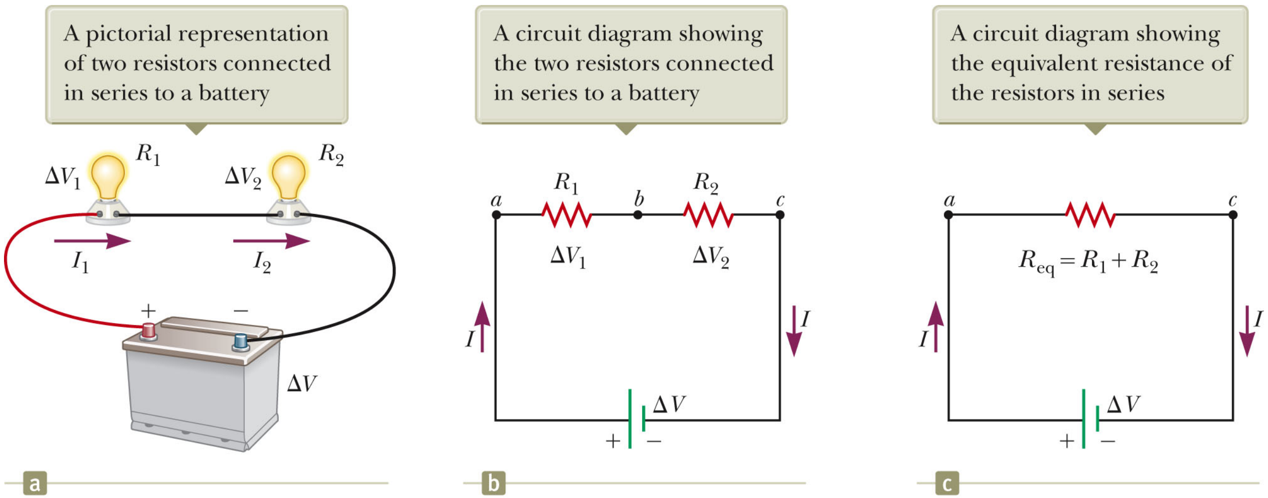
# Equivalent Resistance – Series

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any individual resistance.

If one device in the series circuit creates an open circuit, all devices are inoperative.

# Equivalent Resistance – Series – An Example




All three representations are equivalent.

Two resistors are replaced with their equivalent resistance.

# Some Circuit Notes

A local change in one part of a circuit may result in a global change throughout the circuit.

- For example, changing one resistor will affect the currents and voltages in all the other resistors and the terminal voltage of the battery.

In a series circuit, there is one path for the current to take.

In a parallel circuit, there are multiple paths for the current to take.

# Resistors in Parallel

The potential difference across each resistor is the same because each is connected directly across the battery terminals.

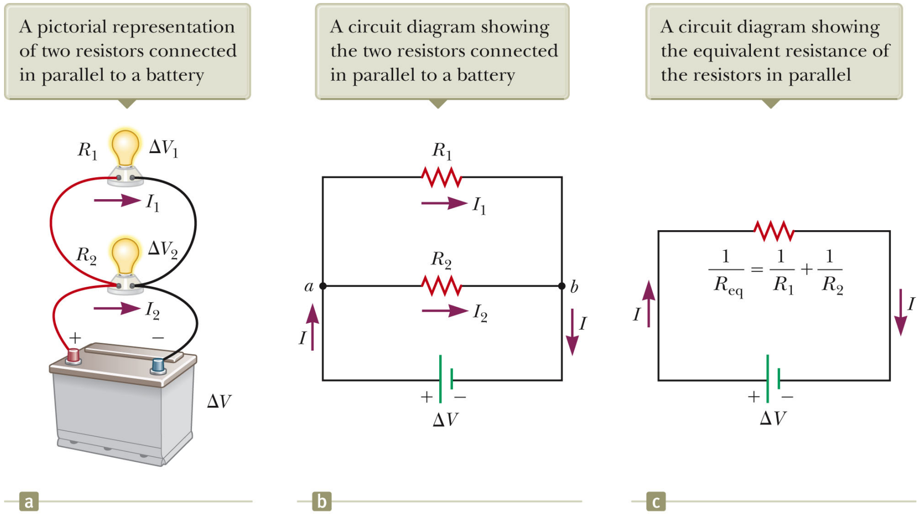
$$\Delta V = \Delta V_1 = \Delta V_2$$

A **junction** is a point where the current can split.

The current, $I$, that enters junction must be equal to the total current leaving that junction.

- $I = I_1 + I_2 = (\Delta V_1 / R_1) + (\Delta V_2 / R_2)$
- The currents are generally not the same.
- Consequence of conservation of electric charge

# Equivalent Resistance – Parallel, Examples




All three diagrams are equivalent.

Equivalent resistance replaces the two original resistances.

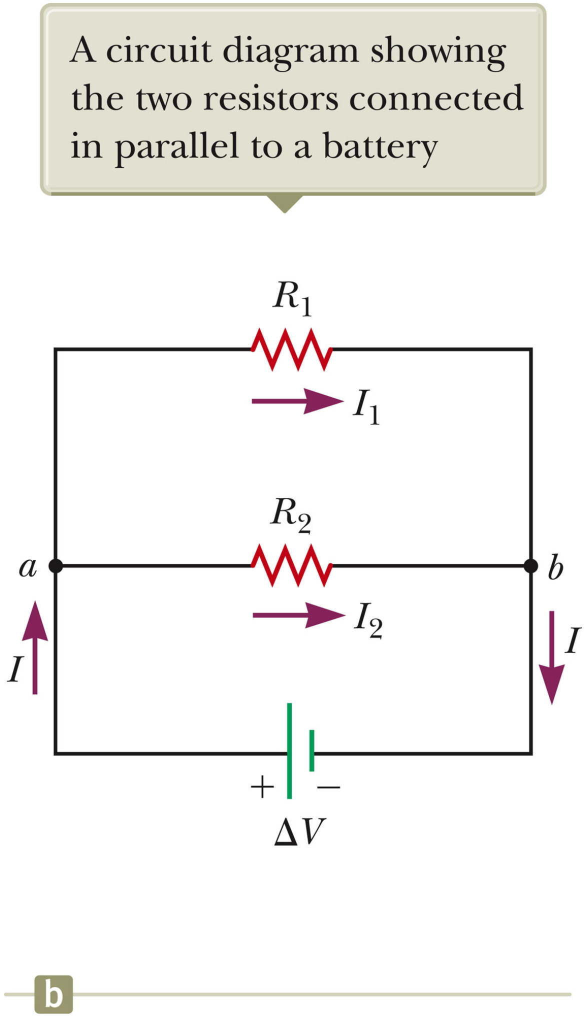
# Equivalent Resistance – Parallel

Equivalent Resistance

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

The inverse of the equivalent resistance of two or more resistors connected in parallel is the algebraic sum of the inverses of the individual resistance.

- The equivalent is always less than the smallest resistor in the group.



## Resistors in Parallel, Final

In parallel, each device operates independently of the others so that if one is switched off, the others remain on.

In parallel, all of the devices operate on the same voltage.

The current takes all the paths.

- The lower resistance will have higher currents.
- Even very high resistances will have some currents.

*Household circuits* are wired so that electrical devices are connected in parallel.

# Example 27.04: Find the Equivalent Resistance

Four resistors are connected as shown in the figure.

(A) Find the equivalent resistance between points $a$ and $c$.

(B) What is the current in each resistor if a potential difference of $42\ \text{V}$ is maintained between $a$ and $c$ ?

(A) $R_{\text{eq}} = 8.0\Omega + 4.0\Omega = 12.0\Omega$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{6.0\Omega} + \frac{1}{3.0\Omega} = \frac{3}{6.0\Omega}$$

$$R_{\text{eq}} = \frac{6.0\Omega}{3} = 2.0\Omega$$

$$R_{\text{eq}} = 12.0\Omega + 2.0\Omega = 14.0\Omega$$

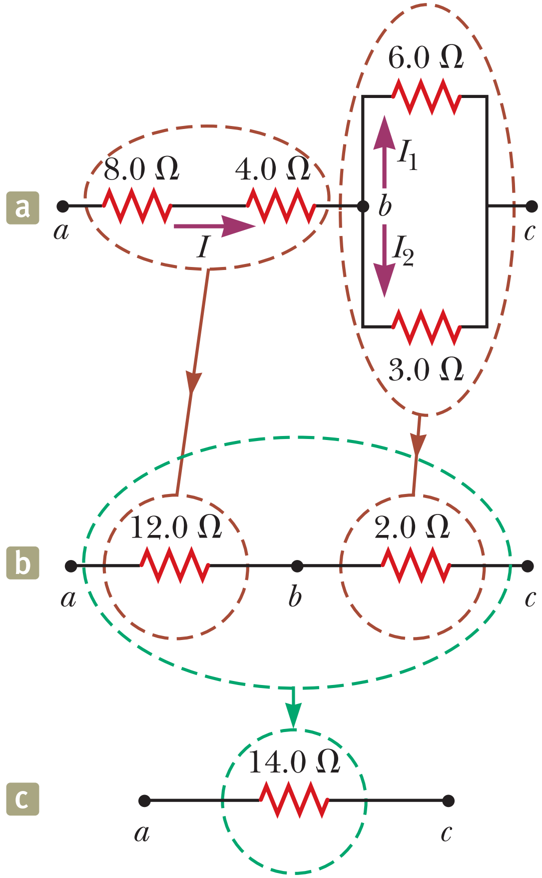
(B)

$$I = \frac{\Delta V_{ac}}{R_{\text{eq}}} = \frac{42\ \text{V}}{14.0\Omega} = 3.0\ \text{A}$$

$$\Delta V_1 = \Delta V_2 \rightarrow (6.0\Omega)I_1 = (3.0\Omega)I_2 \rightarrow I_2 = 2I_1$$

$$I_1 + I_2 = 3.0\ \text{A} \rightarrow I_1 + 2I_1 = 3.0\ \text{A} \rightarrow I_1 = 1.0\ \text{A}$$

$$I_2 = 2I_1 = 2(1.0\ \text{A}) = 2.0\ \text{A}$$



# Example 27.05: Three Resistors in Parallel

Three resistors are connected as shown in the figure. A potential difference of $18.0\ \text{V}$ is maintained between points $a$ and $b$.

(A) Calculate the equivalent resistance of the circuit.

(B) Find the current in each resistor.

(C) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

(A)

$$\frac{1}{R_{eq}} = \frac{1}{3.00\Omega} + \frac{1}{6.00\Omega} + \frac{1}{9.00\Omega} = \frac{11}{18.0\Omega}$$

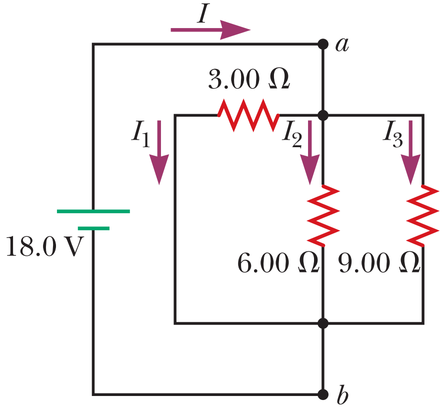
$$R_{eq} = \frac{18.0\Omega}{11} = 1.64\Omega$$

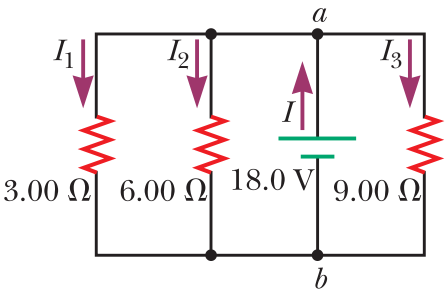
(B)

$$I_1 = \frac{\Delta V}{R_1} = \frac{18.0\ \text{V}}{3.00\Omega} = 6.00\ \text{A}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18.0\ \text{V}}{6.00\Omega} = 3.00\ \text{A}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18.0\ \text{V}}{9.00\Omega} = 2.00\ \text{A}$$




(C)

$$3.00-\Omega : P_1 = I_1^2 R_1 = (6.00\ \text{A})^2(3.00\Omega) = 108\ \text{W}$$

$$6.00-\Omega : P_2 = I_2^2 R_2 = (3.00\ \text{A})^2(6.00\Omega) = 54\ \text{W}$$

$$9.00-\Omega : P_3 = I_3^2 R_3 = (2.00\ \text{A})^2(9.00\Omega) = 36\ \text{W}$$

# Problem 27.09:

A battery with $\varepsilon = 6.00\ \text{V}$ and no internal resistance supplies current to the circuit shown in the figure. When the double-throw switch $S$ is open as shown in the figure, the current in the battery is $1.00\ \text{mA}$ . When the switch is closed in position $a$, the current in the battery is $1.20\ \text{mA}$ . When the switch is closed in position $b$, the current in the battery is $2.00\ \text{mA}$ . Find the resistances (a) $R_1$, (b) $R_2$, and (c) $R_3$.

When $S$ is open, $R_1$, $R_2$ and $R_3$ are in series with the battery. Thus,

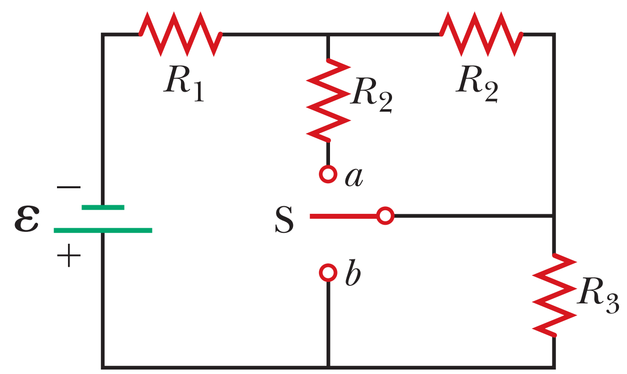
[1] $R_1 + R_2 + R_3 = \dfrac{6\ \text{V}}{10^{-3}\ \text{A}} = 6\text{k}\Omega$

When $S$ is closed in position $a$, the parallel combination of the two $R_2$ 's is in series with $R_1$, $R_3$, and the battery. Thus,

[2] $R_1 + \dfrac{1}{2}R_2 + R_3 = \dfrac{6\ \text{V}}{1.2 \times 10^{-3}\ \text{A}} = 5\text{k}\Omega$

When $S$ is closed in position $b$, $R_1$ and $R_2$ are in series with the battery and $R_3$ is shorted. Thus,

[3] $R_1 + R_2 = \dfrac{6\ \text{V}}{2 \times 10^{-3}\ \text{A}} = 3\text{k}\Omega$




Subtracting [3] from [1] gives $R_3 = 3\text{k}\Omega$.

Subtracting [2] from [1] gives $R_2 = 2\text{k}\Omega$.

Then, from [3],

$R_1 = 1\text{k}\Omega$ .

Answers: (a) $R_1 = 1.00\text{k}\Omega$  (b) $R_2 = 2.00\text{k}\Omega$.
(c) $R_3 = 3.00\text{k}\Omega$

# Problem 27.13:

Calculate the power delivered to each resistor in the circuit shown in the figure.

$$R_P = \frac{1}{(1/1.00\Omega) + (1/3.00\Omega)} = 0.750\Omega$$

$$I = 18.0\ \text{V}/6.75\Omega = 2.67\ \text{A}$$

$$P_2 = I\Delta V = I^2R = (2.67\ \text{A})^2(2.00\Omega) = 14.2\ \text{W}$$

$$P_4 = I^2R = (2.67\ \text{A})^2(4.00\Omega) = 28.4\ \text{W}$$

The voltage across the $0.750 - \Omega$ resistor (a), and across both the $3.00 - \Omega$ and the $1.00 - \Omega$ resistors, is

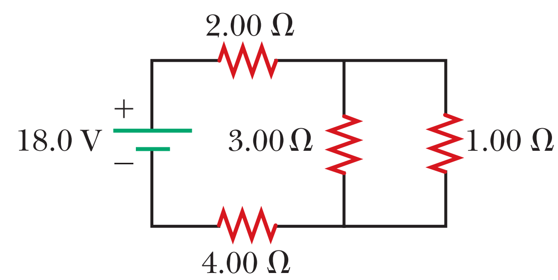
$$\Delta V = IR = (2.67\ \text{A})(0.750\Omega) = 2.00\ \text{V}$$

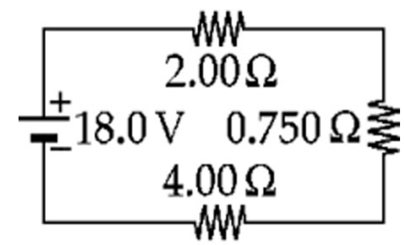
Then for the $3.00 - \Omega$ resistor,

$I = \frac{\Delta V}{R} = \frac{2.00\ \text{V}}{3.00\Omega}$ and the power is $P_3 = I\Delta V = \left(\frac{2.00\ \text{V}}{3.00\Omega}\right)(2.00\ \text{V}) = 1.33\ \text{W}$

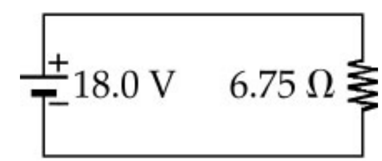
For the $1.00 - \Omega$ resistor,

$I = \frac{2.00\ \text{V}}{1.00\Omega}$ and $P_1 = \left(\frac{2.00\ \text{V}}{1.00\Omega}\right)(2.00\ \text{V}) = 4.00\ \text{W}$



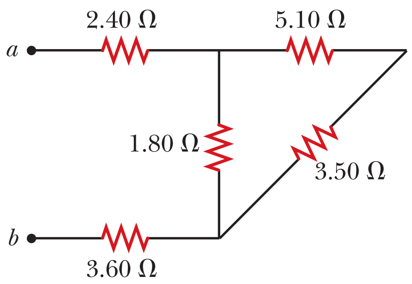
(a)

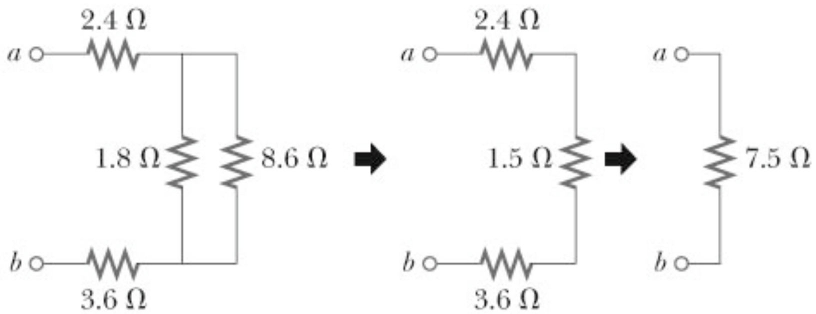


(b)

# Problem 27.33:

Find the equivalent resistance between points $a$ and $b$ in the figure.



# Gustav Kirchhoff

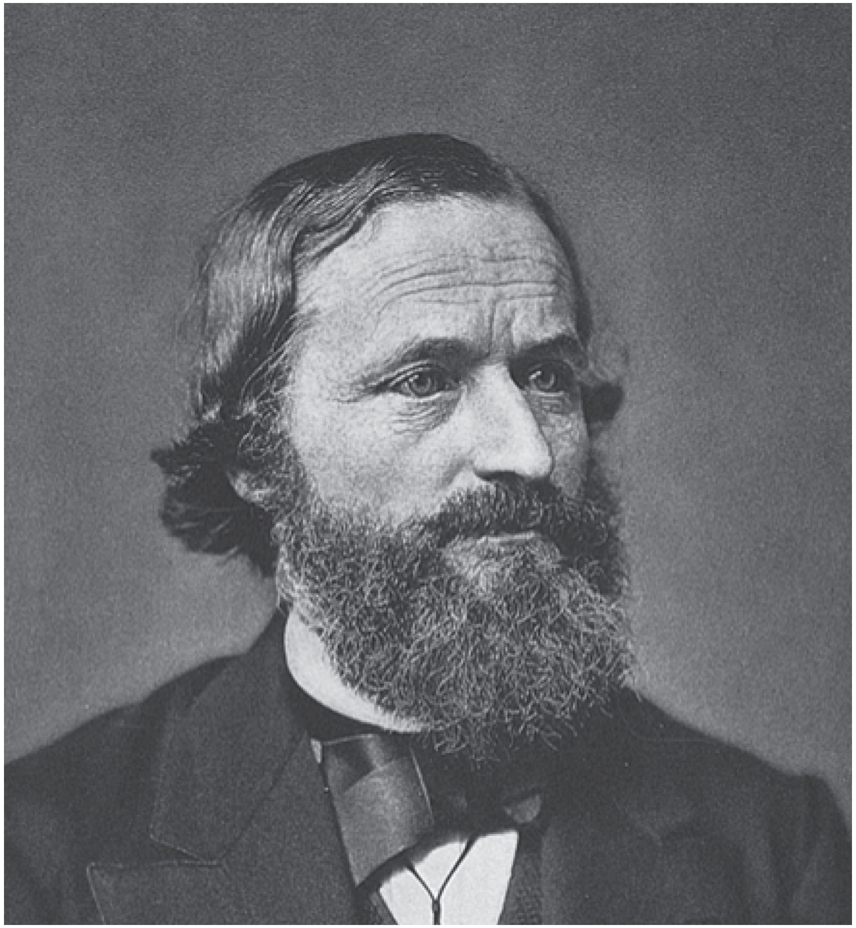
1824 – 1887

German physicist

Worked with Robert Bunsen

Kirchhoff and Bunsen

- Invented the spectroscope and founded the science of spectroscopy
- Discovered the elements cesium and rubidium
- Invented astronomical spectroscopy



# Kirchhoff’s Rules

There are ways in which resistors can be connected so that the circuits formed cannot be reduced to a single equivalent resistor.

Two rules, called **Kirchhoff’s rules**, can be used instead.

# Kirchhoff's Junction Rule

Junction Rule

- The sum of the currents at any junction must equal zero.
  - Currents directed into the junction are entered into the equation as $+I$ and those leaving as $-I$.
  - A statement of Conservation of Charge
- Mathematically,

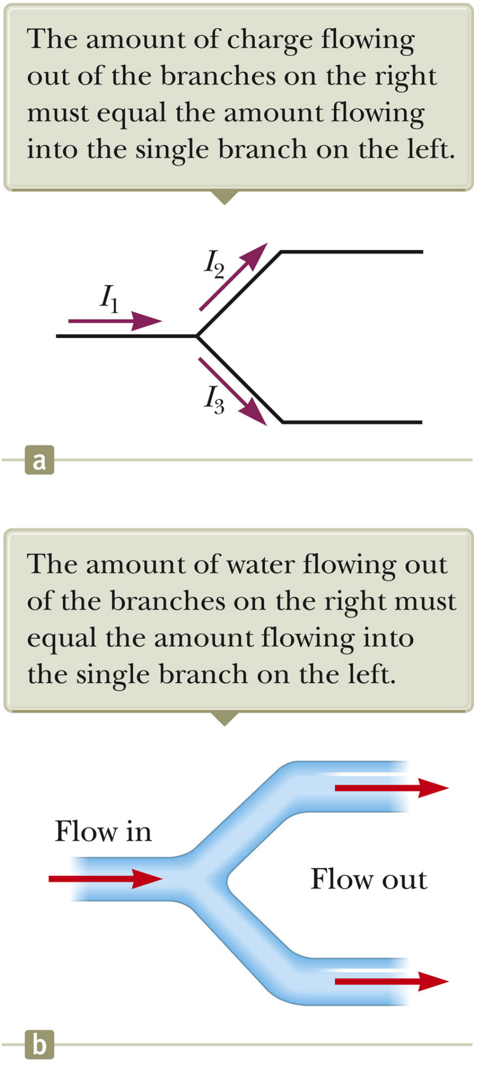
$$\sum_{\text{junction}} I = 0$$

# More about the Junction Rule

$$I_1 - I_2 - I_3 = 0$$

*Required by Conservation of Charge*

Diagram (b) shows a mechanical analog



# Kirchhoff's Loop Rule

Loop Rule

- The sum of the potential differences across all elements around any closed circuit loop must be zero.
  - A statement of Conservation of Energy

Mathematically,

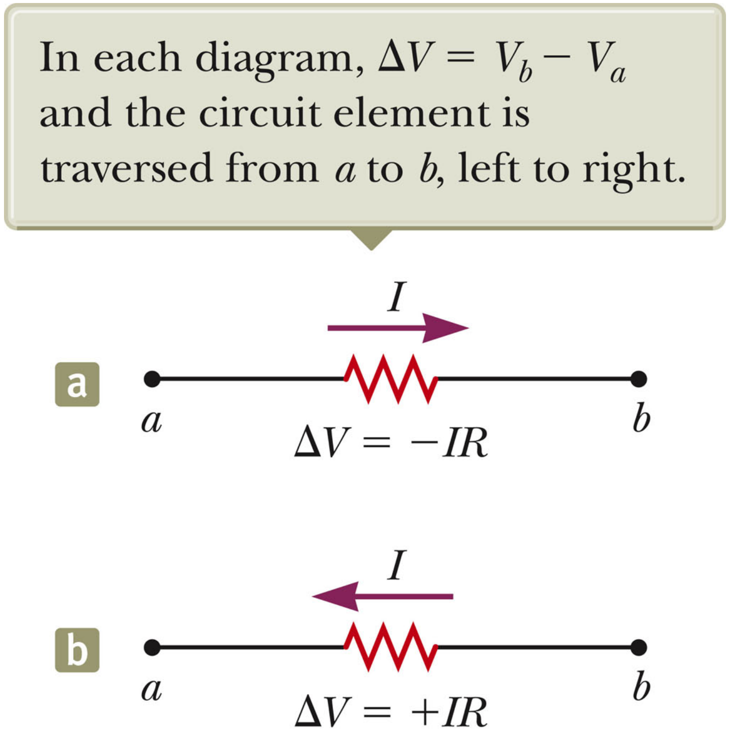
$$\sum_{\text{closed loop}} \Delta V = 0$$

# More about the Loop Rule

Traveling around the loop from *a* to *b*

In (a), the resistor is traversed in the direction of the current, the potential across the resistor is $-IR$.

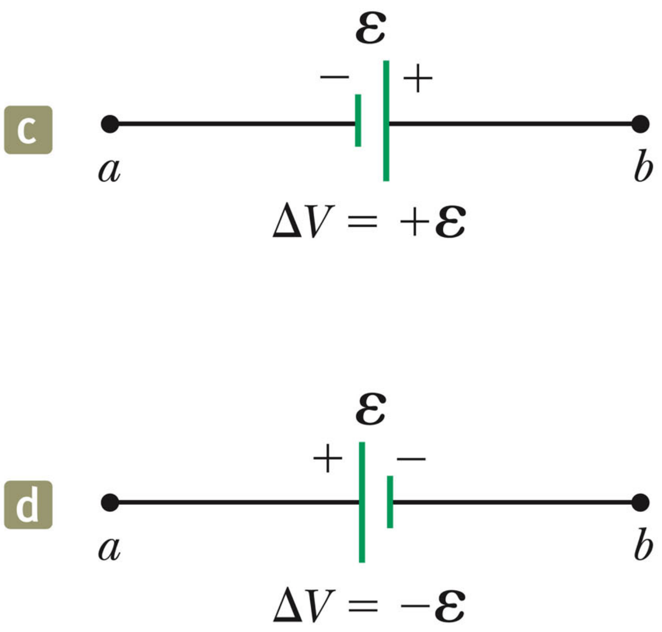
In (b), the resistor is traversed in the direction opposite of the current, the potential across the resistor is is $+IR$.



# Loop Rule, final

In (c), the source of emf is traversed in the direction of the emf (from $-$ to $+$), and the change in the potential difference is $+\varepsilon$.

In (d), the source of emf is traversed in the direction opposite of the emf (from $+$ to $-$), and the change in the potential difference is $-\varepsilon$.



# Equations from Kirchhoff's Rules

Use the junction rule as often as needed, so long as each time you write an equation, you include in it a current that has not been used in a previous junction rule equation.

- In general, the number of times the junction rule can be used is one fewer than the number of junction points in the circuit.

The loop rule can be used as often as needed so long as a new circuit element (resistor or battery) or a new current appears in each new equation.

In order to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents.

Any capacitor acts as an open branch in a circuit.

- The current in the branch containing the capacitor is zero under steady-state conditions.

# Problem-Solving Strategy – Kirchhoff's Rules

*Conceptualize*

- Study the circuit diagram and identify all the elements.
- Identify the polarity of each battery.
- Imagine the directions of the currents in each battery.

*Categorize*

- Determine if the circuit can be reduced by combining series and parallel resistors.
  - If so, proceed with those techniques
  - If not, apply Kirchhoff's Rules

# Problem-Solving Strategy, cont.

*Analyze*

- Assign labels and symbols to all known and unknown quantities.
- Assign directions to the currents.
  - The direction is arbitrary, but you must adhere to the assigned directions when applying Kirchhoff's rules.
- Apply the junction rule to any junction in the circuit that provides new relationships among the various currents.
- Apply the loop rule to as many loops as are needed to solve for the unknowns.
  - To apply the loop rule, you must choose a direction in which to travel around the loop.
  - You must also correctly identify the potential difference as you cross various elements.
- Solve the equations simultaneously for the unknown quantities.

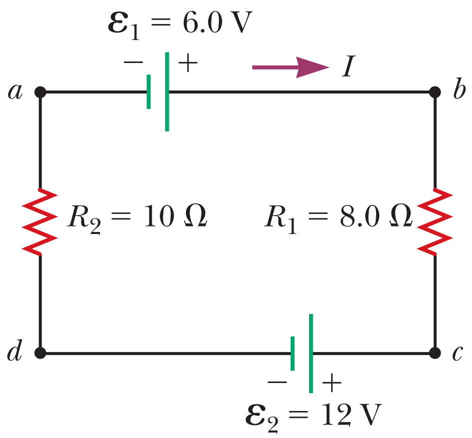
# Problem-Solving Strategy, final

*Finalize*

- Check your numerical answers for consistency.
- If any current value is negative, it means you guessed the direction of that current incorrectly.
  - The magnitude will still be correct.

# Example 27.06: A Single-Loop Circuit

A single-loop circuit contains two resistors and two batteries as shown in the figure. (Neglect the internal resistances of the batteries.) Find the current in the circuit.




Apply Kirchhoff's loop rule to the single loop in the circuit:

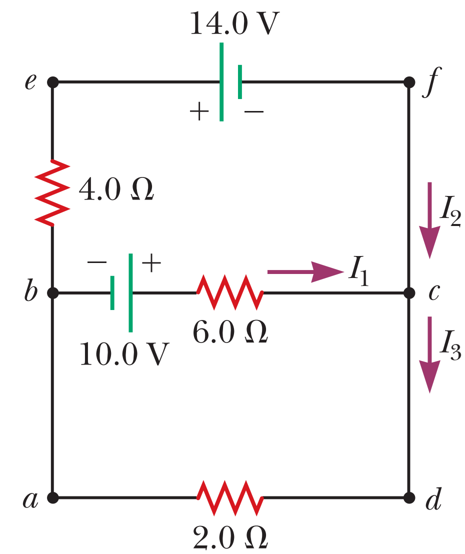
$$\sum \Delta V = 0 \rightarrow \boldsymbol{\varepsilon}_1 - IR_1 - \boldsymbol{\varepsilon}_2 - IR_2 = 0$$

Solve for $I$ and use the values given in the figure:

$$I = \frac{\boldsymbol{\varepsilon}_1 - \boldsymbol{\varepsilon}_2}{R_1 + R_2} = \frac{6.0\ \text{V} - 12\ \text{V}}{8.0\Omega + 10\Omega} = -\,0.33\ \text{A}$$

# Example 27.07: A Multiloop Circuit

Find the currents $I_1$, $I_2$, and $I_3$ in the circuit shown in the figure.




Apply Kirchhoff's junction rule to junction $c$ : (1) $I_1 + I_2 - I_3 = 0$

Apply Kirchhoff's loop rule to loops $abcda$ and $befcb$ :

$$abcda:(2)\quad 10.0\ \text{V} - (6.0\Omega)I_1 - (2.0\Omega)I_3 = 0$$

$$befcb:-(4.0\Omega)I_2 - 14.0\ \text{V} + (6.0\Omega)I_1 - 10.0\ \text{V} = 0$$

$$(3) - 24.0\ \text{V} + (6.0\Omega)I_1 - (4.0\Omega)I_2 = 0$$

Solve Equation (1) for $I_3$ and substitute into Equation (2):

$$10.0\ \text{V} - (6.0\Omega)I_1 - (2.0\Omega)\left(I_1 + I_2\right) = 0$$

$$(4)\quad 10.0\ \text{V} - (8.0\Omega)I_1 - (2.0\Omega)I_2 = 0$$

Multiply each term in Equation (3) by 4 and each term in Equation (4) by 3:

$$(5)\quad -96.0\ \text{V} + (24.0\Omega)I_1 - (16.0\Omega)I_2 = 0$$

$$(6)\quad 30.0\ \text{V} - (24.0\Omega)I_1 - (6.0\Omega)I_2 = 0$$

Add Equation (6) to Equation (5) to eliminate $I_1$ and find $I_2$

$$-66.0\ \text{V} - (22.0\Omega)I_2 = 0$$

$$I_2 = -3.0\ \text{A}$$

Use this value of $I_2$ in Equation (3) to find $I_1$ :

$$-24.0\ \text{V} + (6.0\Omega)I_1 - (4.0\Omega)(-3.0\ \text{A}) = 0$$

$$-24.0\ \text{V} + (6.0\Omega)I_1 + 12.0\ \text{V} = 0$$

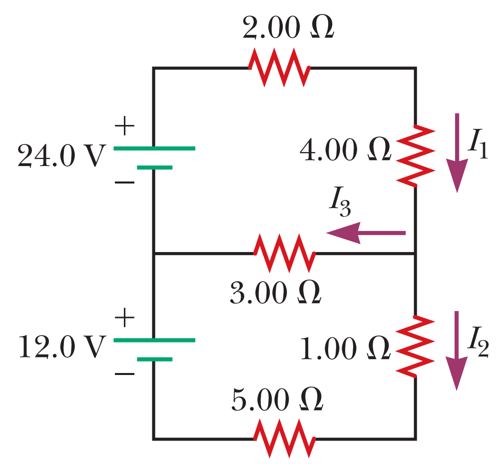
$$I_1 = 2.0\ \text{A}$$

Use Equation (1) to find $I_3$ :

$$I_3 = I_1 + I_2 = 2.0\ \text{A} - 3.0\ \text{A} = -1.0\ \text{A}$$

# Problem 27.21:

(a) Can the circuit shown in the figure be reduced to a single resistor connected to a battery? Explain. Calculate the currents (b) $I_1$, (c) $I_2$, and (d) $I_3$.




(a) No. Some simplification could be made, the $2.0\Omega$ and $4.0\Omega$ resistors are in series, adding to give a total of $6.0Ω$; and the $5.0\Omega$ and $1.0\Omega$ resistors form a series combination with a total resistance of $6.0\Omega$.

The circuit cannot be simplified any further, and Kirchhoff's rules must be used to analyze it.

(b) Applying Kirchhoff's junction rule at junction $a$ gives

(1) $I_1 = I_2 + I_3$

Using Kirchhoff's loop rule on the upper loop yields

$$+24.0\text{ V} - (2.00 + 4.0)I_1 - (3.00)I_3 = 0$$

or (2) $I_3 = 8.00\text{ A} - 2.00I_1$

and for the lower loop,

$$+12.0\text{ V} + (3.00)I_3 - (1.00 + 5.00)I_2 = 0$$

Using equation [2], this reduces to

$$I_2 = \frac{12.0\text{ V} + 3.00\left(8.00\text{ A} - 2.00I_1\right)}{6.00}$$

giving

(3) $I_2 = 6.00\text{ A} - 1.00I_1$

Substituting equations [2] and [3] into [1] gives

$$I_1 = 3.50\text{ A}$$

(c) Then, equation [3] gives

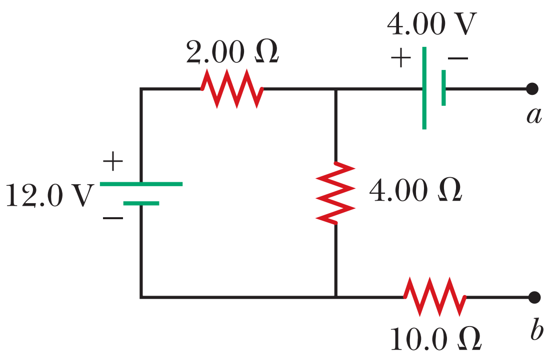
$I_2 = 2.50\text{ A}$, and

(d) Equation [2] yields

$$I_3 = 1.00\text{ A}$$

# Problem 27.37:

(a) Calculate the potential difference between points $a$ and $b$ in the figure and (b) identify which point is at the higher potential.




(a) Using Kirchhoff's loop rule for the closed loop,

$$+12.0 - 2.00I - 4.00I = 0$$

so $I = 2.00\ \text{A}$

Then,

$$V_b - V_a = +4.00\ \text{V} - (2.00\ \text{A})(4.00\Omega) - (0)(10.0\Omega) = -4.00\ \text{V}$$

Thus, $\left|\Delta V_{ab}\right| = 4.00\ \text{V}$

(b) $V_b - V_a = -4.00\ \text{V} \rightarrow V_a = V_b + 4.00\ \text{V}$;

thus, $a$ is at the higher potential.