Chapter 25

Capacitance and Dielectrics

Circuits and Circuit Elements

Electric circuits are the basis for the vast majority of the devices used in society.

Circuit elements can be connected with wires to form electric circuits.

Capacitors are one circuit element.

- Others will be introduced in other chapters

Capacitors

Capacitors are devices that store electric charge.

Examples of where capacitors are used include:

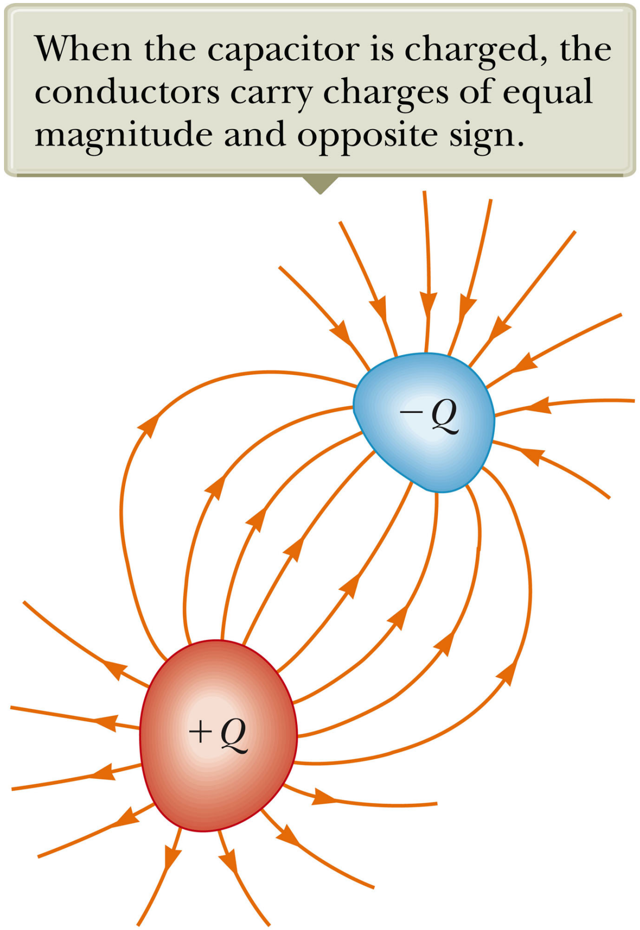
- radio receivers
- filters in power supplies
- to eliminate sparking in automobile ignition systems
- energy-storing devices in electronic flashes

Makeup of a Capacitor

A capacitor consists of two conductors.

- These conductors are called plates.
- When the conductor is charged, the plates carry charges of equal magnitude and opposite directions.

A potential difference exists between the plates due to the charge.



Definition of Capacitance

The **capacitance**, C, of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the potential difference between the conductors.

$$C \equiv \frac{Q}{\Delta V}$$

The SI unit of capacitance is the **farad** (F).

The farad is a large unit, typically you will see microfarads (mF) and picofarads (pF).

Capacitance will always be a positive quantity

The capacitance of a given capacitor is constant.

The capacitance is a measure of the capacitor's ability to store charge .

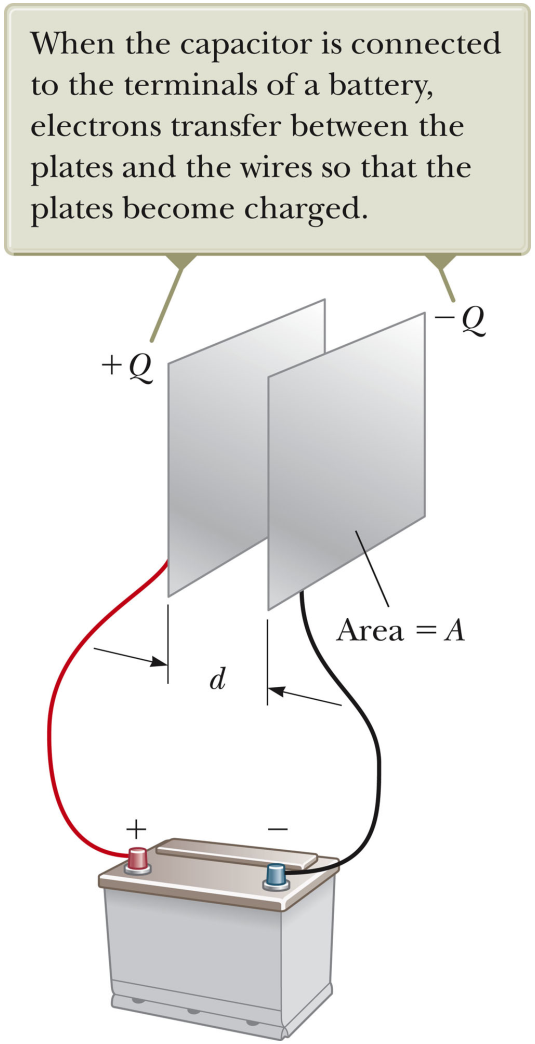
- The capacitance of a capacitor is the amount of charge the capacitor can store per unit of potential difference.

Parallel Plate Capacitor

Each plate is connected to a terminal of the battery.

- The battery is a source of potential difference.

If the capacitor is initially uncharged, the battery establishes an electric field in the connecting wires.



Parallel Plate Capacitor, cont

This field applies a force on electrons in the wire just outside of the plates.

The force causes the electrons to move onto the negative plate.

This continues until equilibrium is achieved.

- The plate, the wire and the terminal are all at the same potential.

At this point, there is no field present in the wire and the movement of the electrons ceases.

The plate is now negatively charged.

A similar process occurs at the other plate, electrons moving away from the plate and leaving it positively charged.

In its final configuration, the potential difference across the capacitor plates is the same as that between the terminals of the battery.

Problem 25.01:

(a) When a battery is connected to the plates of a $3.00-\mu\mathrm{F}$ capacitor, it stores a charge of $27.0\mu\mathrm{C}$. What is the voltage of the battery? (b) If the same capacitor is connected to another battery and $36.0\mu\mathrm{C}$ of charge is stored on the capacitor, what is the voltage of the battery?

(a) From the definition of capacitance, $C=\frac{Q}{\Delta V}$, we have

$$\Delta V=\frac{Q}{C}=\frac{27.0\mu C}{3.00\mu\ \mathrm{F}}=9.00\ \mathrm{V}$$

(b) Similarly,

$$\Delta V=\frac{Q}{C}=\frac{36.0\mu C}{3.00\mu\ \mathrm{F}}=12.0\ \mathrm{V}$$

Capacitance – Parallel Plates

The charge density on the plates is $\sigma = Q/A$.

- A is the area of each plate, the area of each plate is equal
- Q is the charge on each plate, equal with opposite signs

The electric field is uniform between the plates and zero elsewhere.

The capacitance is proportional to the area of its plates and inversely proportional to the distance between the plates.

$$C = \frac{Q}{\Delta V} = \frac{Q}{Ed} = \frac{Q}{Qd/\varepsilon_0 A} = \frac{\varepsilon_0 A}{d}$$

Problem 25.03:

When a potential difference of $150\ \mathrm{V}$ is applied to the plates of a parallel-plate capacitor, the plates carry a surface charge density of $30.0\mathrm{nC/cm^2}$. What is the spacing between the plates?

We have $Q = C\Delta V$ and $C = \epsilon_0 A/d$. Thus, $Q = \epsilon_0 A\Delta V/d$
The surface charge density on each plate has the same magnitude, given by

$$\sigma = \frac{Q}{A} = \frac{\epsilon_0 \Delta V}{d}$$

Thus,

$$d = \frac{\epsilon_0 \Delta V}{Q/A} = \frac{\left(8.85\times 10^{-12}\mathrm{C^2/N\cdot\ m^2}\right)(150\ \mathrm{V})}{\left(30.0\times 10^{-9}\mathrm{C/cm^2}\right)} = 4.43\ \mu\mathrm{m}$$

Problem 25.04:

An air-filled parallel-plate capacitor has plates of area $2.30\ \text{cm}^2$ separated by $1.50\ \text{mm}$. (a) Find the value of its capacitance. The capacitor is connected to a $12.0-\text{V}$ battery. (b) What is the charge on the capacitor? (c) What is the magnitude of the uniform electric field between the plates?

(a)

$$C=\frac{\kappa\epsilon_0 A}{d}=\frac{(1.00)\left(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\ \text{m}^2\right)\left(2.30\times10^{-4}\ \text{m}^2\right)}{1.50\times10^{-3}\ \text{m}}$$
$$=1.36\times10^{-12}\ \text{F}=1.36\text{pF}$$

(b) $Q=C\Delta V=(1.36\text{pF})(12.0\ \text{V})=16.3\text{pC}$

(c) $E=\frac{\Delta V}{d}=\frac{12.0\ \text{V}}{1.50\times10^{-3}\ \text{m}}=8.00\times10^3\ \text{V/m}$

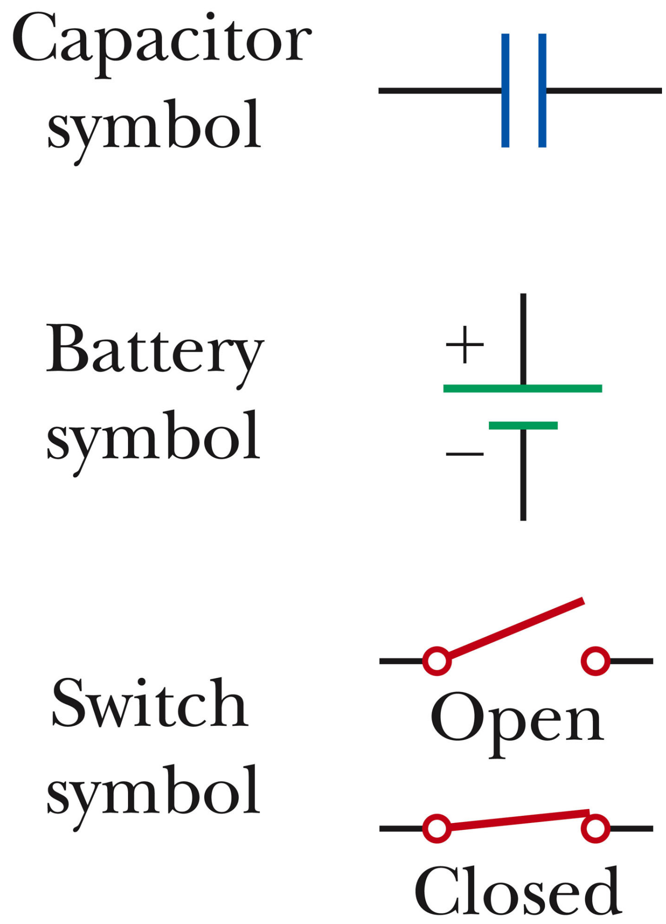
Circuit Symbols

A circuit diagram is a simplified representation of an actual circuit.

Circuit symbols are used to represent the various elements.

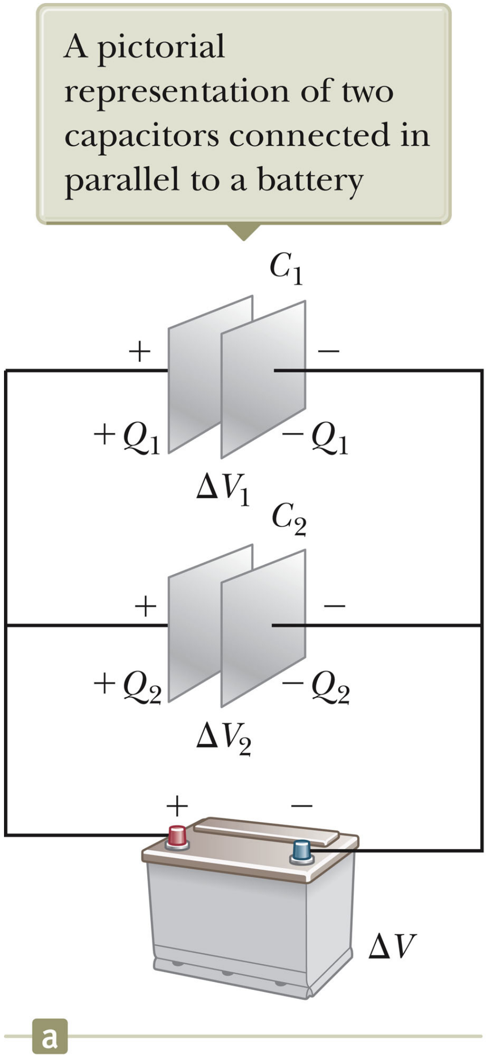
Lines are used to represent wires.

The battery's positive terminal is indicated by the longer line.



Capacitors in Parallel

When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged.



Capacitors in Parallel, 2

The flow of charges ceases when the voltage across the capacitors equals that of the battery.

The potential difference across the capacitors is the same.

- And each is equal to the voltage of the battery
- $\Delta V_1 = \Delta V_2 = \Delta V$
  - ΔV is the battery terminal voltage

The capacitors reach their maximum charge when the flow of charge ceases.

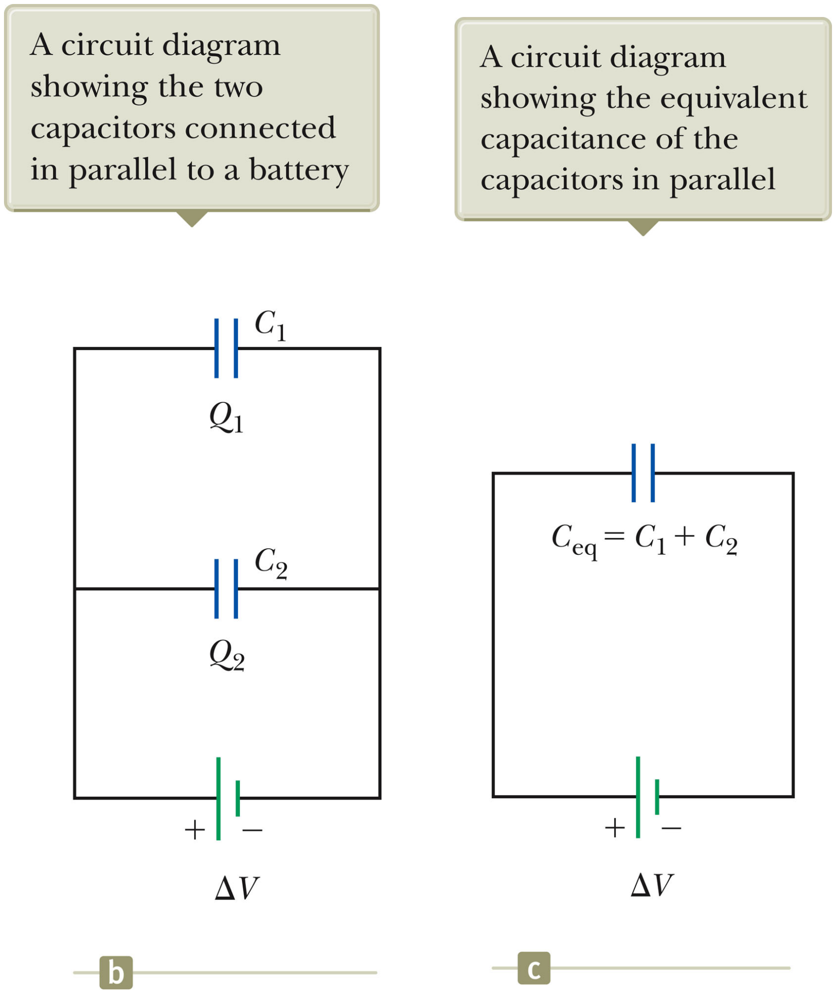
The total charge is equal to the sum of the charges on the capacitors.

- $Q_{\text{tot}} = Q_1 + Q_2$

Capacitors in Parallel, 3

The capacitors can be replaced with one capacitor with a capacitance of C_{eq}.

- The *equivalent capacitor* must have exactly the same external effect on the circuit as the original capacitors.



Capacitors in Parallel, final

$$C_{eq} = C_1 + C_2 + C_3 + \ldots$$

The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors.

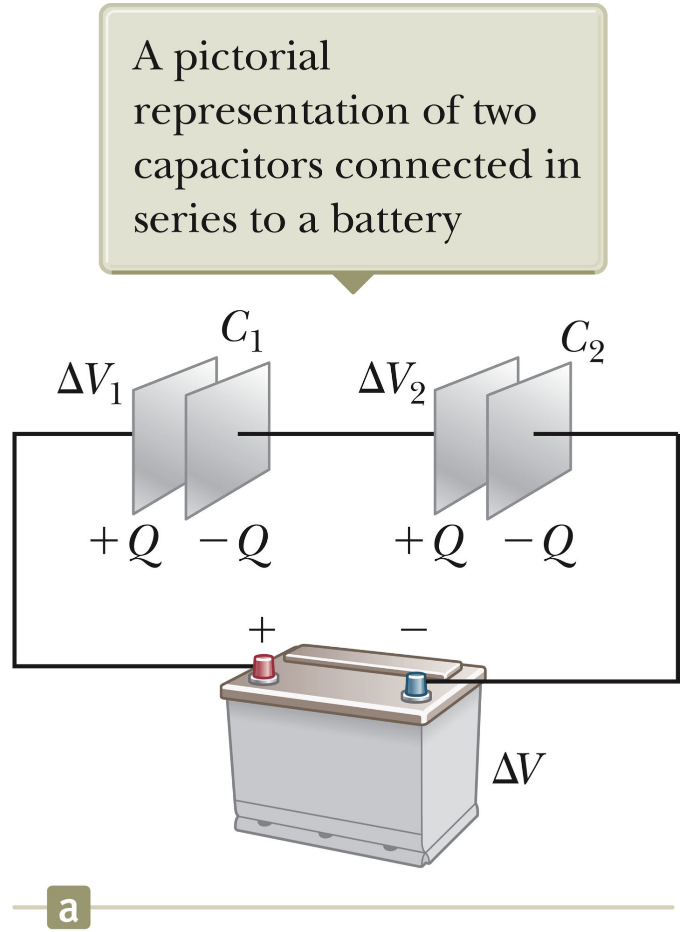
- Essentially, the areas are combined

Capacitors in Series

When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery.

As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is removed from the left plate of C_2, leaving it with an excess positive charge.

All of the right plates gain charges of $-Q$ and all the left plates have charges of $+Q$.

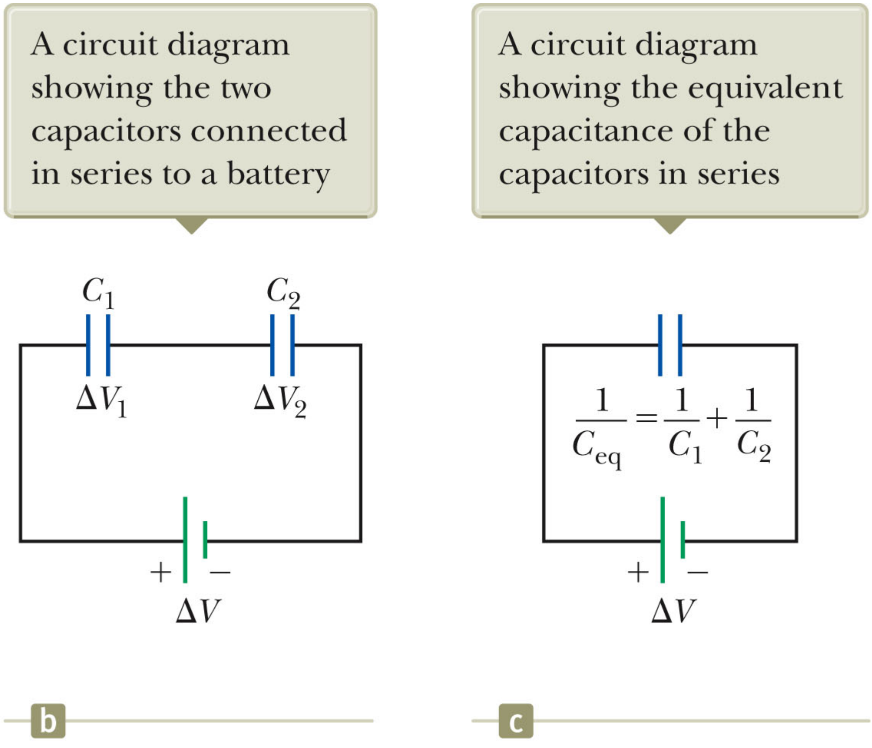


Capacitors in Series, cont.

An equivalent capacitor can be found that performs the same function as the series combination.

The charges are all the same.

$$Q_1 = Q_2 = Q$$



Capacitors in Series, final

The potential differences add up to the battery voltage.

$$\Delta V_{\text{tot}} = \Delta V_1 + \Delta V_2 + \dots$$

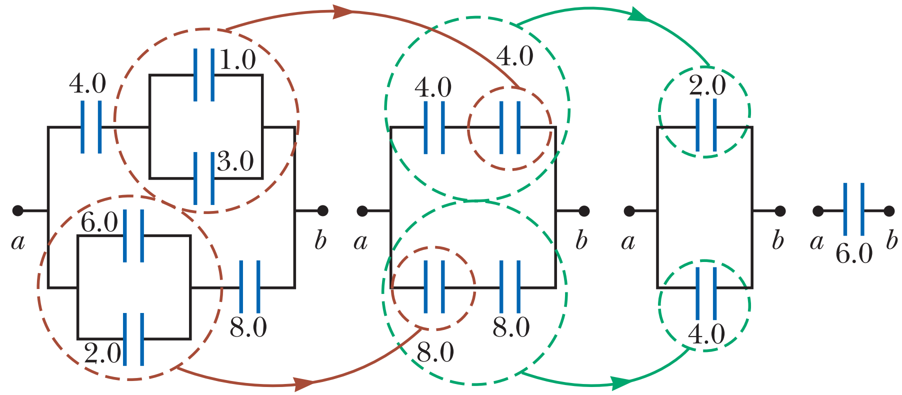
The equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

The equivalent capacitance of a series combination is always less than any individual capacitor in the combination.

Example 25.03: Equivalent Capacitance

Find the equivalent capacitance between a and b for the combination of capacitors shown in the figure. All capacitances are in microfarads.



$$C_{eq} = C_1 + C_2 = 4.0\mu \text{ F}$$

$$C_{eq} = C_1 + C_2 = 8.0\mu \text{ F}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.0\mu \text{ F}} + \frac{1}{4.0\mu \text{ F}} = \frac{1}{2.0\mu \text{ F}}$$

$$C_{eq} = 2.0\mu \text{ F}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{8.0\mu \text{ F}} + \frac{1}{8.0\mu \text{ F}} = \frac{1}{4.0\mu \text{ F}}$$

$$C_{eq} = 4.0\mu \text{ F}$$

$$C_{eq} = C_1 + C_2 = 6.0\mu \text{ F}$$

Problem 25.07:

Find the equivalent capacitance of a $4.20-\mu\mathrm{F}$ capacitor and an $8.50-\mu\mathrm{F}$ capacitor when they are connected (a) in series and (b) in parallel.

(a) When connected in series, the equivalent capacitance is

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$, or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.20\mu\ \mathrm{F}} + \frac{1}{8.50\mu\ \mathrm{F}} \rightarrow C_{eq} = 2.81\mu\ \mathrm{F}$$

(b) When connected in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 = 4.20\mu\ \mathrm{F} + 8.50\mu\ \mathrm{F} = 12.70\mu\ \mathrm{F}$$

Problem 25.09:

A group of identical capacitors is connected first in series and then in parallel. The combined capacitance in parallel is 100 times larger than for the series connection. How many capacitors are in the group?

Call C the capacitance of one capacitor and n the number of capacitors. The equivalent capacitance for n capacitors in parallel is

$$C_p = C_1 + C_2 + \cdots + C_n = nC$$

The relationship for n capacitors in series is

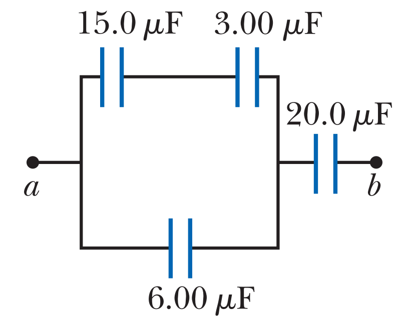
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} = \frac{n}{C}$$

Therefore,

$$\frac{C_p}{C_s} = \frac{nC}{C/n} = n^2 \quad \text{or} \quad n = \sqrt{C_p/C_s} = \sqrt{100} = 10$$

Problem 25.11:

Four capacitors are connected as shown in the figure. (a) Find the equivalent capacitance between points a and b. (b) Calculate the charge on each capacitor, taking $\Delta V_{ab} = 15.0\,\text{V}$.



(a) $$\frac{1}{(1/15.0\ \mu\text{F}) + (1/3.00\ \mu\text{F})} = 2.50\ \mu\text{F}$$

$$(2.5\ \mu\text{F}) + (6\ \mu\text{F}) = 8.50\ \mu\text{F}$$

$$\frac{1}{(1/8.50\ \mu\text{F}) + (1/20.00\ \mu\text{F})} = 5.96\ \mu\text{F}$$

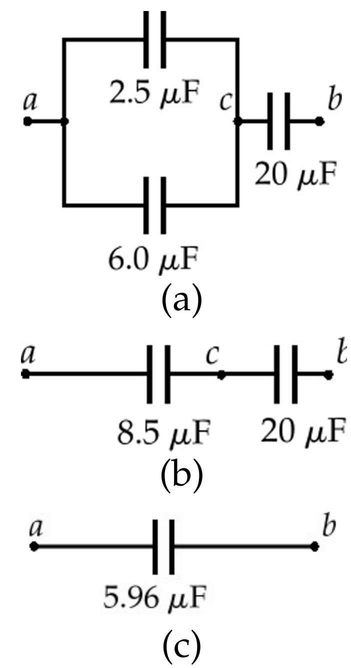
(b) We find the charge on each capacitor and the voltage across each by working backwards through solution figures (c) - (a), alternately applying $Q = C\Delta V$ and $\Delta V = Q/C$ to every capacitor, real or equivalent.

$$Q_{20} = C\Delta V = (5.96\ \mu\text{F})(15.0\ \text{V}) = 89.5\mu\text{C}$$

$$\Delta V_{ac} = \frac{Q}{C} = \frac{89.5\mu\text{C}}{8.50\mu\ \text{F}} = 10.5\ \text{V},\ \ \Delta V_{cb} = \frac{Q}{C} = \frac{89.5\mu\text{C}}{20.0\mu\ \text{F}} = 4.47\ \text{V}$$

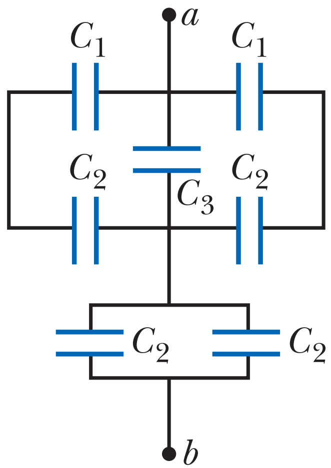
$$Q_3 = Q_{15} = C\Delta V = (2.50\ \mu\text{F})(10.5\ \text{V}) = 26.3\,\mu\text{C}$$

$$Q_6 = C\Delta V = (6.00\ \mu\text{F})(10.5\ \text{V}) = 63.2\,\mu\text{C}$$



Problem 25.12:

(a) Find the equivalent capacitance between points a and b for the group of capacitors connected as shown in the figure. Take $C_1 = 5.00\mu\ \mathrm{F}$, $C_2 = 10.0\mu\ \mathrm{F}$, and $C_3 = 2.00\mu\ \mathrm{F}$. (b) What charge is stored on C_3 if the potential difference between points a and b is $60.0\ \mathrm{V}$?



$$C_s = \left(\frac{1}{5.00} + \frac{1}{10.0}\right)^{-1} = 3.33\ \mu\mathrm{F}$$

$$C_{\text{upper}} = 2(3.33) + 2.00 = 8.67\ \mu\mathrm{F}$$

$$C_{\text{lower}} = 2(10.0) = 20.0\ \mu\mathrm{F}$$

$$C_{\text{eq}} = \left(\frac{1}{8.67} + \frac{1}{20.0}\right)^{-1} = 6.05\ \mu\mathrm{F}$$

$$Q_{\text{upper}} = Q_{\text{eq}} = C_{\text{eq}}\Delta V = (6.05\mu F)(60.0\ \mathrm{V}) \simeq 363\mu C$$

$$\Delta V_{\text{upper}} = \frac{Q_{\text{eq}}}{C_{\text{eq}}} = \frac{363\mu C}{8.67\mu F} \simeq 41.9\ \mathrm{V}$$

Therefore, the charge on C_3 is

$$Q_3 = C_3\Delta V_3 \simeq (2.00\mu F)(41.9\ \mathrm{V}) = 83.7\mu\mathrm{C}$$

Energy in a Capacitor – Overview

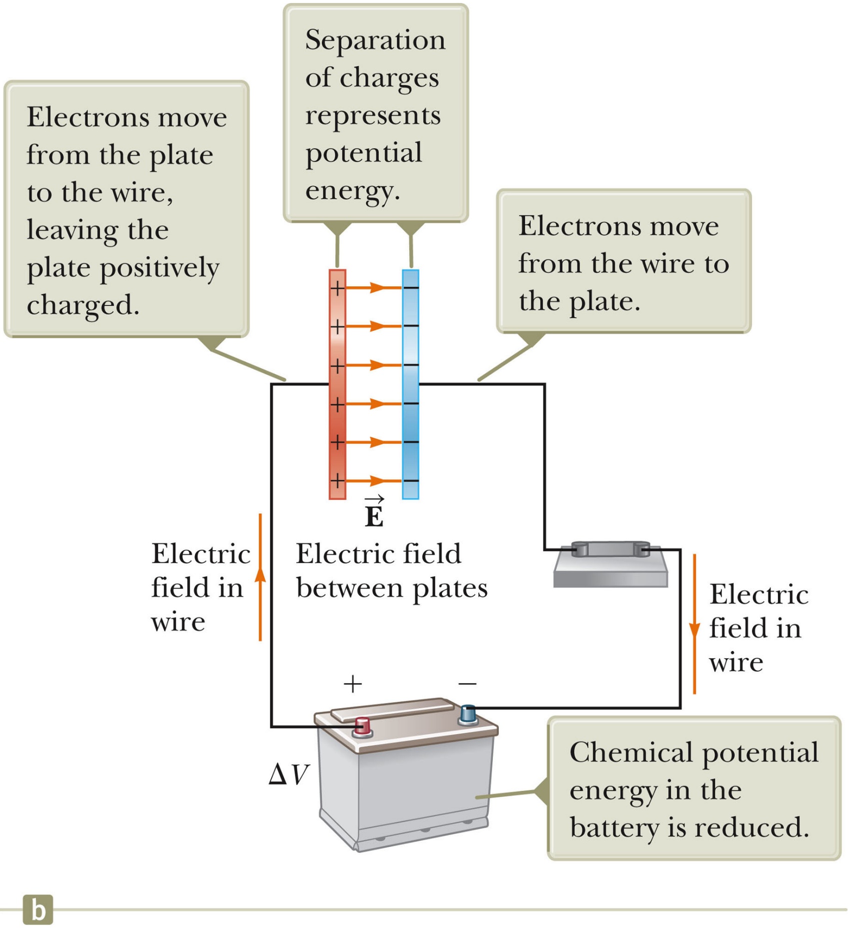
Consider the circuit to be a system.

Before the switch is closed, the energy is stored as chemical energy in the battery.

When the switch is closed, the energy is transformed from chemical potential energy to electric potential energy.

The electric potential energy is related to the separation of the positive and negative charges on the plates.

A capacitor can be described as a device that stores energy as well as charge.



Energy Stored in a Capacitor

Assume the capacitor is being charged and, at some point, has a charge q on it.

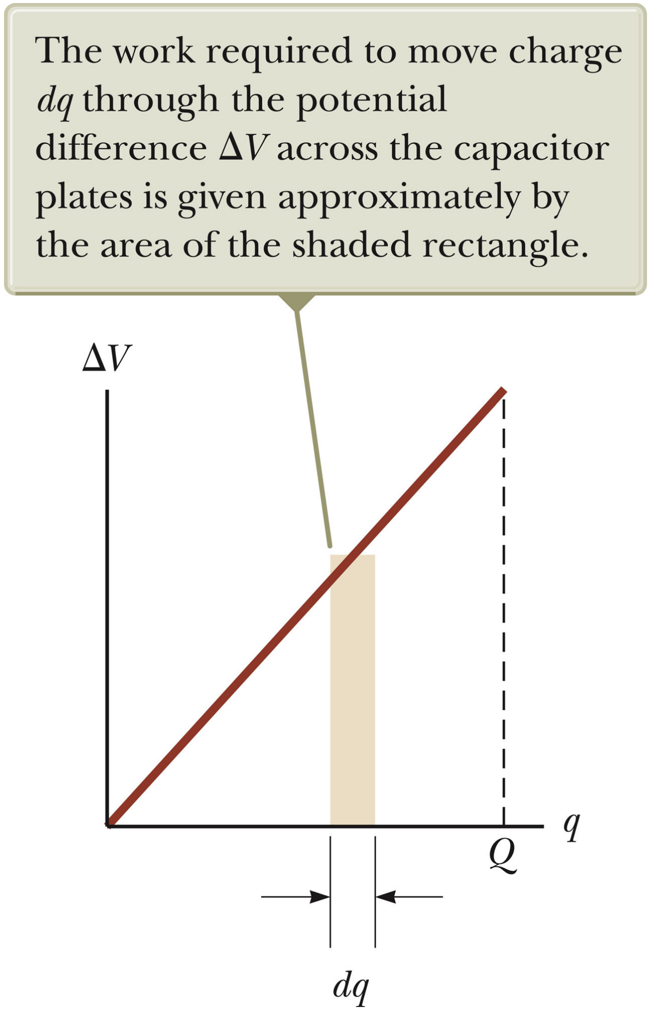
The work needed to transfer a charge from one plate to the other is

$$dW = \Delta V dq = \frac{q}{C} dq$$

The work required is the area of the tan rectangle.

The total work required is

$$W = \int_0^Q \frac{q}{C} dq = \frac{1}{C} \int_0^Q q dq = \frac{Q^2}{2C}$$



Energy, cont

The work done in charging the capacitor appears as electric potential energy U:

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$$

This applies to a capacitor of any geometry.

The energy stored increases as the charge increases and as the potential difference increases.

In practice, there is a maximum voltage before discharge occurs between the plates.

Energy, final

The energy can be considered to be stored in the electric field .

For a parallel-plate capacitor, the energy can be expressed in terms of the field as:

$$U_E = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2 = \frac{1}{2}\left(\epsilon_0 A d\right) E^2$$

It can also be expressed in terms of the energy density (energy per unit volume).

$$u_E = \frac{1}{2}\epsilon_0 E^2$$

Some Uses of Capacitors

Defibrillators

- When cardiac fibrillation occurs, the heart produces a rapid, irregular pattern of beats
- A fast discharge of electrical energy through the heart can return the organ to its normal beat pattern.

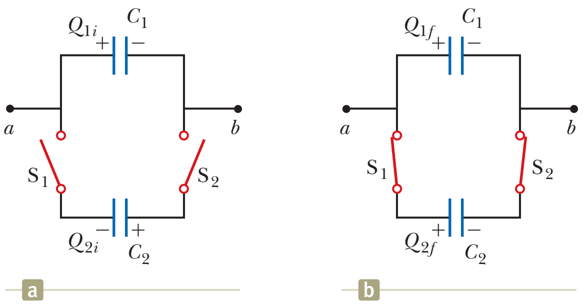
In general, capacitors act as energy reservoirs that can be slowly charged and then discharged quickly to provide large amounts of energy in a short pulse.

Example 25.04: Rewiring Two Charged Capacitors

Two capacitors C_1 and C_2 (where $C_1 > C_2$) are charged to the same initial potential difference ΔV_i. The charged capacitors are removed from the battery, and their plates are connected with opposite polarity as in the figure a. The switches S_2 and S_2 are then closed as in the figure b.

(A) Find the final potential difference ΔV_f between a and b after the switches are closed.

(B) Find the total energy stored in the capacitors before and after the switches are closed and determine the ratio of the final energy to the initial energy.



(1) $Q_i = Q_{1i} + Q_{2i} = C_1\Delta V_i - C_2\Delta V_i = (C_1 - C_2)\,\Delta V_i$

(2)

$$Q_f = Q_{1f} + Q_{2f} = C_1\Delta V_f + C_2\Delta V_f = (C_1 + C_2)\,\Delta V_f$$

$$Q_f = Q_i \rightarrow (C_1 + C_2)\,\Delta V_f = (C_1 - C_2)\,\Delta V_i$$

(3) $\Delta V_f = \left(\dfrac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i$

(4)

$$U_i = \tfrac{1}{2}C_1(\Delta V_i)^2 + \tfrac{1}{2}C_2(\Delta V_i)^2 = \tfrac{1}{2}(C_1 + C_2)(\Delta V_i)^2$$

$$U_f = \frac{1}{2}C_1\left(\Delta V_f\right)^2 + \frac{1}{2}C_2\left(\Delta V_f\right)^2 = \frac{1}{2}(C_1 + C_2)\left(\Delta V_f\right)^2$$

(5)

$$U_f = \frac{1}{2}(C_1 + C_2)\left[\left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i\right]^2 = \frac{1}{2}\frac{(C_1 - C_2)^2(\Delta V_i)^2}{C_1 + C_2}$$

$$\frac{U_f}{U_i} = \frac{\frac{1}{2}(C_1 - C_2)^2(\Delta V_i)^2/(C_1 + C_2)}{\frac{1}{2}(C_1 + C_2)(\Delta V_i)^2}$$

(6) $\dfrac{U_f}{U_i} = \left(\dfrac{C_1 - C_2}{C_1 + C_2}\right)^2$

Problem 25.17:

A $3.00-\mu\text{F}$ capacitor is connected to a $12.0-\text{V}$ battery. How much energy is stored in the capacitor? (b) Had the capacitor been connected to a $6.00-\text{V}$ battery, how much energy would have been stored?

(a) $U_E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\mu\ \text{F})(12.0\ \text{V})^2 = 216\mu\ \text{J}$

(b) $U_E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\mu\ \text{F})(6.00\ \text{V})^2 = 54.0\mu\ \text{J}$

Problem 25.35:

A uniform electric field $E = 3000\ \mathrm{V/m}$ exists within a certain region. What volume of space contains an energy equal to $1.00 \times 10^{-7}\ \mathrm{J}$? Express your answer in cubic meters and in liters.

$$u_E = \frac{U_E}{V} = \frac{1}{2}\epsilon_0 E^2$$

Solving for the volume gives

$$V = \frac{U_E}{\frac{1}{2}\epsilon_0 E^2} = \frac{1.00 \times 10^{-7}\ \mathrm{J}}{\frac{1}{2}\left(8.85 \times 10^{-12} \mathrm{C^2/N \cdot\ m^2}\right)(3000\ \mathrm{V/m})^2}$$

$$= 2.51 \times 10^{-3}\ \mathrm{m^3} = \left(2.51 \times 10^{-3}\ \mathrm{m^3}\right)\left(\frac{1000\ \mathrm{L}}{\mathrm{m^3}}\right) = 2.51\ \mathrm{L}$$

Capacitors with Dielectrics

A dielectric is a nonconducting material that, when placed between the plates of a capacitor, increases the capacitance.

- Dielectrics include rubber, glass, and waxed paper

With a dielectric, the capacitance becomes $C = \kappa C_o$.

- The capacitance increases by the factor κ when the dielectric completely fills the region between the plates.
- κ is the dielectric constant of the material.

If the capacitor remains connected to a battery, the voltage across the capacitor necessarily remains the same.

If the capacitor is disconnected from the battery, the capacitor is an isolated system and the charge remains the same.

Dielectrics, cont

For a parallel-plate capacitor, $C = \kappa(\varepsilon_o A)/d$

In theory, d could be made very small to create a very large capacitance.

In practice, there is a limit to d.

- d is limited by the electric discharge that could occur though the dielectric medium separating the plates.

For a given d, the maximum voltage that can be applied to a capacitor without causing a discharge depends on the **dielectric strength** of the material.

Dielectrics, final

Dielectrics provide the following advantages:

- Increase in capacitance
- Increase the maximum operating voltage
- Possible mechanical support between the plates
  - This allows the plates to be close together without touching.
  - This decreases d and increases C.

TABLE 25.1 Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

Material	Dielectric Constant κ	Dielectric Strength[a] (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polyethylene	2.30	18
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—

[a]The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. These values depend strongly on the presence of impurities and flaws in the materials.

Example 25.05: Energy Stored Before and After

A parallel-plate capacitor is charged with a battery to a charge Q_0. The battery is then removed, and a slab of material that has a dielectric constant κ is inserted between the plates. Identify the system as the capacitor and the dielectric. Find the energy stored in the system before and after the dielectric is inserted.

$$U_0 = \frac{{Q_0}^2}{2C_0}$$

$$U_E = \frac{{Q_0}^2}{2C}$$

$$U_E = \frac{{Q_0}^2}{2\kappa C_0} = \frac{U_0}{\kappa}$$

Problem 25.25:

Determine (a) the capacitance and (b) the maximum potential difference that can be applied to a Teflon-filled parallel-plate capacitor having a plate area of $1.75\ \text{cm}^2$ and a plate separation of $0.04\,\text{mm}$.

(a)

$$C = \frac{\kappa \in_0 A}{d} = \frac{2.10\left(8.85 \times 10^{-12}\ \text{F/m}\right)\left(1.75 \times 10^{-4}\ \text{m}^2\right)}{4.00 \times 10^{-5}\ \text{m}} = 8.13 \times 10^{-11}\ \text{F}$$

$$= 81.3\text{pF}$$

(b)

$$\Delta V_{\text{max}} = E_{\text{max}} d = \left(60.0 \times 10^6\ \text{V/m}\right)\left(4.00 \times 10^{-5}\ \text{m}\right) = 2.40\text{kV}$$