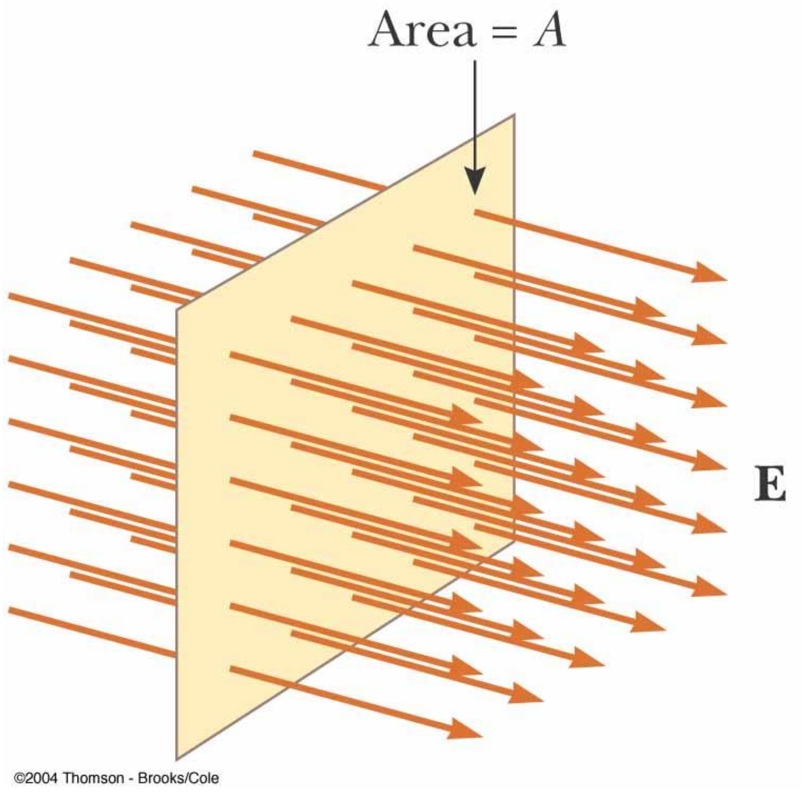
Chapter 24: Gauss's Law

Electric Flux

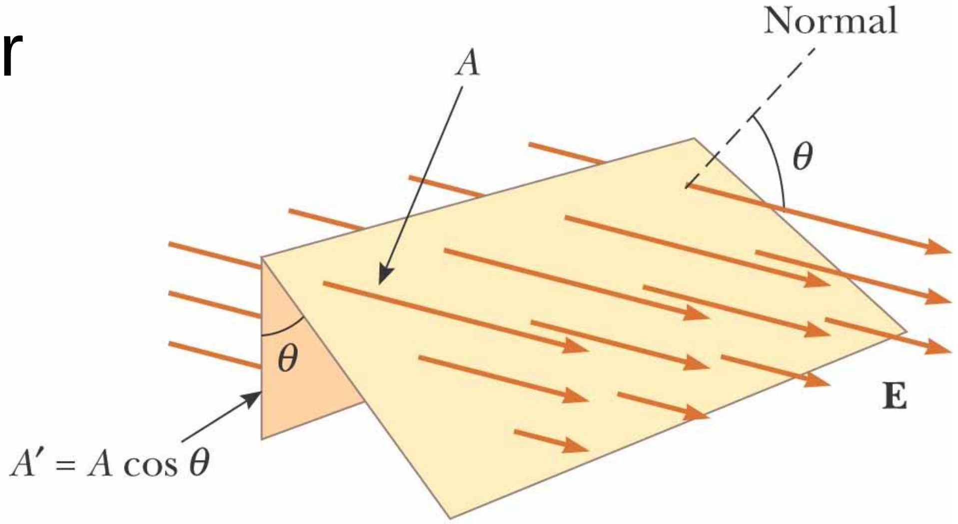
Electric Flux

- ***Electric flux*** is the product of the magnitude of the electric field and the surface area, *A*, perpendicular to the field
- $\Phi_E = EA$



Electric Flux, General Area

- The field lines may make some angle θ with the perpendicular to the surface
- Then $\Phi_E = EA \cos \theta$



Electric Flux, Interpreting the Equation

- The flux is a maximum when the surface is perpendicular to the field
- The flux is zero when the surface is parallel to the field
- If the field varies over the surface, $\Phi = EA \cos \theta$ is valid for only a small element of the area

Example

EXAMPLE 24.1 Flux Through a Sphere

What is the electric flux through a sphere that has a radius of 1.00 m and carries a charge of +1.00 μC at its center?

Solution The magnitude of the electric field 1.00 m from this charge is given by Equation 23.4,

$$E = k_e \frac{q}{r^2} = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \frac{1.00 \times 10^{-6}\ \mathrm{C}}{(1.00\ \mathrm{m})^2}$$

$$= 8.99 \times 10^3\ \mathrm{N/C}$$

Example

$$\Phi_E = EA = (8.99 \times 10^3 \text{ N/C})(12.6 \text{ m}^2)$$

$$= 1.13 \times 10^5 \text{ N}\cdot\text{m}^2/\text{C}$$

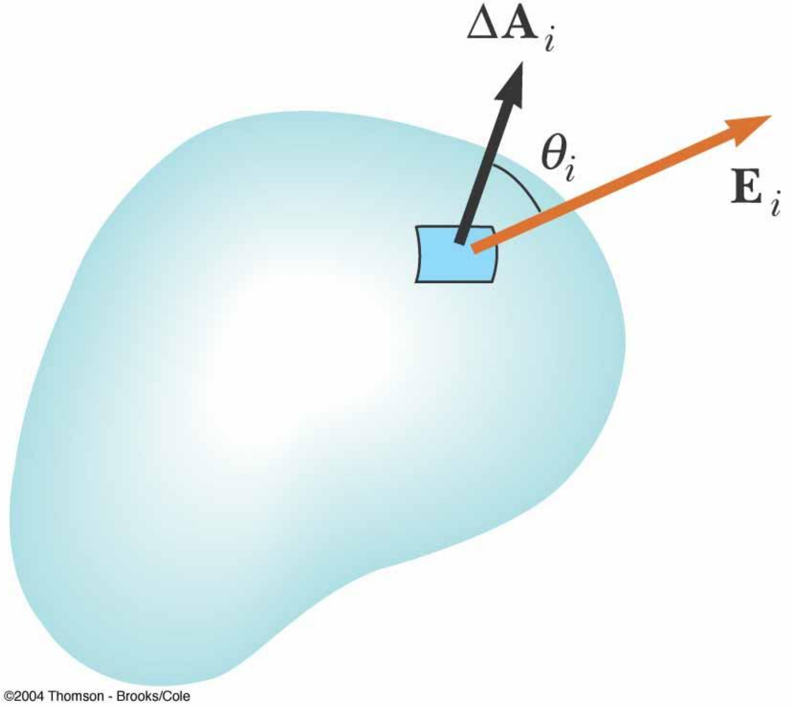
Exercise What would be the (a) electric field and (b) flux through the sphere if it had a radius of 0.500 m?

Answer (a) 3.60×10^4 N/C; (b) 1.13×10^5 N·m^2/C.

Electric Flux, General

- In the more general case, look at a small area element

$$\Delta\Phi_E = E_i \Delta A_i \cos\theta_i = \mathbf{E}_i \cdot \Delta\mathbf{A}_i$$



- In general, this becomes

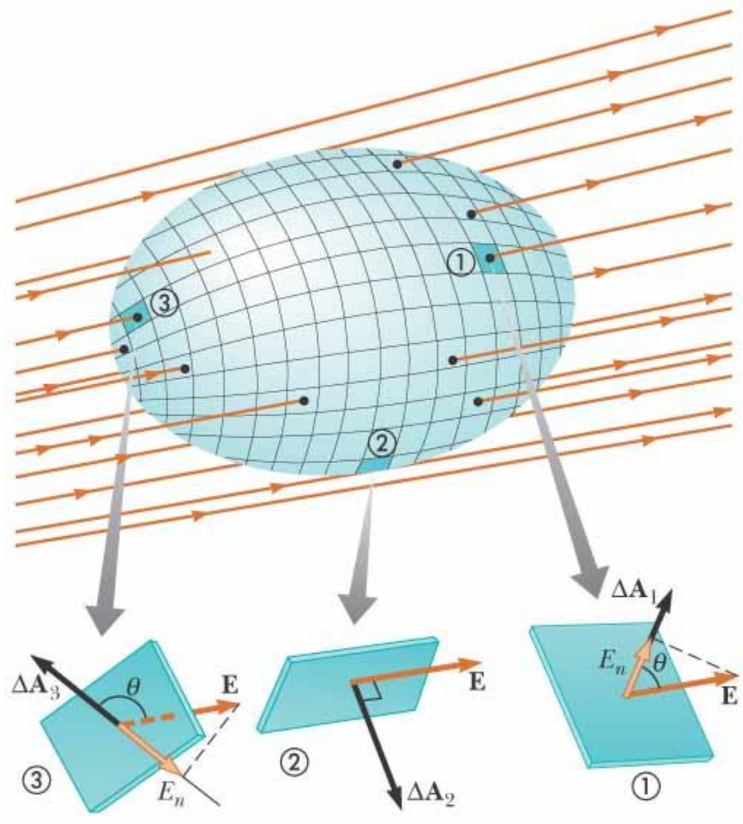
$$\Phi_E = \lim_{\Delta A_i \to 0} \sum E_i \cdot \Delta A_i = \int_{\text{surface}} \mathbf{E} \cdot d\mathbf{A}$$

Electric Flux, final

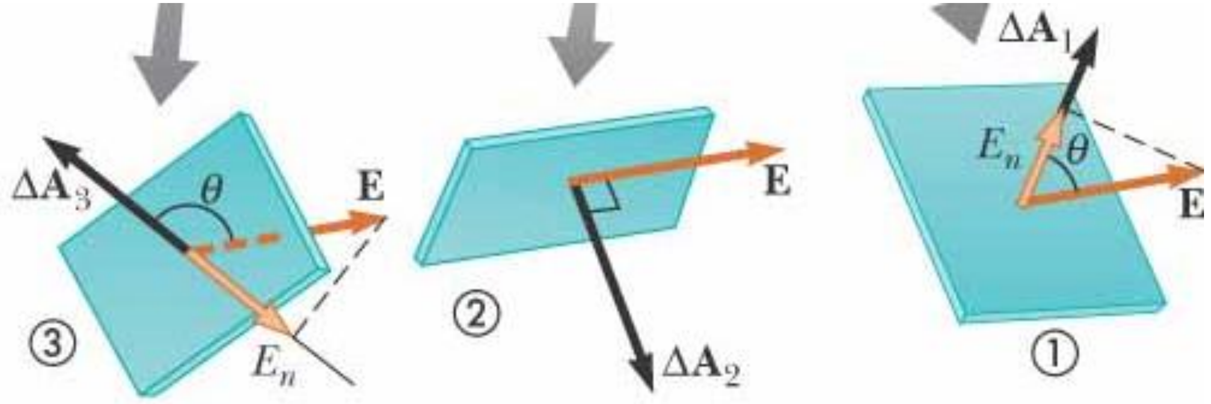
- The surface integral means the integral must be evaluated over the surface in question
- In general, the value of the flux will depend both on the field pattern and on the surface
- The units of electric flux will be $N \cdot m^2/C^2$

Electric Flux, Closed Surface

- Assume a closed surface
- The vectors $\Delta \boldsymbol{A}_i$ point in different directions
  - At each point, they are perpendicular to the surface
  - By convention, they point outward



Flux Through Closed Surface, cont.



- At (1), the field lines are crossing the surface from the inside to the outside; $\theta < 90^o$, Φ is positive
- At (2), the field lines graze surface; $\theta = 90^o$, Φ = 0
- At (3), the field lines are crossing the surface from the outside to the inside; $180^o > \theta > 90^o$, Φ is negative

Flux Through Closed Surface, final

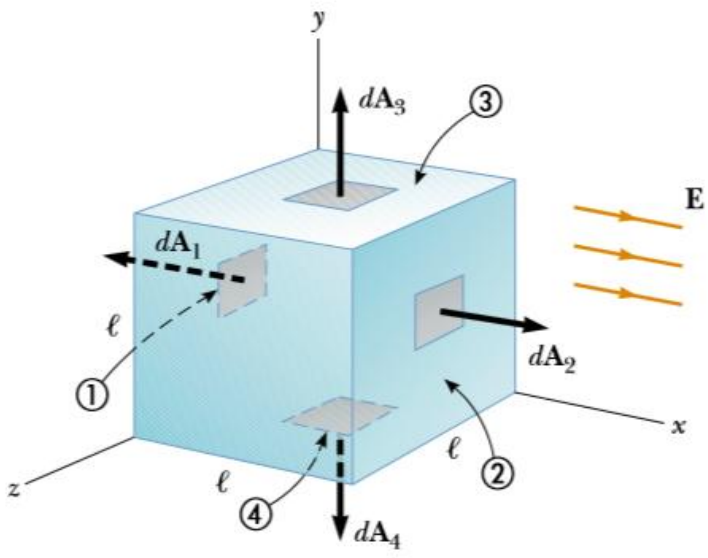
- The ***net*** flux through the surface is proportional to the net number of lines leaving the surface
  - This net number of lines is the number of lines leaving the surface minus the number entering the surface
- If E_n is the component of **E** perpendicular to the surface, then

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E_n dA$$

Example 24.2

EXAMPLE 24.2 **Flux Through a Cube**

Consider a uniform electric field **E** oriented in the x direction. Find the net electric flux through the surface of a cube of edges ℓ, oriented as shown in Figure 24.5.



Solution of Example 24.2

For ①, $\mathbf{E}$ is constant and directed inward but $d\mathbf{A}_1$ is directed outward ($\theta = 180°$); thus, the flux through this face is

$$\int_1 \mathbf{E} \cdot d\mathbf{A} = \int_1 E(\cos 180°)\, dA = -E \int_1 dA = -EA = -E\ell^2$$

because the area of each face is $A = \ell^2$.

For ②, $\mathbf{E}$ is constant and outward and in the same direction as $d\mathbf{A}_2 (\theta = 0°)$; hence, the flux through this face is

$$\int_2 \mathbf{E} \cdot d\mathbf{A} = \int_2 E(\cos 0°)\, dA = E \int_2 dA = +EA = E\ell^2$$

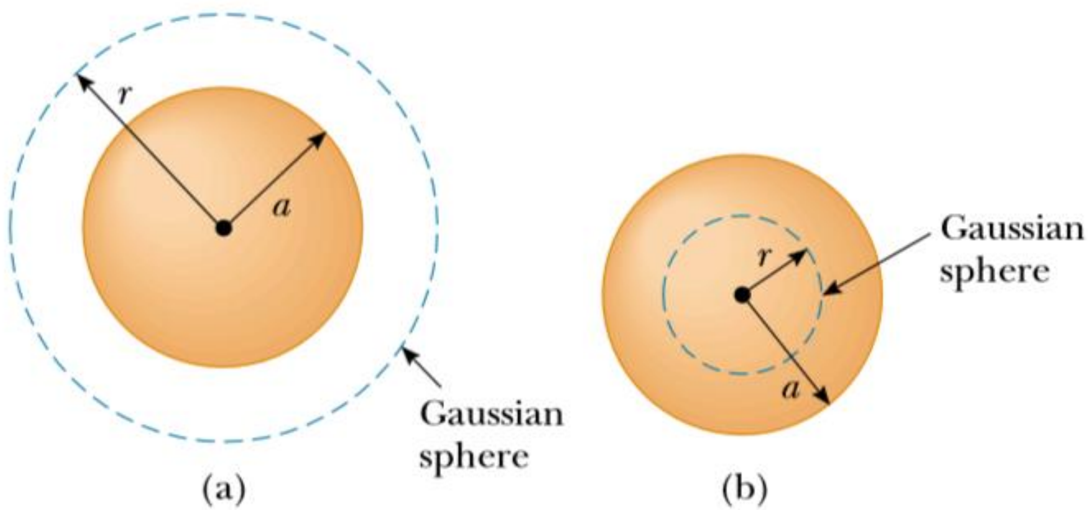
Therefore, the net flux over all six faces is

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = \boxed{0}$$

Example 24.5

An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q (Fig. 24.11). (a) Calculate the magnitude of the electric field at a point outside the sphere.

(b) Find the magnitude of the electric field at a point inside the sphere.



(a)

$$E = k_e \frac{Q}{r^2} \qquad \text{(for } r > a\text{)}$$

(b)

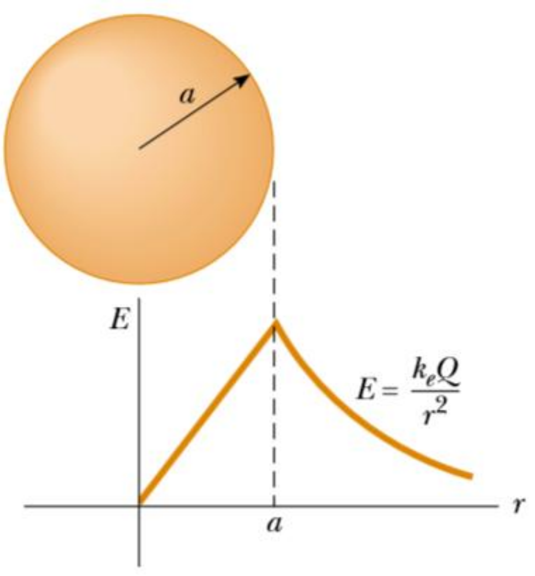
$$q_{\text{in}} = \rho V' = \rho\left(\tfrac{4}{3}\pi r^3\right)$$

$$\oint E\, dA = E \oint dA = E(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E = \frac{q_{\text{in}}}{4\pi\epsilon_0 r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0}\, r$$

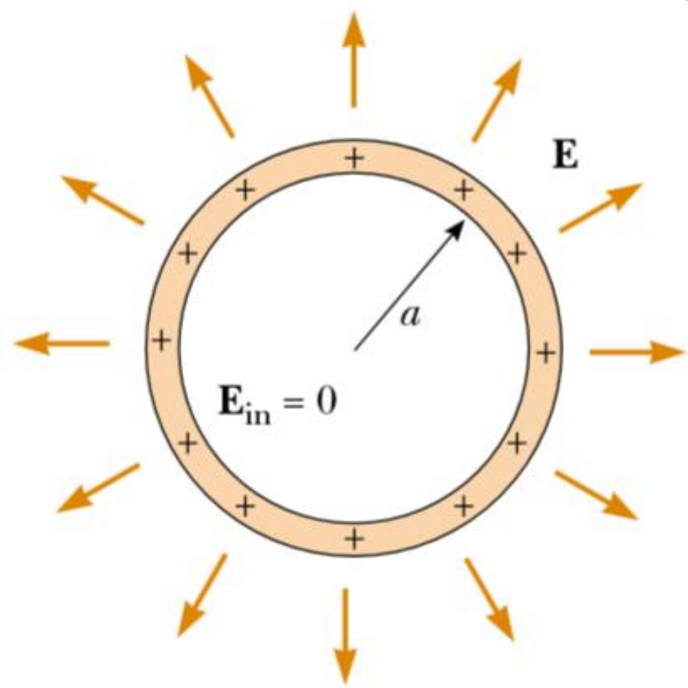
Because $\rho = Q/\frac{4}{3}\pi a^3$ by definition and since $k_e = 1/(4\pi\epsilon_0)$, this expression for E can be written as

$$E = \frac{Qr}{4\pi\epsilon_0 a^3} = \frac{k_e Q}{a^3}\, r \qquad (\text{for } r < a)$$

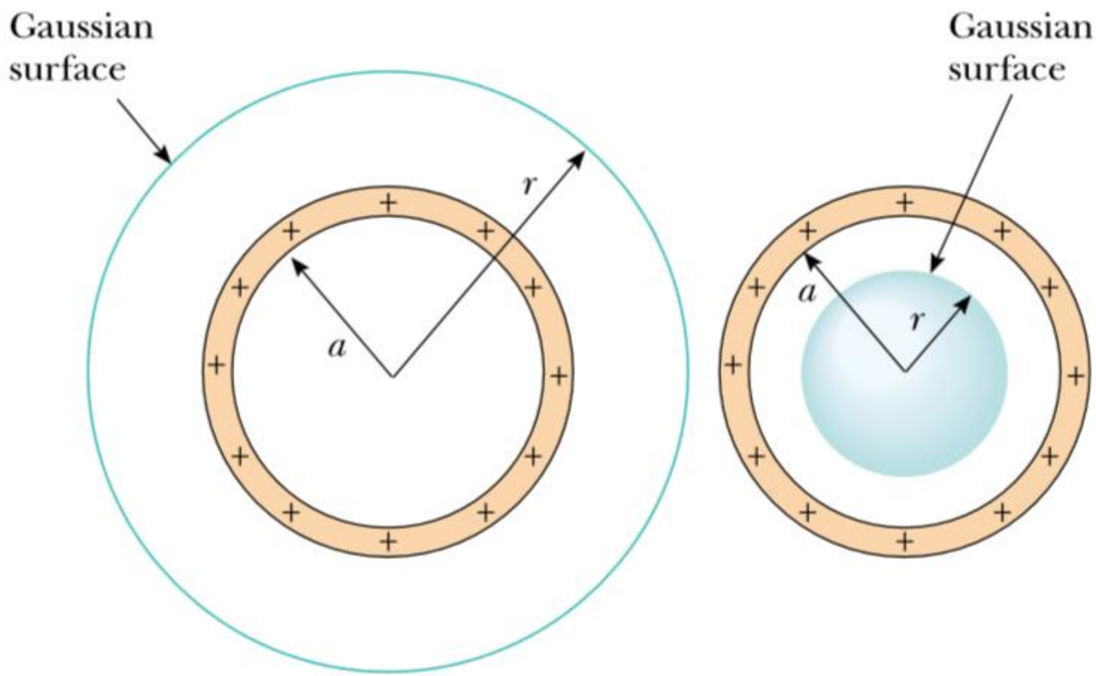


EXAMPLE 24.6 The Electric Field Due to a Thin Spherical Shell

A thin spherical shell of radius a has a total charge Q distributed uniformly over its surface (Fig. 24.13a). Find the electric field at points (a) outside and (b) inside the shell.



EXAMPLE 24.6 The Electric Field Due to a Thin Spherical Shell



EXAMPLE 24.6 The Electric Field Due to a Thin Spherical Shell

(a) Outside the ring

$$E = k_e \frac{Q}{r^2} \qquad (\text{for } r > a)$$

(b) E=0 inside the ring