# Chapter 23

## Continuous Charge Distributions and Gauss's Law

# Gauss' Law

Gauss' Law can be used as an alternative procedure for calculating electric fields.

It is convenient for calculating the electric field of highly symmetric charge distributions.

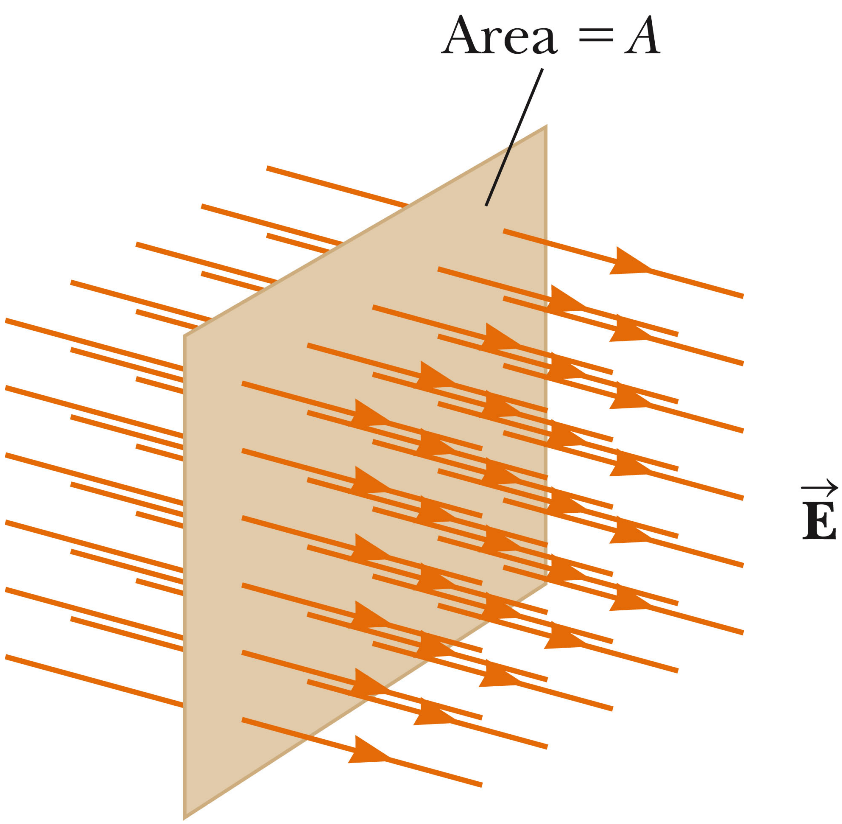
Gauss' Law is important in understanding and verifying the properties of conductors in electrostatic equilibrium.

# Electric Flux

***Electric flux*** is the product of the magnitude of the electric field and the surface area, $A$, perpendicular to the field.

$$\Phi_E = EA$$

Units: $\mathrm{N \cdot m^2/C}$

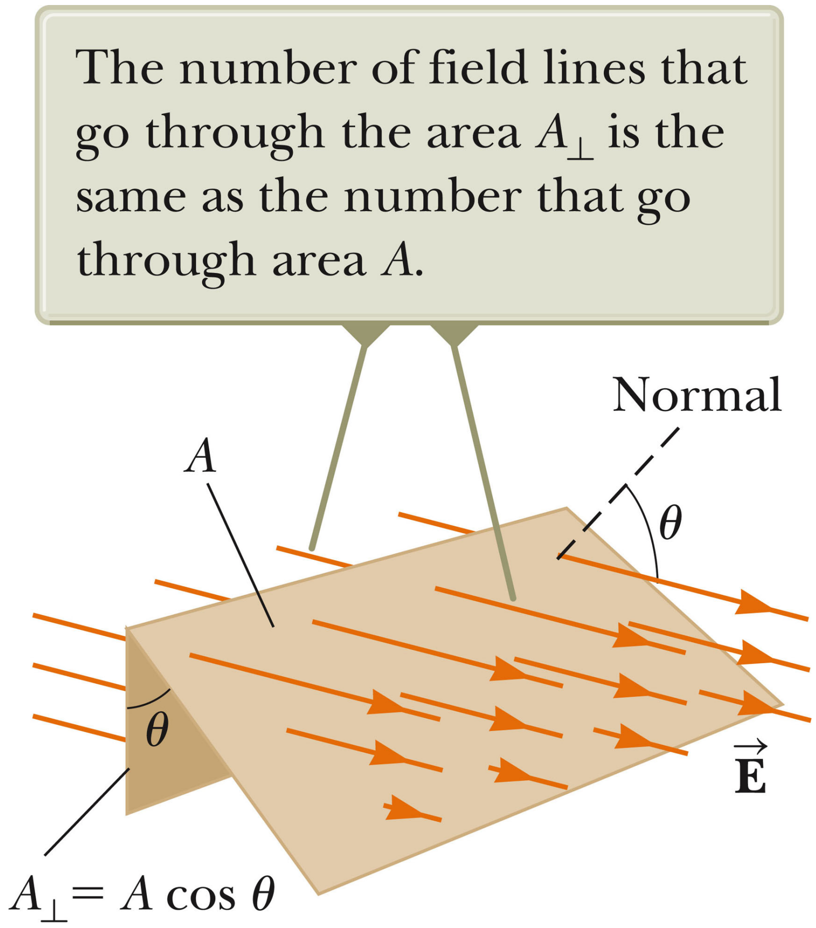


# Electric Flux, General Area

The electric flux is proportional to the **number of electric field lines** penetrating some surface.

The field lines may make some angle $\theta$ with the perpendicular to the surface.

Then $\Phi_E = EA\cos\theta$



# Electric Flux, Interpreting the Equation

The flux is a maximum when the surface is perpendicular to the field.

- $\theta = 0°$

The flux is zero when the surface is parallel to the field.

- $\theta = 90°$

If the field varies over the surface, $\Phi_E = EA\cos\theta$ is valid for only a small element of the area.

# Electric Flux, General

In the more general case, look at a small area element.

$$\Phi_{E,i} = E_i \Delta A_i \cos\theta_i = \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i$$

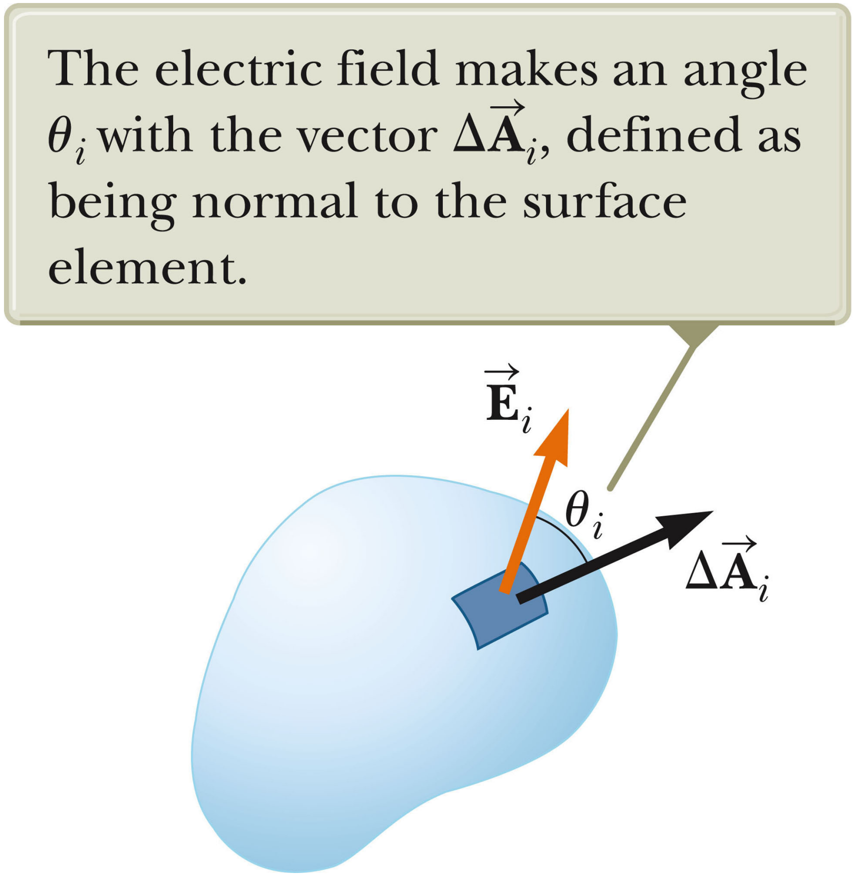
In general, this becomes

$$\Phi_E \approx \sum \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i$$

$$\Phi_E \equiv \int_{\text{surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

- The surface integral means the integral must be evaluated over the surface in question.

In general, the value of the flux will depend both on the field pattern and on the surface.

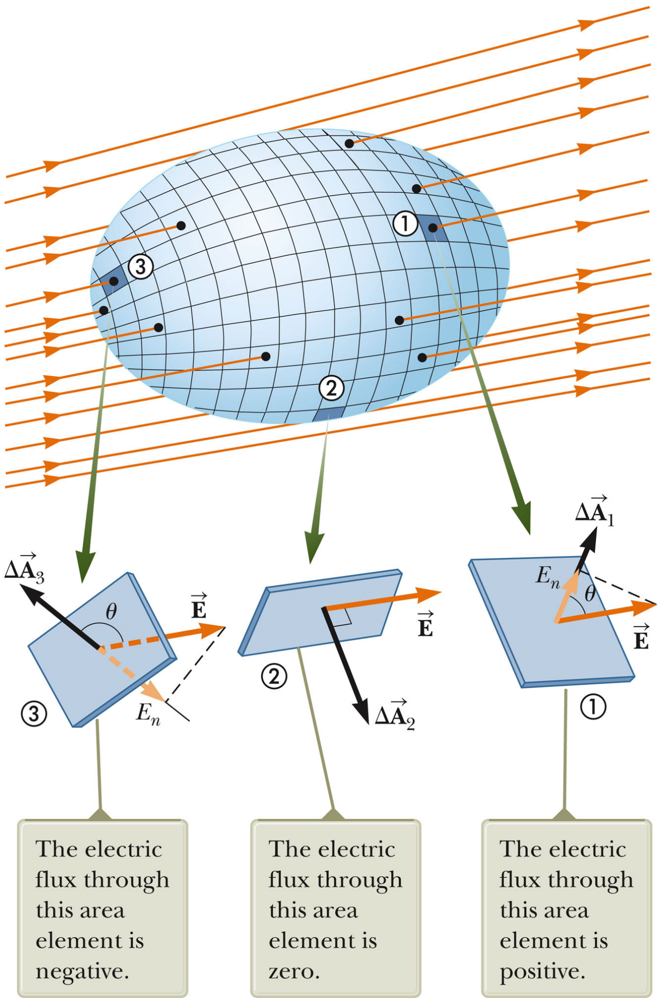


# Electric Flux, Closed Surface

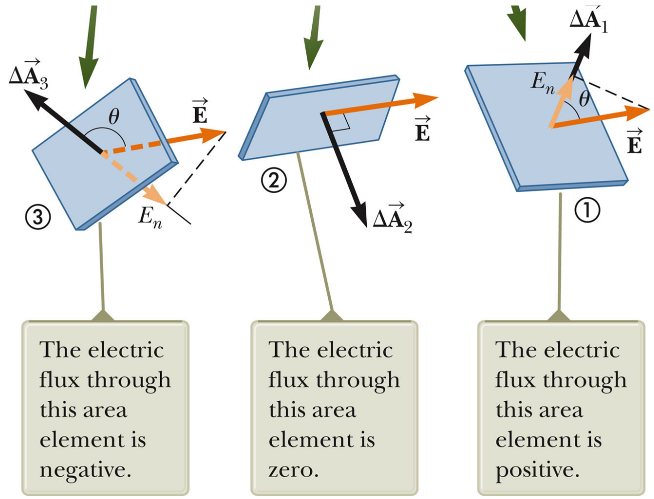
Assume a closed surface

The vectors $\Delta\vec{\mathbf{A}}_i$ point in different directions.

- At each point, they are perpendicular to the surface.
- By convention, they point outward.



# Flux Through Closed Surface, cont.




At (1), the field lines are crossing the surface from the inside to the outside; $\theta < 90^\circ, \Phi$ is positive.

At (2), the field lines graze surface; $\theta = 90^\circ, \Phi = 0$

At (3), the field lines are crossing the surface from the outside to the inside; $180^\circ > \theta > 90^\circ, \Phi$ is negative.

# Flux Through Closed Surface, final

The ***net*** flux through the surface is proportional to the net number of lines leaving the surface.

- This net number of lines is the number of lines leaving the surface minus the number entering the surface.

If $E_n$ is the component of the field perpendicular to the surface, then

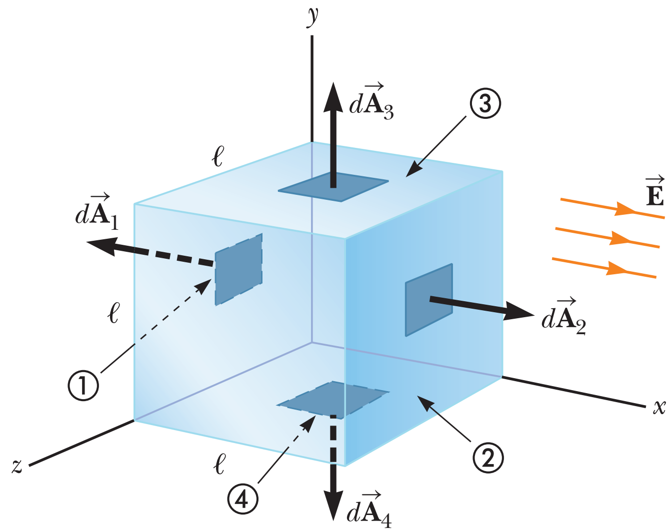
- The integral is over a closed surface.

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E_n dA$$

# Example 23.04: Flux Through a Cube

Consider a uniform electric field $\vec{\mathbf{E}}$ oriented in the $x$ direction in empty space. A cube of edge length $\ell$ is placed in the field, oriented as shown in the figure. Find the net electric flux through the surface of the cube.

The flux through four of the faces (3), (4), and the unnumbered faces) is zero because $\vec{\mathbf{E}}$ is parallel to the four faces and therefore perpendicular to $d\vec{\mathbf{A}}$ on these faces.




$$\Phi_E = \int_1 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} + \int_2 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

$$\int_1 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_1 E\,(\cos 180°)\,dA = -E\int_1 dA = -EA = -E\ell^2$$

$$\int_2 \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int_2 E\,(\cos 0°)\,dA = E\int_2 dA = +EA = E\ell^2$$

$$\Phi_E = -E\ell^2 + E\ell^2 + 0 + 0 + 0 + 0 = 0$$

# Problem 23.10:

A vertical electric field of magnitude $2.00 \times 10^4\ \mathrm{N/C}$ exists above the Earth's surface on a day when a thunderstorm is brewing. A car with a rectangular size of $6.00\ \mathrm{m}$ by $3.00\ \mathrm{m}$ is traveling along a dry gravel roadway sloping downward at $10.0°$. Determine the electric flux through the bottom of the car.

The electric flux through the bottom of the car is given by

$$\Phi_E = EA\cos\theta = \left(2.00 \times 10^4\ \mathrm{N/C}\right)(3.00\ \mathrm{m})(6.00\ \mathrm{m})\cos 10.0° = 355\mathrm{kN} \cdot\ \mathrm{m}^2/\mathrm{C}$$

# Problem 23.11:

A flat surface of area $3.20\ \text{m}^2$ is rotated in a uniform electric field of magnitude $E = 6.20 \times 10^5\ \text{N/C}$. Determine the electric flux through this area (a) when the electric field is perpendicular to the surface and (b) when the electric field is parallel to the surface.

For a uniform electric field passing through a plane surface, $\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = EA\cos\theta$, where $\$\theta\$$is the angle between the electric field and the normal to the surface.

(a) The electric field is perpendicular to the surface, so $\theta = 0°$ :

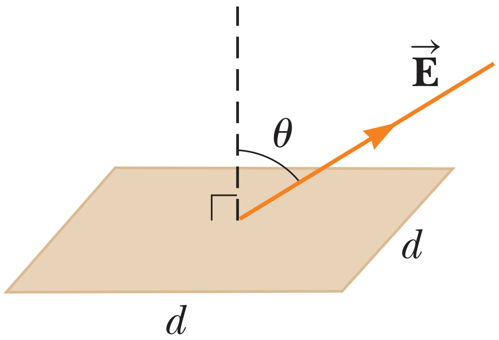
$$\Phi_E = \left(6.20 \times 10^5\ \text{N/C}\right)\left(3.20\ \text{m}^2\right)\cos 0°$$
$$\Phi_E = 1.98 \times 10^6\ \text{N} \cdot\ \text{m}^2/\text{C}$$

(b) The electric field is parallel to the surface: $\theta = 90°$, so $\cos\theta = 0$, and the flux is zero.

# Problem 23.37:

Find the electric flux through the plane surface shown in the figure if $\theta = 60.0^\circ$, $E = 350\ \mathrm{N/C}$, and $d = 5.00\ \mathrm{cm}$. The electric field is uniform over the entire area of the surface.

The electric field makes an angle of $60.0^\circ$ with to the normal. The square has side $d = 5.00\ \mathrm{cm}$.




$$\Phi_E = EA\cos\theta = \left(3.50 \times 10^2\ \mathrm{N/C}\right)\left(5.00 \times 10^{-2}\ \mathrm{m}\right)^2 \cos 60.0^\circ$$
$$= 0.438\ \mathrm{N \cdot\ m^2/C}$$

# Karl Friedrich Gauss

1777 – 1855

Made contributions in

- Electromagnetism
- Number theory
- Statistics
- Non-Euclidean geometry
- Cometary orbital mechanics
- A founder of the German Magnetic Union
  - Studies the Earth's magnetic field

# Gauss's Law, Introduction

Gauss's law is an expression of the general relationship between the net electric flux through a closed surface and the charge enclosed by the surface.

- The closed surface is often called a *gaussian surface.*

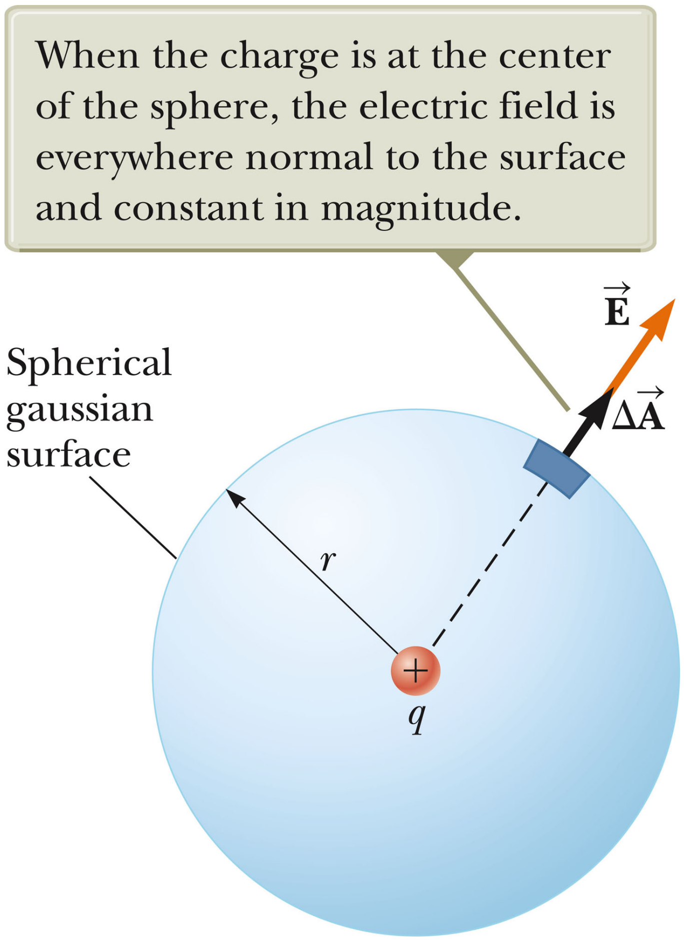
Gauss's law is of fundamental importance in the study of electric fields.

# Gauss's Law – General

A positive point charge, $q$, is located at the center of a sphere of radius $r$.

The magnitude of the electric field everywhere on the surface of the sphere is

$$E = k_e q / r^2$$



# Gauss's Law – General, cont.

The field lines are directed radially outward and are perpendicular to the surface at every point.

$$\Phi_E = \oint \mathbf{E} \cdot d\mathbf{A} = \oint E dA = E \oint dA$$

This will be the net flux through the gaussian surface, the sphere of radius $r$.

We know $E = k_e q / r^2$ and $A_{\text{sphere}} = 4\pi r^2$,

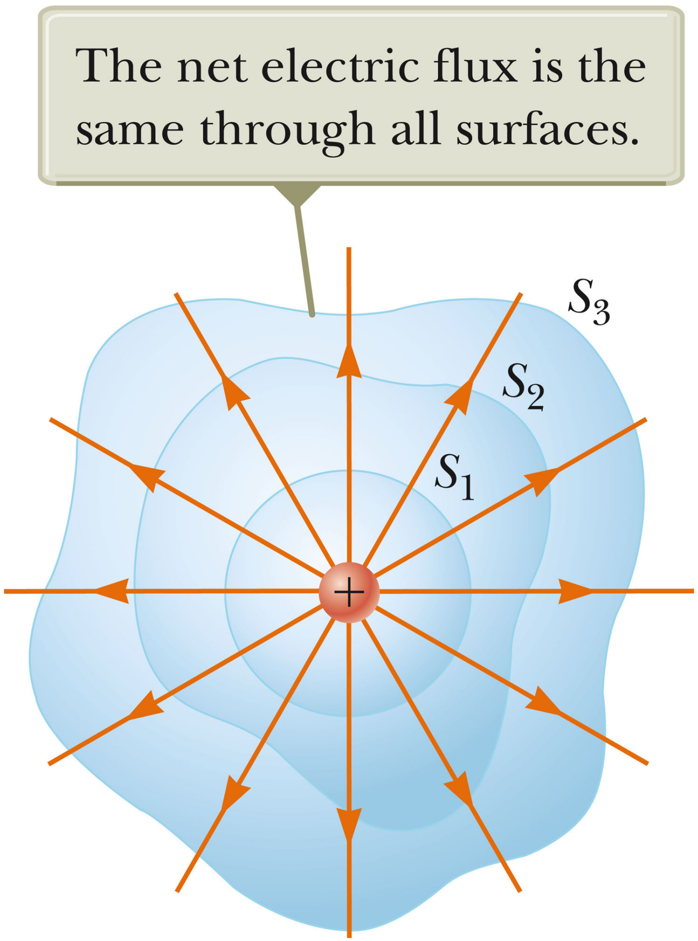
$$\Phi_E = k_e \frac{q}{r^2} \left(4\pi r^2\right) = 4\pi k_e q = \frac{q}{\epsilon_0}$$

# Gaussian Surface, Example

Closed surfaces of various shapes can surround the charge.

- Only $S_1$ is spherical

Verifies the net flux through any closed surface surrounding a point charge q is given by $q/\varepsilon_o$ and is independent of the shape of the surface.

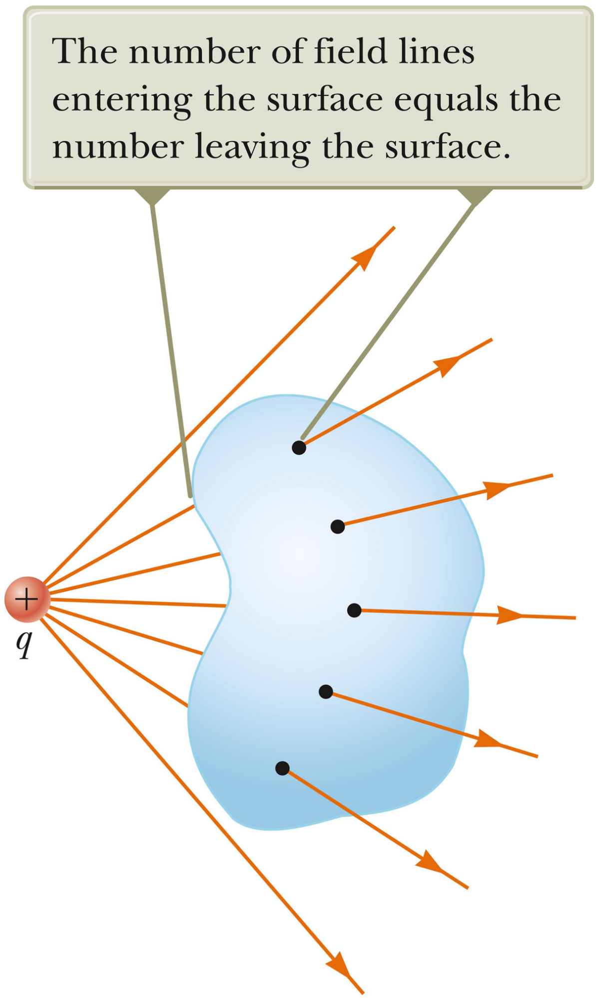


# Gaussian Surface, Example 2

The charge is *outside* the closed surface with an arbitrary shape.

Any field line entering the surface leaves at another point.

Verifies the electric flux through a closed surface that surrounds no charge is zero.



# Gauss's Law – General, notes

The net flux through any closed surface surrounding a point charge, $q$, is given by $q/\varepsilon_o$ and is independent of the shape of that surface.

The net electric flux through a closed surface that surrounds no charge is zero.

Since the electric field due to many charges is the vector sum of the electric fields produced by the individual charges, the flux through any closed surface can be expressed as

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint \left( \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \cdots \right) \cdot d\vec{\mathbf{A}}$$

# Gauss's Law – Final

The mathematical form of Gauss's law states

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

- $q_{\text{in}}$ is the net charge inside the surface.

represents the electric field at any point on the surface.

- $\vec{\mathbf{E}}$ is the *total electric field* and may have contributions from charges both inside and outside of the surface.

Although Gauss's law can, in theory, be solved to find $\vec{\mathbf{E}}$ for any charge configuration, in practice it is limited to symmetric situations.

# Example 23.05: Flux Due to a Point Charge

A spherical gaussian surface surrounds a point charge q. Describe what happens to the total flux through the surface if

(A) the charge is tripled,

The flux through the surface is tripled because flux is proportional to the amount of charge inside the surface.

(B) the radius of the sphere is doubled,

The flux does not change because all electric field lines from the charge pass through the sphere, regardless of its radius.

(C) the surface is changed to a cube,

The flux does not change when the shape of the gaussian surface changes because all electric field lines from the charge pass through the surface, regardless of its shape.

(D) the charge is moved to another location inside the surface.

The flux does not change when the charge is moved to another location inside that surface because Gauss's law refers to the total charge enclosed, regardless of where the charge is located inside the surface.

# Problem 23.13:

An uncharged, nonconducting, hollow sphere of radius $10.0\,\text{cm}$ surrounds a $10.0-\mu\text{C}$ charge located at the origin of a Cartesian coordinate system. A drill with a radius of $1.00\,\text{mm}$ is aligned along the $z$ axis, and a hole is drilled in the sphere. Calculate the electric flux through the hole.

The electric flux through the hole is given by Gauss's Law as

$$\Phi_{E,\text{hole}} = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}_{\text{hole}} = \left(\frac{k_e Q}{R^2}\right)\left(\pi r^2\right)$$

$$= \left(\frac{\left(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right)\left(10.0 \times 10^{-6}\text{C}\right)}{(0.100\ \text{m})^2}\right)$$

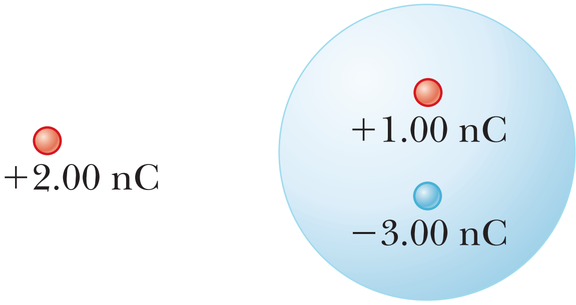
$$= 28.2\ \text{N} \cdot \text{m}^2/\text{C}$$

# Problem 23.14:

Find the net electric flux through the spherical closed surface shown in the Figure. The two charges on the right are inside the spherical surface.

The gaussian surface encloses the $+1.00-\mathrm{nC}$ and $-3.00-\mathrm{nC}$ charges, but not the $+2.00-\mathrm{nC}$ charge. The electric flux is therefore

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\left(1.00\times10^{-9}\mathrm{C} - 3.00\times10^{-9}\mathrm{C}\right)}{8.85\times10^{-12}\mathrm{C}^2/\mathrm{N}\cdot\ \mathrm{m}^2} = -\ 226\ \mathrm{N}\cdot\ \ \mathrm{m}^2/\mathrm{C}$$



# Problem 23.15:

Four closed surfaces, $S_1$ through $S_4$, together with the charges $-2Q$, $Q$, and $-Q$ are sketched in the figure. (The colored lines are the intersections of the surfaces with the page.) Find the electric flux through each surface.

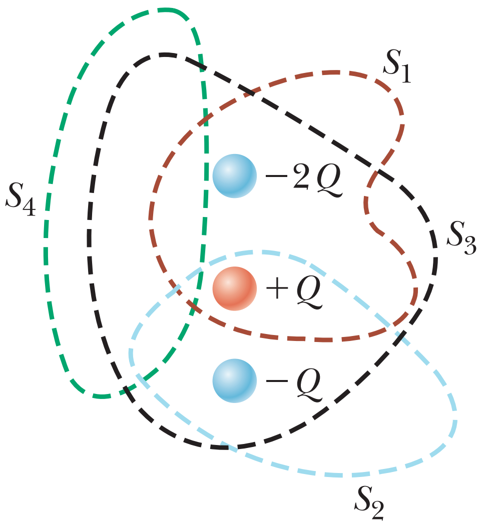
The electric flux through each of the surfaces is given by $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0}$.

Flux through $S_1$ : $\Phi_E = \frac{-2Q + Q}{\epsilon_0} = -\frac{Q}{\epsilon_0}$

Flux through $S_2$ : $\Phi_E = \frac{+Q - Q}{\epsilon_0} = 0$

Flux through $S_3$ : $\Phi_E = \frac{-2Q + Q - Q}{\epsilon_0} = -\frac{2Q}{\epsilon_0}$

Flux through $S_4$ : $\Phi_E = 0$



# Problem 23.16:

A charge of $170 \mu C$ is at the center of a cube of edge $80.0\,\text{cm}$ . No other charges are nearby. (a) Find the flux through each face of the cube. (b) Find the flux through the whole surface of the cube. (c) What If? Would your answers to either part (a) or part (b) change if the charge were not at the center? Explain.

The total flux through the surface of the cube is

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{170 \times 10^{-6}\text{C}}{8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\ \text{m}^2} = 1.92 \times 10^7\ \text{N}\cdot\ \text{m}^2/\text{C}$$

(a) By symmetry, the flux through each face of the cube is the same.

$$\left(\Phi_E\right)_{\text{one face}} = \frac{1}{6}\Phi_E = \frac{1}{6}\frac{q_{\text{in}}}{\epsilon_0}$$

$$\left(\Phi_E\right)_{\text{one face}} = \frac{1}{6}\left(\frac{170 \times 10^{-6}\text{C}}{8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\ \text{m}^2}\right)$$

$$= 3.20 \times 10^6\ \text{N}\cdot\ \text{m}^2/\text{C}$$

(b) $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \left(\frac{170 \times 10^{-6}\text{C}}{8.85 \times 10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2}\right) = 1.92 \times 10^7\ \text{N}\cdot\ \text{m}^2/\text{C}$

(c) The answer to part (a) would change because the charge could now be at different distances from each face of the cube. The answer to part (b) would be unchanged because the flux through the entire closed surface depends only on the total charge inside the surface.

# Problem 23.18:

A particle with charge of $12.0\mu\text{C}$ is placed at the center of a spherical shell of radius $22.0\,\text{cm}$. What is the total electric flux through (a) the surface of the shell and (b) any hemispherical surface of the shell? (c) Do the results depend on the radius? Explain.

(a) The total electric flux through the surface of the shell is

$$\Phi_{E,\text{ shell}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{12.0 \times 10^{-6}}{8.85 \times 10^{-12}} = 1.36 \times 10^6\ \text{N} \cdot \ \text{m}^2/\text{C}$$
$$= 1.36\text{MN} \cdot \ \text{m}^2/\text{C}$$

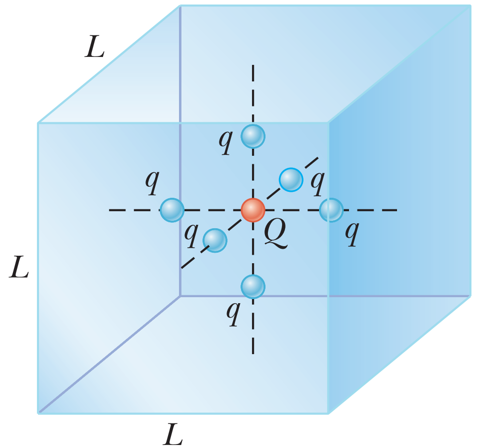
(b) Through any hemispherical surface of the shell, by symmetry,

$$\Phi_{E,\text{ half shell}} = \frac{1}{2}\left(1.36 \times 10^6\ \text{N} \cdot \ \text{m}^2/\text{C}\right) = 6.78 \times 10^5\ \text{N} \cdot \ \text{m}^2/\text{C}$$
$$= 678\text{kN} \cdot \ \text{m}^2/\text{C}$$

(c) No , the same number of field lines will pass through each surface, no matter how the radius changes.

# Problem 23.19:

A particle with charge $Q = 5.00\mu\text{C}$ is located at the center of a cube of edge $L = 0.100\ \text{m}$. In addition, six other identical charged particles having $q = -1.00\mu\text{C}$ are positioned symmetrically around $Q$ as shown in the figure. Determine the electric flux through one face of the cube.




The total charge is $Q - 6|q|$. The total outward flux from the cube is $\frac{Q - 6|q|}{\epsilon_0}$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \frac{Q - 6|q|}{6\epsilon_0}$$

$$(\Phi_E)_{\text{one face}} = \frac{Q - 6|q|}{6\epsilon_0} = \frac{(5.00 - 6.00)\times 10^{-6}\text{C} \cdot \text{N} \cdot \text{m}^2}{6 \times 8.85 \times 10^{-12}\text{C}^2}$$

$$= -18.8\text{kN} \cdot \text{m}^2/\text{C}$$

# Applying Gauss's Law

To use Gauss's law, you want to choose a gaussian surface over which the surface integral can be simplified and the electric field determined.

Take advantage of symmetry.

Remember, the gaussian surface is a surface you choose, it does not have to coincide with a real surface.

# Conditions for a Gaussian Surface

Try to choose a surface that satisfies one or more of these conditions:

- 1. The value of the electric field can be argued by symmetry to be constant over the portion of the surface.
- 2. The dot product $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$ can be expressed as a simple algebraic product $EdA$ because $\vec{\mathbf{E}}$ and $d\vec{\mathbf{A}}$ are parallel.
- 3. The dot product is zero because $\vec{\mathbf{E}}$ and $d\vec{\mathbf{A}}$ are perpendicular.
- 4. The electric field is zero over the portion of the surface.

If the charge distribution does not have sufficient symmetry such that a gaussian surface that satisfies these conditions can be found, Gauss' law is not useful for determining the electric field for that charge distribution.

# Example 23.06: A Spherically Symmetric Charge Distribution

An insulating solid sphere of radius $a$ has a uniform volume charge density $\rho$ and carries a total positive charge $Q$.

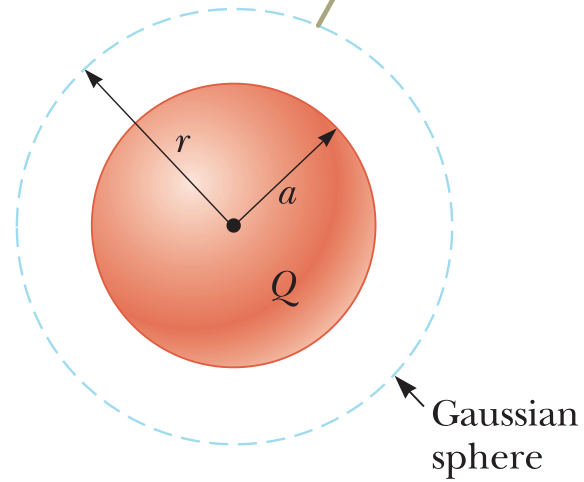
(A) Calculate the magnitude of the electric field at a point outside the sphere.

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\,dA = \frac{Q}{\epsilon_0}$$

$$\oint E\,dA = E\oint dA = E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}$$

$$\text{(1)}\ E = \frac{Q}{4\pi\epsilon_0 r^2} = k_e\frac{Q}{r^2} \quad (\text{for } r > a)$$

For points outside the sphere, a large, spherical gaussian surface is drawn concentric with the sphere.



# Example 23.06: A Spherically Symmetric Charge Distribution

An insulating solid sphere of radius $a$ has a uniform volume charge density $\rho$ and carries a total positive charge $Q$.

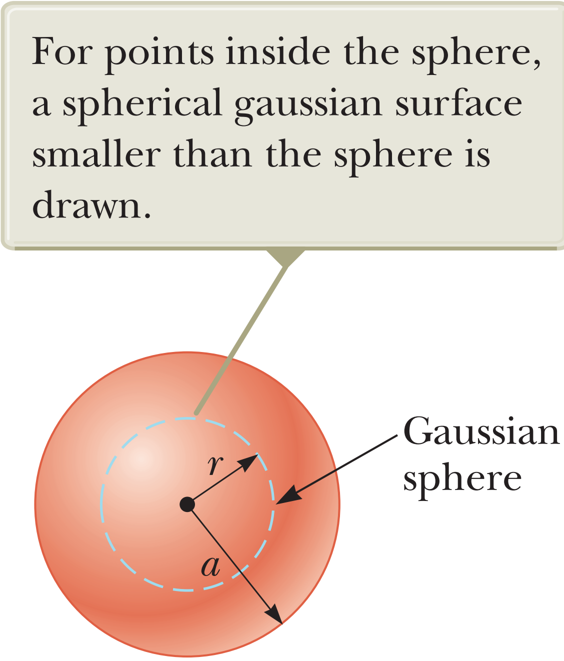
(B) Find the magnitude of the electric field at a point inside the sphere.

$$q_{\text{in}} = \rho V' = \rho\left(\frac{4}{3}\pi r^3\right)$$

$$\oint E\,dA = E\oint dA = E\left(4\pi r^2\right) = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E = \frac{q_{\text{in}}}{4\pi\epsilon_0 r^2} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0}r$$

$$\text{(2) } E = \frac{Q/\frac{4}{3}\pi a^3}{3\left(1/4\pi k_e\right)}r = k_e\frac{Q}{a^3}r \quad (\text{for } r < a)$$



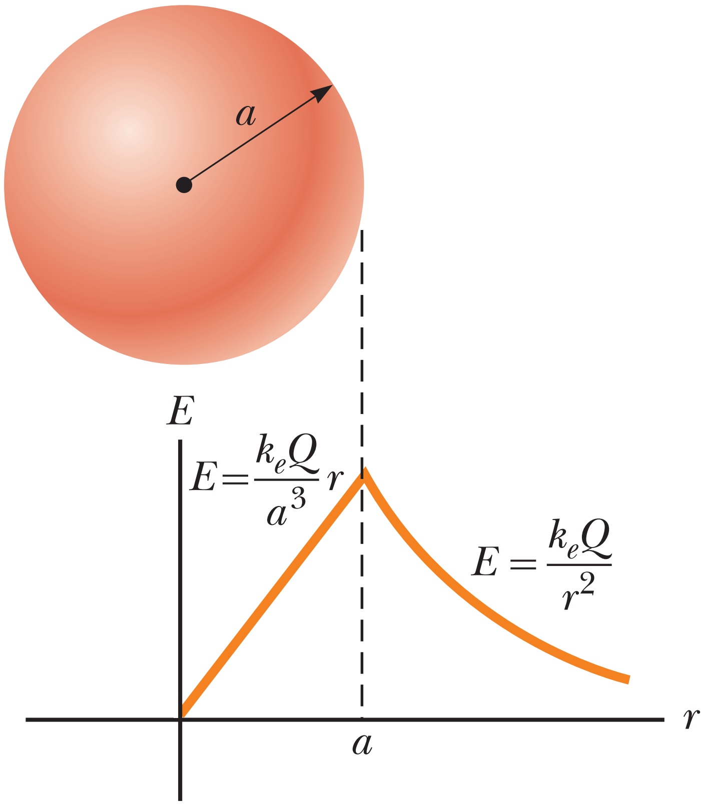
# Example 23.06: A Spherically Symmetric Charge Distribution

An insulating solid sphere of radius $a$ has a uniform volume charge density $\rho$ and carries a total positive charge $Q$.

Inside the sphere, $E$ varies linearly with $r$

$E \to 0$ as $r \to 0$

The field outside the sphere is equivalent to that of a point charge located at the center of the sphere.



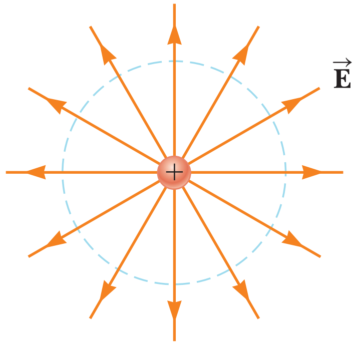
# Example 23.07: A Cylindrically Symmetric Charge Distribution

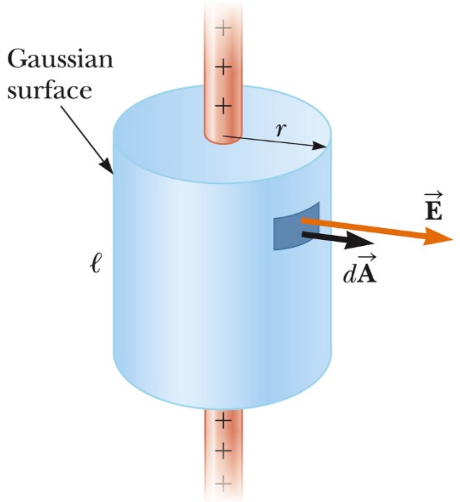
Find the electric field a distance $r$ from a line of positive charge of infinite length and constant charge per unit length $\lambda$.

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\oint dA = EA = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\lambda \ell}{\epsilon_0}$$

$$E(2\pi r \ell) = \frac{\lambda \ell}{\epsilon_0}$$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} = 2k_e\frac{\lambda}{r}$$



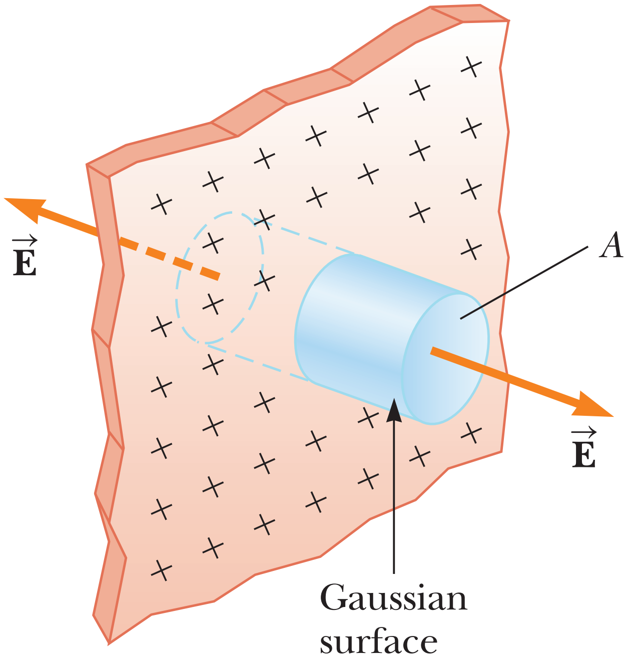



# Example 23.08: A Plane of Charge

Find the electric field due to an infinite plane of positive charge with uniform surface charge density $\sigma$.

$$\Phi_E = 2EA = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$



# Example 23.09: Don't Use Gauss's Law Here!

Explain why Gauss's law cannot be used to calculate the electric field near an electric dipole, a charged disk, or a triangle with a point charge at each corner.

The charge distributions of all these configurations do not have sufficient symmetry to make the use of Gauss's law practical.

We cannot find a closed surface surrounding any of these distributions for which all portions of the surface satisfy one or more of conditions (1) through (4) listed at the beginning of this section.

# Problem 23.24:

Determine the magnitude of the electric field at the surface of a lead-$208$ nucleus, which contains $82$ protons and $126$ neutrons. Assume the lead nucleus has a volume $208$ times that of one proton and consider a proton to be a sphere of radius $1.20 \times 10^{-15}$ m.

The charge distributed through the nucleus creates a field at the surface equal to that of a point charge at its center: $E = \frac{k_e q}{r^2}$.

$$E = \frac{\left(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right)\left(82 \times 1.60 \times 10^{-19}\text{C}\right)}{\left[(208)^{1/3}\left(1.20 \times 10^{-15}\ \text{m}\right)\right]^2}$$

$E = 2.33 \times 10^{21}$ N/C away from the nucleus

# Problem 23.27:

A large, flat, horizontal sheet of charge has a charge per unit area of $9.00\mu\mathrm{C/m^2}$. Find the electric field just above the middle of the sheet.

For a large uniformly charged sheet, $\vec{E}$ will be perpendicular to the sheet, and will have a magnitude of

$$E = \frac{\sigma}{2\epsilon_0} = 2\pi k_e \sigma$$

$$= (2\pi)\left(8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}\right)\left(9.00 \times 10^{-6}\mathrm{C/m^2}\right)$$

so $\vec{\mathbf{E}} = 5.08 \times 10^5\ \mathrm{N/C}\hat{\mathbf{j}}$

# Problem 23.29:

A uniformly charged, straight filament $7.00\ \mathrm{m}$ in length has a total positive charge of $2.00\mu\mathrm{C}$. An uncharged cardboard cylinder $2.00\ \mathrm{cm}$ in length and $10.0\ \mathrm{cm}$ in radius surrounds the filament at its center, with the filament as the axis of the cylinder. Using reasonable approximations, find (a) the electric field at the surface of the cylinder and (b) the total electric flux through the cylinder.

The approximation in this case is that the filament length is so large when compared to the cylinder length that the "infinite line" of charge can be assumed.
(a) We have

$$E=\frac{2k_e\lambda}{r}$$

where the linear charge density is

$$\lambda=\frac{2.00\times10^{-6}\mathrm{C}}{7.00\ \mathrm{m}}=2.86\times10^{-7}\mathrm{C/m}$$

so

$$E=\frac{(2)\left(8.99\times10^{9}\ \mathrm{N}\cdot\ \mathrm{m}^2/\mathrm{C}\right)\left(2.86\times10^{-7}\mathrm{C/m}\right)}{0.100\ \mathrm{m}}$$
$$=51.4\mathrm{kN/C}\text{ radially outward}$$

(b) We can find the flux by multiplying the field and the lateral surface area of the cylinder:

$$\Phi_E=2\pi rLE=2\pi rL\left(\frac{2k_e\lambda}{r}\right)=4\pi k_e\lambda L$$

so

$$\Phi_E=4\pi\left(8.99\times10^{9}\ \mathrm{N}\cdot\ \mathrm{m}^2/\mathrm{C}^2\right)\left(2.86\times10^{-7}\mathrm{C/m}\right)(0.0200\ \mathrm{m})$$
$$=6.46\times10^{2}\ \mathrm{N}\cdot\ \mathrm{m}^2/\mathrm{C}$$

# Problem 23.33:

A solid sphere of radius 40.0 cm has a total positive charge of $26.0\mu\mathrm{C}$ uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) $0\,\mathrm{cm}$, (b) $10\,\mathrm{cm}$, (c) $40\,\mathrm{cm}$, and (d) $60\,\mathrm{cm}$ from the center of the sphere.

(a) At the center of the sphere, the total charge is zero, so

$$E = \frac{k_e Q r}{a^3} = 0$$

(b) At a distance of $10.0\ \mathrm{cm} = 0.100\ \mathrm{m}$ from the center,

$$E = \frac{k_e Q r}{a^3} = \frac{\left(8.99\times 10^9\ \mathrm{N\cdot\ m^2/C}\right)\left(26.0\times 10^{-6}\mathrm{C}\right)(0.100\ \mathrm{m})}{(0.400\ \mathrm{m})^3}$$

$$= 365\mathrm{kN/C}$$

(c) At a distance of $40.0\ \mathrm{cm} = 0.400\ \mathrm{m}$ from the center, all of the charge is enclosed, so

$$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times 10^9\ \mathrm{N\cdot\ m^2/C}\right)\left(26.0\times 10^{-6}\mathrm{C}\right)}{(0.400\ \mathrm{m})^2}$$

$$= 1.46\mathrm{MN/C}$$

(d) At a distance of $60.0\ \mathrm{cm} = 0.600\ \mathrm{m}$ from the center,

$$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times 10^9\ \mathrm{N\cdot\ m^2/C}\right)\left(26.0\times 10^{-6}\mathrm{C}\right)}{(0.600\ \mathrm{m})^2}$$

$$= 649\mathrm{kN/C}$$

The direction for each electric field is radially outward

# Problem 23.34:

A cylindrical shell of radius $7.00\text{ cm}$ and length $2.40\text{ m}$ has its charge uniformly distributed on its curved surface. The magnitude of the electric field at a point $19.0\text{ cm}$ radially outward from its axis (measured from the midpoint of the shell) is $36.0\text{kN/C}$. Find (a) the net charge on the shell and (b) the electric field at a point $4.00\text{ cm}$ from the axis, measured radially outward from the midpoint of the shell.

(a) The electric field is given by

$$E = \frac{2k_e\lambda}{r} = \frac{2k_e(Q/\ell)}{r}$$

Solving for the charge $Q$ gives

$$Q = \frac{Er\ell}{2k_e} = \frac{\left(3.60\times10^4\ \text{N/C}\right)(0.190\ \text{m})(2.40\ \text{m})}{2\left(8.99\times10^9\ \text{N}\cdot\ \text{m}^2/\text{C}\right)} =$$

$$Q = +9.13\times10^{-7}\text{C} = +913\text{nC}$$

(b) Since the charge is uniformly distributed on the surface of the cylindrical shell, a gaussian surface in the shape of a cylinder of $4.00\text{ cm}$ in radius encloses no charge, and $\vec{E} = 0$.

# Problem 23.38:

Three solid plastic cylinders all have radius $2.50\,\text{cm}$ and length $6.00\,\text{cm}$. Find the charge of each cylinder given the following additional information about each one. Cylinder (a) carries charge with uniform density $15.0\text{nC/m}^2$ everywhere on its surface. Cylinder (b) carries charge with uniform density $15.0\text{nC/m}^2$ on its curved lateral surface only. Cylinder (c) carries charge with uniform density $500\text{nC/m}^3$ throughout the plastic.

(a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r + L)$.

$$
\begin{aligned}
Q &= \sigma A \\
&= \left(15.0 \times 10^{-9}\text{C/m}^2\right) 2\pi(0.0250\ \text{m})[0.0250\ \text{m} + 0.0600\ \text{m}] \\
&= 2.00 \times 10^{-10}\text{C}
\end{aligned}
$$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$$
\begin{aligned}
Q &= \sigma A = \left(15.0 \times 10^{-9}\text{C/m}^2\right)[2\pi(0.0250\ \text{m})(0.0600\ \text{m})] \\
&= 1.41 \times 10^{-10}\text{C}
\end{aligned}
$$

(c)

$$
\begin{aligned}
Q &= \rho V = \rho\pi r^2 L = \left(500 \times 10^{-9}\text{C/m}^3\right)\left[\pi(0.0250\ \text{m})^2(0.0600\ \text{m})\right] \\
&= 5.89 \times 10^{-11}\text{C}
\end{aligned}
$$