# Chapter 24

## Electric Potential

# Electric Potential

Electromagnetism has been connected to the study of forces in previous chapters.

In this chapter, electromagnetism will be linked to energy.

By using an energy approach, problems could be solved that were insoluble using forces.

The concept of potential energy is of great value in the study of electricity.

Because the electrostatic force is conservative, electrostatic phenomena can be conveniently described in terms of an electric potential energy.

This will enable the definition of *electric potential.*

# Electrical Potential Energy

When a test charge is placed in an electric field, it experiences a force.

- $\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$
- The force is conservative.

If the test charge is moved in the field by some external agent, the work done by the field is the negative of the work done by the external agent.

$d\vec{\mathbf{s}}$ is an infinitesimal displacement vector that is oriented tangent to a path through space.

- The path may be straight or curved and the integral performed along this path is called either a *path integral* or a *line integral.*

# Electric Potential Energy, cont

The work done within the charge-field system by the electric field on the charge is

$$W_{\text{int}} = \vec{\mathbf{F}}_e \cdot d\vec{\mathbf{s}} = q\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

As this work is done by the field, the potential energy of the charge-field system is changed by $dU_E = -W_{\text{int}} = -q\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$

For a finite displacement of the charge from A to B, the change in potential energy of the system is

$$\Delta U_E = -q\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

Because the force is conservative, the line integral does not depend on the path taken by the charge.

# Electric Potential

The potential energy per unit charge, $U/q$, is the **electric potential.**

- The potential is characteristic of the field only.
  - The potential energy is characteristic of the charge-field system.
- The potential is independent of the value of $q$.
- The potential has a value at every point in an electric field.

The electric potential is

$$V = \frac{U_E}{q}$$

# Electric Potential, cont.

The potential is a scalar quantity.

- Since energy is a scalar

As a charged particle moves in an electric field, it will experience a change in potential.

$$\Delta V \equiv \frac{\Delta U_E}{q} = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

The infinitesimal displacement is interpreted as the displacement between two points in space rather than the displacement of a point charge.

# Electric Potential, final

The difference in potential is the meaningful quantity.

We often take the value of the potential to be zero at some convenient point in the field.

Electric potential is a scalar characteristic of an electric field, independent of any charges that may be placed in the field.

The potential difference between two points exists solely because of a source charge and depends on the source charge distribution.

- For a potential energy to exist, there must be a system of two or more charges.
- The potential energy belongs to the system and changes only if a charge is moved relative to the rest of the system.

# Work and Electric Potential

Assume a charge moves in an electric field without any change in its kinetic energy.

The work performed on the charge is

$$W = \Delta U = q\Delta V$$

Units: $1\ \mathrm{V} \equiv \mathrm{J/C}$

- $\mathrm{V}$ is a volt.
- It takes one joule of work to move a 1-coulomb charge through a potential difference of 1 volt.

In addition, $1\ \mathrm{N/C} = 1\ \mathrm{V/m}$

- This indicates we can interpret the electric field as a measure of the rate of change of the electric potential with respect to position.

# Voltage

Electric potential is described by many terms.

The most common term is *voltage.*

A voltage applied to a device or across a device is the same as the potential difference across the device.

- The voltage is not something that moves through a device.

## Electron-Volts

Another unit of energy that is commonly used in atomic and nuclear physics is the electron-volt.

One ***electron-volt*** is defined as the energy a charge-field system gains or loses when a charge of magnitude *e* (an electron or a proton) is moved through a potential difference of 1 volt.

- $1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$

# Problem 24.01:

How much work is done (by a battery, generator, or some other source of potential difference) in moving Avogadro's number of electrons from an initial point where the electric potential is $9.00\text{ V}$ to a point where the electric potential is $-5.00\text{ V}$? (The potential in each case is measured relative to a common reference point.)

The potential difference is

$$\Delta V = V_f - V_i = -5.00\text{ V} - 9.00\text{ V} = -14.0\text{ V}$$

and the total charge to be moved is

$$Q = -N_A e = -\left(6.02 \times 10^{23}\right)\left(1.60 \times 10^{-19}\text{C}\right) = -9.63 \times 10^{4}\text{C}$$

Now, from $\Delta V = \frac{W}{Q}$, we obtain

$$W = Q\Delta V = \left(-9.63 \times 10^{4}\text{C}\right)(-14.0\text{ J/C}) = 1.35\text{MJ}$$

# Problem 24.02:

(a) Find the electric potential difference $\Delta V_e$ required to stop an electron (called a "stopping potential") moving with an initial speed of $2.85 \times 10^7\ \text{m/s}$. (b) Would a proton traveling at the same speed require a greater or lesser magnitude of electric potential difference? Explain. (c) Find a symbolic expression for the ratio of the proton stopping potential and the electron stopping potential, $\Delta V_p / \Delta V_e$.

(a) The electron-electric field is an isolated system:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m_e v_i^2 + (-e)V_i = 0 + (-e)V_f$$

$$e\left(V_f - V_i\right) = -\frac{1}{2} m_e v_i^2$$

The potential difference is then

$$\Delta V_e = -\frac{m_e v_i^2}{2e} = -\frac{(9.11 \times 10^{-31}\ \text{kg})\left(2.85 \times 10^7\ \text{m/s}\right)^2}{2\left(1.60 \times 10^{-19}\text{C}\right)}$$

$$= -2.31 \times 10^3\ \text{V} = -2.31\text{kV}$$

(b)From (a), we see that the stopping potential is proportional to the kinetic energy of the particle.

Because a proton is more massive than an electron, a proton traveling at the same speed as an electron has more initial kinetic energy and requires a greater magnitude stopping potential.

(c) The proton-electric field is an isolated system:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m_p v_i^2 + eV_i = 0 + eV_f$$

$$e\left(V_f - V_i\right) = \frac{1}{2} m_p v_i^2$$

The potential difference is

$$\Delta V_p = \frac{m_p v_i^2}{2e}$$

Therefore, from (a),

$$\frac{\Delta V_p}{\Delta V_e} = \frac{m_p v_i^2 / 2e}{-m_e v_i^2 / 2e} \rightarrow \Delta V_p / \Delta V_e = -m_p / m_e$$

# Potential Difference in a Uniform Field

The equations for electric potential between two points $A$ and $B$ can be simplified if the electric field is uniform:

$$V_B - V_A = \Delta V = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\int_A^B E ds (\cos 0°) = -\int_A^B E ds = -Ed$$

The displacement points from $A$ to $B$ and is parallel to the field lines.

The negative sign indicates that the electric potential at point $B$ is lower than at point $A$.

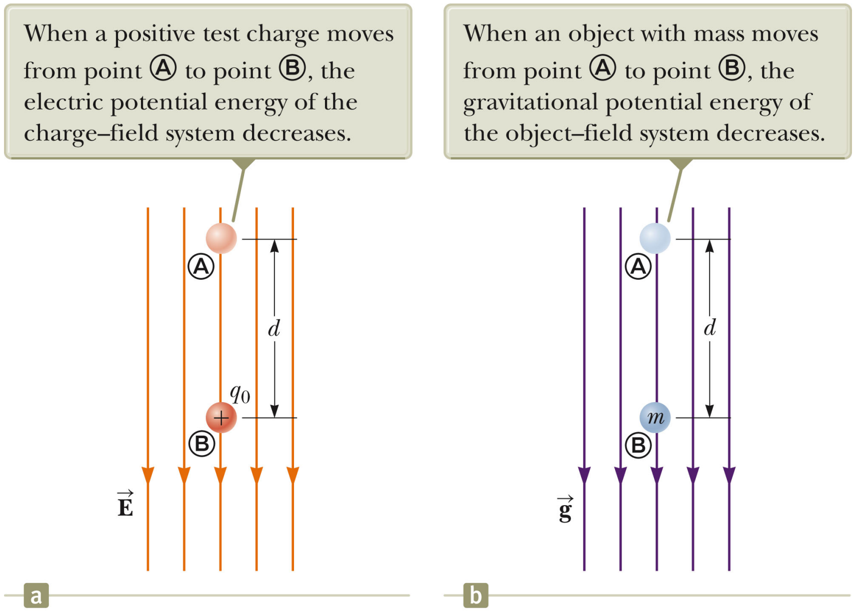
- Electric field lines always point in the direction of decreasing electric potential.

# Energy and the Direction of Electric Field

When the electric field is directed downward, point $B$ is at a lower potential than point $A$.

When a positive test charge moves from $A$ to $B$, the charge-field system loses potential energy.

Electric field lines always point in the direction of decreasing electric potential.



# More About Directions

A system consisting of a positive charge and an electric field **loses** electric potential energy when the charge moves in the direction of the field.

- An electric field does work on a positive charge when the charge moves in the direction of the electric field.

The charged particle gains kinetic energy and the potential energy of the charge-field system decreases by an equal amount.

- Another example of Conservation of Energy

# Directions, cont.

If $q$ is negative, then $\Delta U$ is positive.

A system consisting of a negative charge and an electric field *gains* potential energy when the charge moves in the direction of the field.

- In order for a negative charge to move in the direction of the field, an external agent must do positive work on the charge.

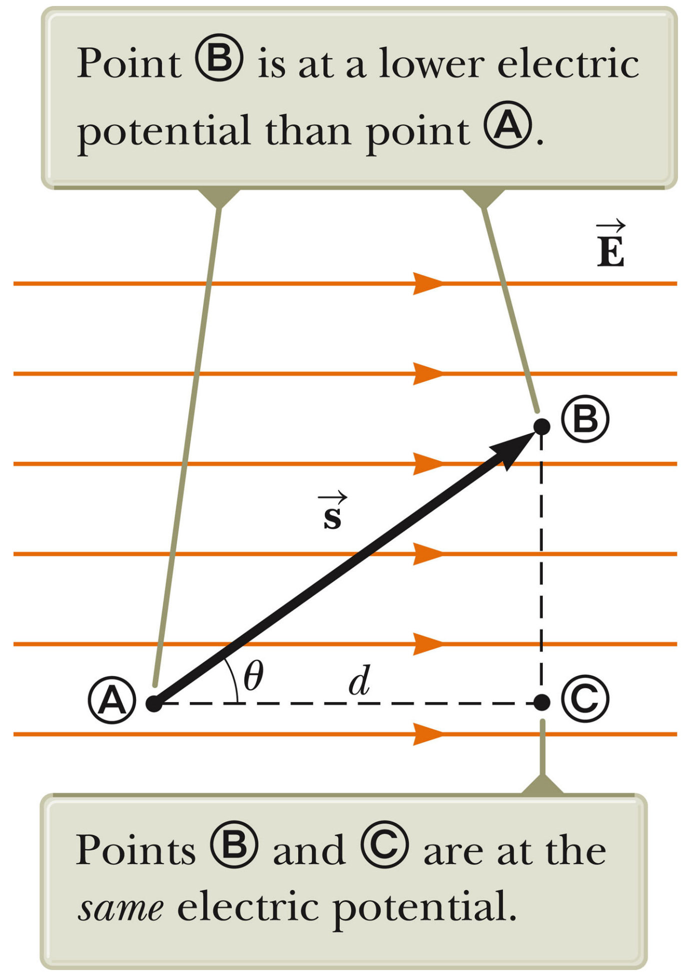
# Equipotentials

Point $B$ is at a lower potential than point $A$.

Points $B$ and $C$ are at the same potential.

- All points in a plane perpendicular to a uniform electric field are at the same electric potential.

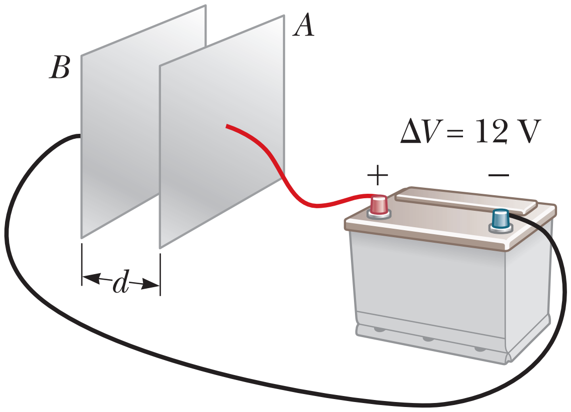
The name **equipotential surface** is given to any surface consisting of a continuous distribution of points having the same electric potential.



# Example 24.01: The Electric Field Between Two Parallel Plates of Opposite Charge

A battery has a specified potential difference $\Delta V$ between its terminals and establishes that potential difference between conductors attached to the terminals. A $12-\mathrm{V}$ battery is connected between two parallel plates as shown in the figure. The separation between the plates is $d = 0.30\ \mathrm{cm}$, and we assume the electric field between the plates to be uniform. (This assumption is reasonable if the plate separation is small relative to the plate dimensions and we do not consider locations near the plate edges.) Find the magnitude of the electric field between the plates.

$$E = \frac{\left|V_B - V_A\right|}{d} = \frac{12\ \mathrm{V}}{0.30 \times 10^{-2}\ \mathrm{m}} = 4.0 \times 10^{3}\ \mathrm{V/m}$$



# Example 24.02: Motion of a Proton in a Uniform Electric Field

A proton is released from rest at point (A) in a uniform electric field that has a magnitude of $8.0 \times 10^4$ V/m. The proton undergoes a displacement of magnitude $d = 0.50$ m to point (B) in the direction of $\vec{\mathbf{E}}$. Find the speed of the proton after completing the displacement.

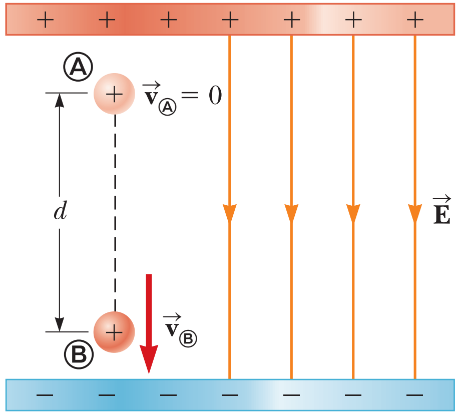
$$\Delta K + \Delta U_E = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + e\Delta V = 0$$

$$v = \sqrt{\frac{-2e\Delta V}{m}} = \sqrt{\frac{-2e(-Ed)}{m}} = \sqrt{\frac{2eEd}{m}}$$

$$v = \sqrt{\frac{2\left(1.6 \times 10^{-19}\text{C}\right)\left(8.0 \times 10^4\ \text{V}\right)(0.50\ \text{m})}{1.67 \times 10^{-27}\ \text{kg}}}$$

$$= 2.8 \times 10^6\ \text{m/s}$$



# Problem 24.03:

Oppositely charged parallel plates are separated by $5.33\ \text{mm}$ . A potential difference of $600\ \text{V}$ exists between the plates. (a) What is the magnitude of the electric field between the plates? (b) What is the magnitude of the force on an electron between the plates? (c) How much work must be done on the electron to move it to the negative plate if it is initially positioned $2.90\ \text{mm}$ from the positive plate?

(a)

$$E = \frac{|\Delta V|}{d} = \frac{600\ \text{J/C}}{5.33 \times 10^{-3}\ \text{m}} = 1.13 \times 10^{5}\ \text{N/C}$$

(b) The force on an electron is given by

$$F = |q|E = \left(1.60 \times 10^{-19}\text{C}\right)\left(1.13 \times 10^{5}\ \text{N/C}\right) = 1.80 \times 10^{-14}\ \text{N}$$

(c) Because the electron is repelled by the negative plate, the force used to move the electron must be applied in the direction of the electron's displacement. The work done to move the electron is

$$W = F \cdot s \cos\theta = \left(1.80 \times 10^{-14}\ \text{N}\right)\left[(5.33 - 2.00) \times 10^{-3}\ \text{m}\right]\cos 0^{\circ}$$
$$= 4.37 \times 10^{-17}\ \text{J}$$

# Potential and Point Charges

An isolated positive point charge produces a field directed radially outward.

The potential difference between points $A$ and $B$ will be

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

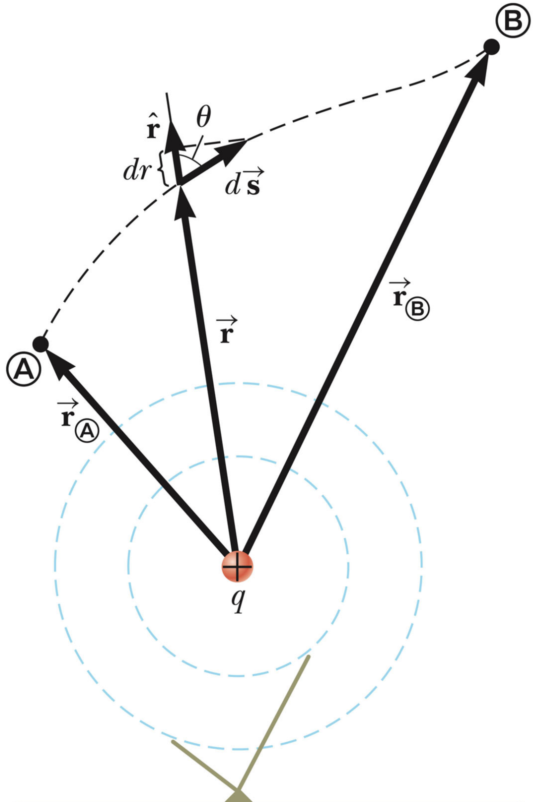
We have

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \cdot d\vec{\mathbf{s}}$$

We can use $ds \cos\theta = dr$

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = k_e \frac{q}{r} \Big|_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[ \frac{1}{r_B} - \frac{1}{r_A} \right]$$



The two dashed circles represent intersections of spherical equipotential surfaces with the page.

# Potential and Point Charges, cont.

The electric potential is independent of the path between points $A$ and $B$.

It is customary to choose a reference potential of $V = 0$ at $r_A = \infty$.

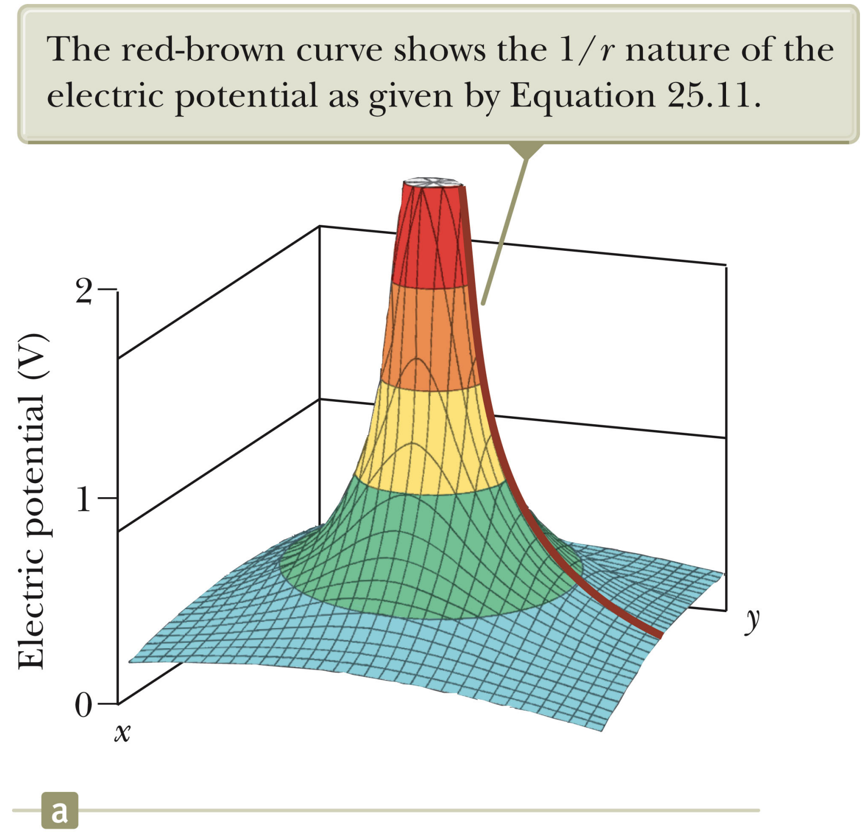
Then the potential due to a point charge at some point $r$ is

$$V = k_e \frac{q}{r}$$

# Electric Potential of a Point Charge

The electric potential in the plane around a single point charge is shown.

The red line shows the $1/r$ nature of the potential.



# Electric Potential with Multiple Charges

The electric potential due to several point charges is the sum of the potentials due to each individual charge.

- This is another example of the superposition principle.
- The sum is the algebraic sum

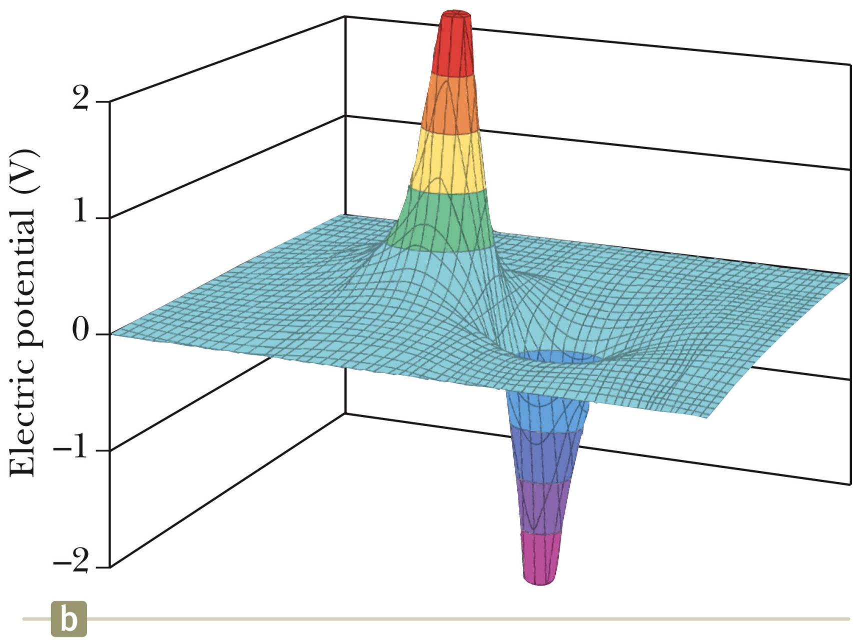
$$V = k_e \sum_i \frac{q_i}{r_i}$$

  - $V = 0$ at $r = \infty$

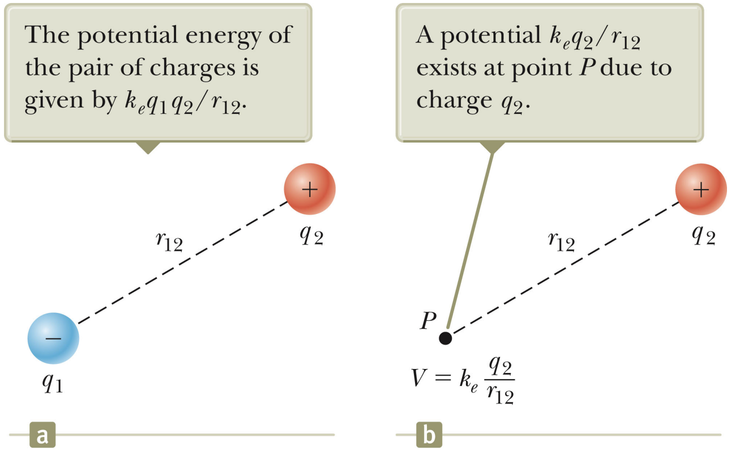
# Electric Potential of a Dipole

The graph shows the potential (y-axis) of an electric dipole.

The steep slope between the charges represents the strong electric field in this region.



# Potential Energy of Multiple Charges




The potential energy of the system is

$$\Delta U_E = W = q_2 \Delta V \rightarrow U_E - 0 = q_2 \left( k_e \frac{q_1}{r_{12}} - 0 \right)$$

$$U_E = k_e \frac{q_1 q_2}{r_{12}}$$

If the two charges are the same sign, $U$ is positive and work must be done to bring the charges together.

If the two charges have opposite signs, $U$ is negative and work is done to keep the charges apart.

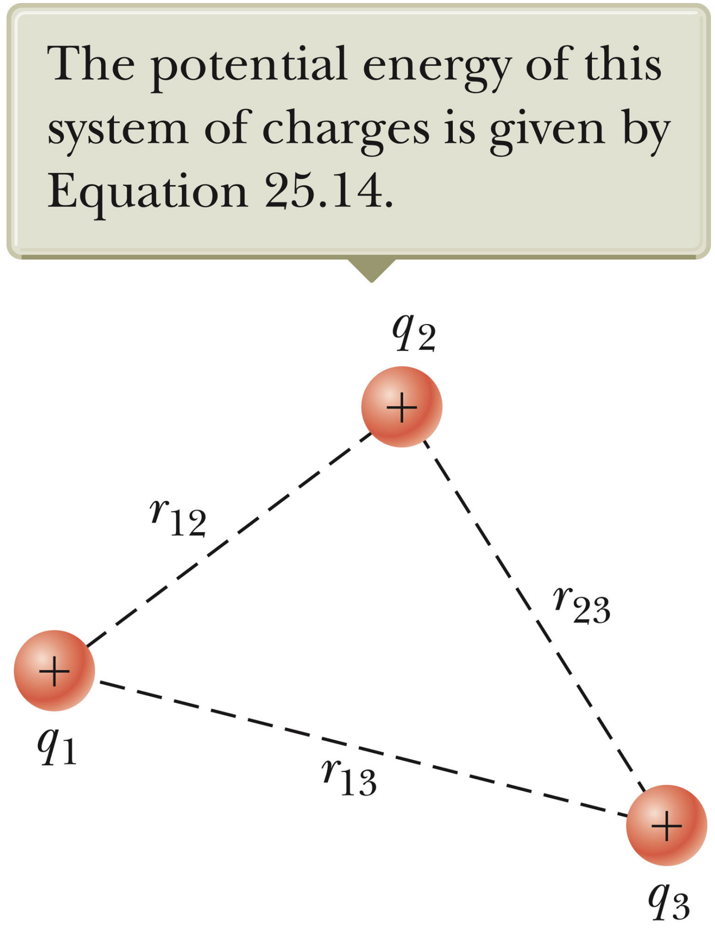
# $U$ with Multiple Charges, final

If there are more than two charges, then find $U$ for each pair of charges and add them.

For three charges:

$$U_E = k_e \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

- The result is independent of the order of the charges.



# Example 24.03: The Electric Potential Due to Two Point Charges

As shown in the figure, a charge $q_1 = 2.00\mu\text{C}$ is located at the origin and a charge $q_2 = -6.00\mu\text{C}$ is located at $(0,3.00)$ m.

(A) Find the total electric potential due to these charges at the point $P$, whose coordinates are $(4.00,0)$m.

(B) Find the change in potential energy of the system of two charges plus a third charge $q_3 = 3.00\mu\text{C}$ as the latter charge moves from infinity to point $P$.

(a) $V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$

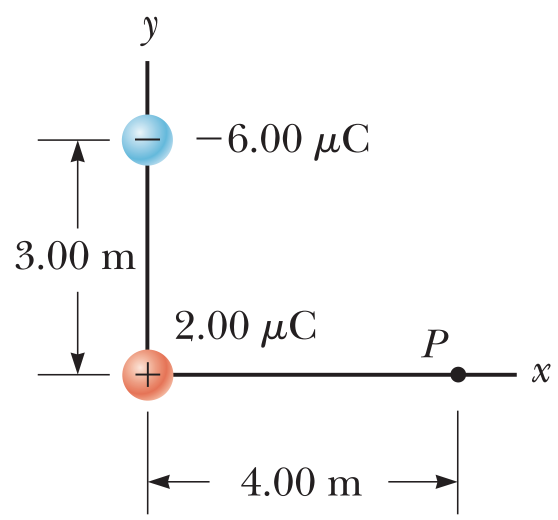
$$V_P = \left(8.988 \times 10^9 \text{ N}\cdot\text{ m}^2/\text{C}^2\right)\left(\frac{2.00 \times 10^{-6}\text{C}}{4.00 \text{ m}} + \frac{-6.00 \times 10^{-6}\text{C}}{5.00 \text{ m}}\right)$$

$$= -6.29 \times 10^3 \text{ V}$$

(b) $U_f = q_3 V_P$

$$\Delta U_E = U_f - U_i = q_3 V_P - 0 = \left(3.00 \times 10^{-6}\text{C}\right)\left(-6.29 \times 10^3 \text{ V}\right)$$

$$= -1.89 \times 10^{-2} \text{ J}$$



# Problem 24.08:

Two point charges $Q_1 = +5.00\text{nC}$ and $Q_2 = -3.00\text{nC}$ are separated by $35.0\,\text{cm}$ . (a) What is the electric potential at a point midway between the charges? (b) What is the potential energy of the pair of charges? What is the significance of the algebraic sign of your answer?

(a) The electric potential due to the two charges is

$$V = k_e \sum_i \frac{q_i}{r_i} = \left(8.99 \times 10^9\ \text{N} \cdot\ \text{m}^2/\text{C}^2\right)$$

$$\times \left( \frac{5.00 \times 10^{-9}\text{C}}{0.175\ \text{m}} + \frac{-3.00 \times 10^{-9}\text{C}}{0.175\ \text{m}} \right) = 103\ \text{V}$$

(b) The potential energy of the pair of charges is

$$U = \frac{k_e q_1 q_2}{r_{12}} = \left(8.99 \times 10^9\ \text{N} \cdot\ \text{m}^2/\text{C}^2\right)$$

$$\times \frac{\left(5.00 \times 10^{-9}\text{C}\right)\left(-3.00 \times 10^{-9}\text{C}\right)}{0.350\ \text{m}}$$

$$= -3.85 \times 10^{-7}\ \text{J}$$

The negative sign means that positive work must be done to separate the charges by an infinite distance (that is, to bring them to a state of zero potential energy).

# Properties of a Conductor in Electrostatic Equilibrium

When there is no net motion of charge within a conductor, the conductor is said to be in **electrostatic equilibrium.**

1- The electric field is zero everywhere inside the conductor.

- Whether the conductor is solid or hollow

2- If the conductor is isolated and carries a charge, the charge resides on its surface.

3- The electric field at a point just outside a charged conductor is perpendicular to the surface and has a magnitude of $\sigma/\varepsilon_o$.

- $\sigma$ is the surface charge density at that point.

4- On an irregularly shaped conductor, the surface charge density is greatest at locations where the radius of curvature is the smallest.

# Property 1: $\text{Field}_{\text{inside}} = 0$

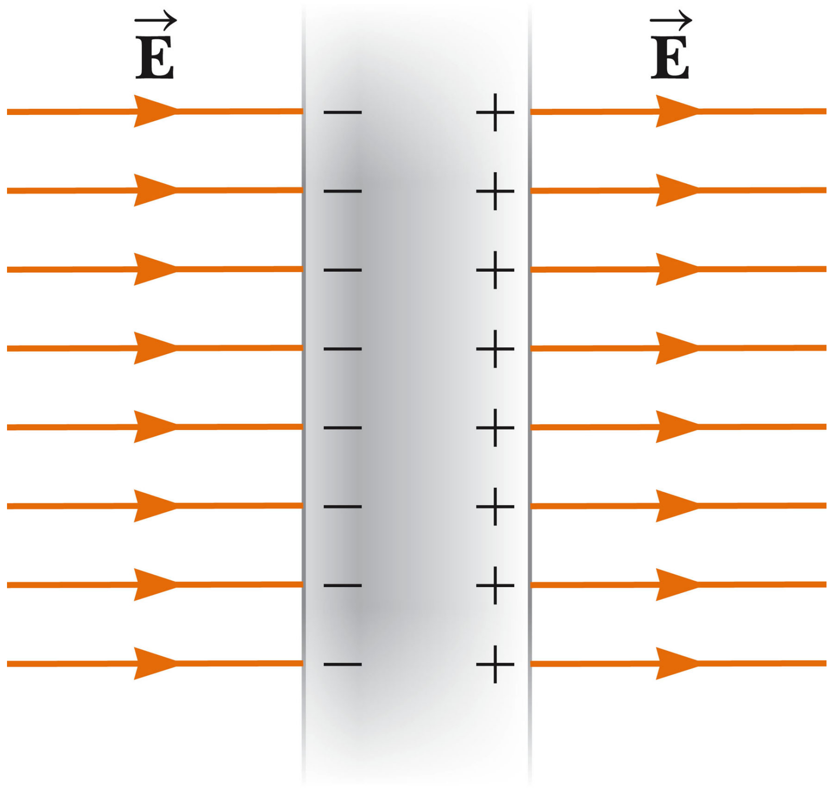
Consider a conducting slab in an external field.

If the field inside the conductor were not zero, free electrons in the conductor would experience an electrical force.

These electrons would accelerate.

These electrons would not be in equilibrium.

Therefore, there cannot be a field inside the conductor.



# Property 1: $\text{Field}_{\text{inside}} = 0$, cont.

Before the external field is applied, free electrons are distributed throughout the conductor.

When the external field is applied, the electrons redistribute until the magnitude of the internal field equals the magnitude of the external field.

There is a net field of zero inside the conductor.

This redistribution takes about $10^{-16}\,\text{s}$ and can be considered instantaneous.

If the conductor is hollow, the electric field inside the conductor is also zero.

- Either the points in the conductor or in the cavity within the conductor can be considered.

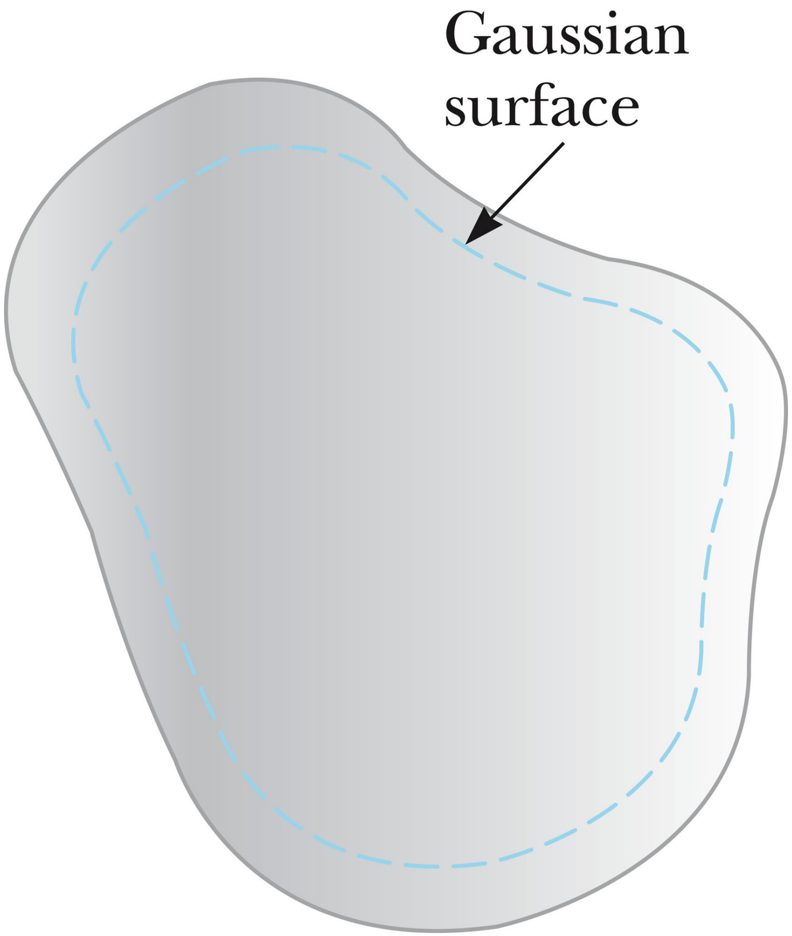
# Property 2: Charge Resides on the Surface

Choose a gaussian surface inside but close to the actual surface.

The electric field inside is zero (property 1).

There is no net flux through the gaussian surface.

Because the gaussian surface can be as close to the actual surface as desired, there can be no charge inside the surface.



# Property 2: Charge Resides on the Surface, cont.

Since no net charge can be inside the surface, any net charge must reside ***on*** the surface.

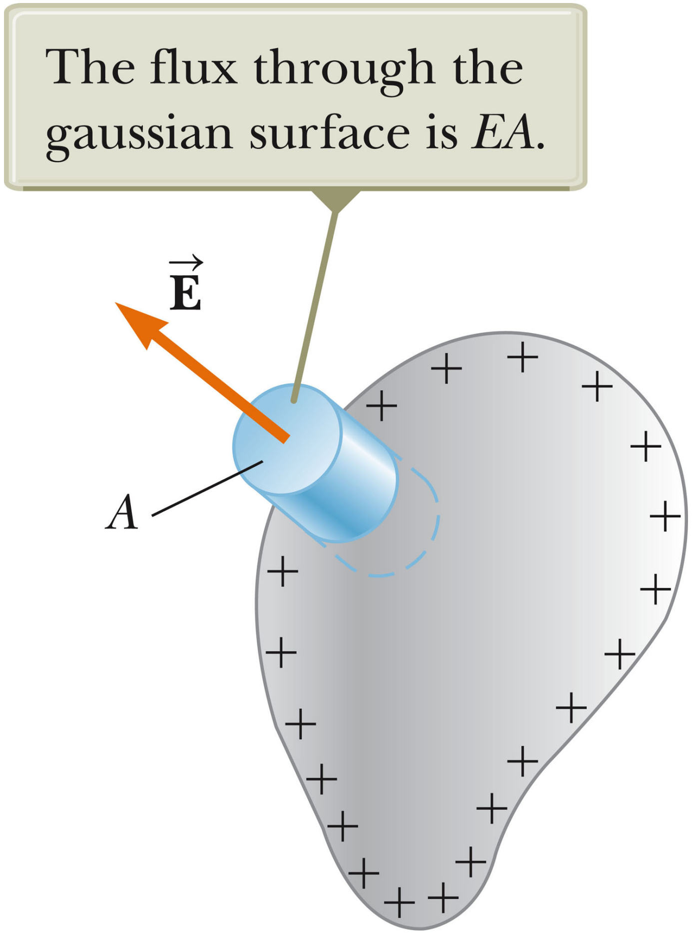
Gauss's law does not indicate the distribution of these charges, only that it must be on the surface of the conductor.

# Property 3: Field's Magnitude and Direction

Choose a cylinder as the gaussian surface.

The field must be perpendicular to the surface.

- If there were a parallel component to $\vec{\mathbf{E}}$, charges would experience a force and accelerate along the surface and it would not be in equilibrium.



# Property 3: Field's Magnitude and Direction, cont.

The net flux through the gaussian surface is through only the flat face outside the conductor.

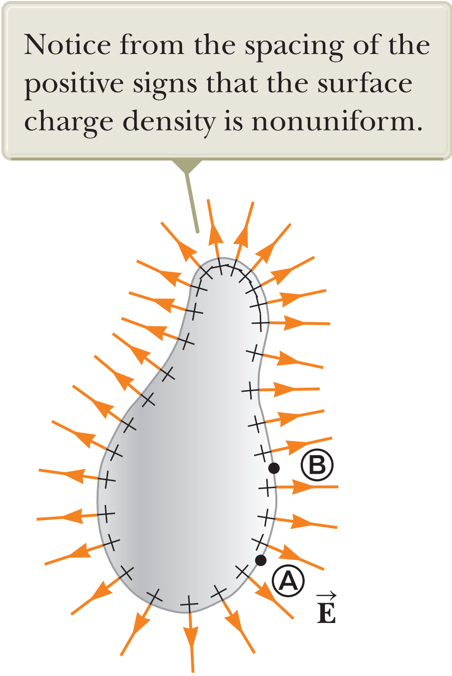
- The field here is perpendicular to the surface.

Applying Gauss's law

$$\Phi_E = EA = \frac{\sigma A}{\varepsilon_o} \text{ and } E = \frac{\sigma}{\varepsilon_0}$$

# Property 3: Electric potential in conductors

The surface of any charged conductor in electrostatic equilibrium is an equipotential surface: every point on the surface of a charged conductor in equilibrium is at the same electric potential. Furthermore, because the electric field is zero inside the conductor, the electric potential is constant everywhere inside the conductor and equal to its value at the surface.



# Problem 24.34:

A solid conducting sphere of radius $2.00\,\text{cm}$ has a charge of $8.00\mu\text{C}$. A conducting spherical shell of inner radius $4.00\,\text{cm}$ and outer radius $5.00\,\text{cm}$ is concentric with the solid sphere and has a charge of $-4.00\mu\text{C}$. Find the electric field at (a) $r = 1.00\ \text{cm}$, (b) $r = 3.00\ \text{cm}$, (c) $r = 4.50\ \text{cm}$, and (d) $r = 7.00\ \text{cm}$ from the center of this charge configuration.

(a) $\vec{\text{E}} = 0$

(b) $$E = \frac{k_e Q}{r^2} = \frac{\left(8.99 \times 10^9\ \text{N} \cdot\ \text{m}^2/\text{C}^2\right)\left(8.00 \times 10^{-6}\text{C}\right)}{(0.0300\ \text{m})^2} = 7.99 \times 10^7\ \text{N/C}$$

$\vec{\text{E}} = 79.9\,\text{MN/C}$

radially outward

(c) $\vec{\text{E}} = 0$

(d)

$$E = \frac{k_e Q}{r^2} = \frac{\left(8.99 \times 10^9\ \text{N} \cdot\ \text{m}^2/\text{C}^2\right)\left(4.00 \times 10^{-6}\text{C}\right)}{(0.0700\ \text{m})^2} = 7.34 \times 10^6\ \text{N/C}$$

$\vec{\text{E}} = 7.34\text{MN/C}$ radially outward

# Problem 24.35:

A spherical conductor has a radius of $14.0\,\mathrm{cm}$ and a charge of $26.0\mu\mathrm{C}$. Calculate the electric field and the electric potential at (a) $r = 10.0$ cm, (b) $r = 20.0$ cm, and (c) $r = 14.0$ cm from the center.

For points on the surface and outside, the sphere of charge behaves like a charged particle at its center, both for creating field and potential.

(a) Inside a conductor when charges are not moving, the electric field is zero and the potential is uniform, the same as on the surface, and

$E = 0.$

$$V = \frac{k_e q}{R} = \frac{\left(8.99 \times 10^9\ \mathrm{N \cdot\ m^2/C^2}\right)\left(26.0 \times 10^{-6}\mathrm{C}\right)}{0.140\ \mathrm{m}} = 1.67\mathrm{MV}$$

(b)

$$E = \frac{k_e q}{r^2} = \frac{\left(8.99 \times 10^9\ \mathrm{N \cdot\ m^2/C^2}\right)\left(26.0 \times 10^{-6}\mathrm{C}\right)}{(0.200\ \mathrm{m})^2}$$
$$= 5.84\mathrm{MN/C}\text{ away}$$

$$V = \frac{k_e q}{R} = \frac{\left(8.99 \times 10^9\ \mathrm{N \cdot\ m^2/C^2}\right)\left(26.0 \times 10^{-6}\mathrm{C}\right)}{0.200\ \mathrm{m}} = 1.17\mathrm{MV}$$

(c)

$$E = \frac{k_e q}{R^2} = \frac{\left(8.99 \times 10^9\ \mathrm{N \cdot\ m^2/C^2}\right)\left(26.0 \times 10^{-6}\mathrm{C}\right)}{(0.140\ \mathrm{m})^2}$$
$$= 11.9\mathrm{MN/C}\text{ away}$$

$$V = \frac{k_e q}{R} = 1.67\mathrm{MV}$$