Chapter 22

Electric Fields

Electricity and Magnetism

The laws of electricity and magnetism play a central role in the operation of many modern devices.

The interatomic and intermolecular forces responsible for the formation of solids and liquids are electric in nature.

Electricity and Magnetism – Forces

The concept of force links the study of electromagnetism to previous study.

The electromagnetic force between charged particles is one of the fundamental forces of nature.

Electric Charges

There are two kinds of electric charges

- Called positive and negative
  - Negative charges are the type possessed by electrons.
  - Positive charges are the type possessed by protons.

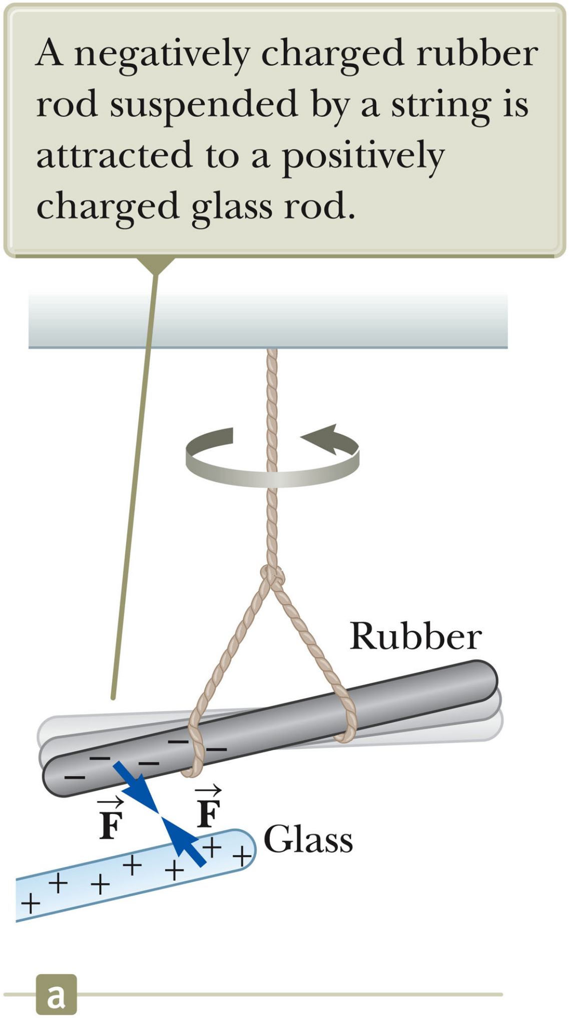
Charges of the same sign repel one another and charges with opposite signs attract one another.

Electric Charges, 2

The rubber rod is negatively charged.

The glass rod is positively charged.

The two rods will attract.

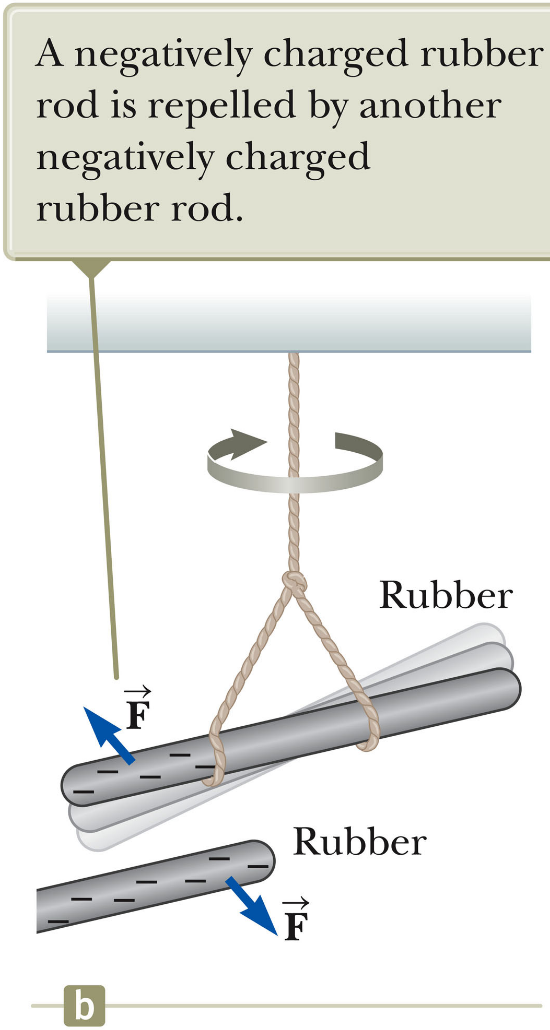


Electric Charges, 3

The rubber rod is negatively charged.

The second rubber rod is also negatively charged.

The two rods will repel.



More About Electric Charges

Electric charge is always conserved in an isolated system.

- For example, charge is not created in the process of rubbing two objects together.
- The electrification is due to a transfer of charge from one object to another.

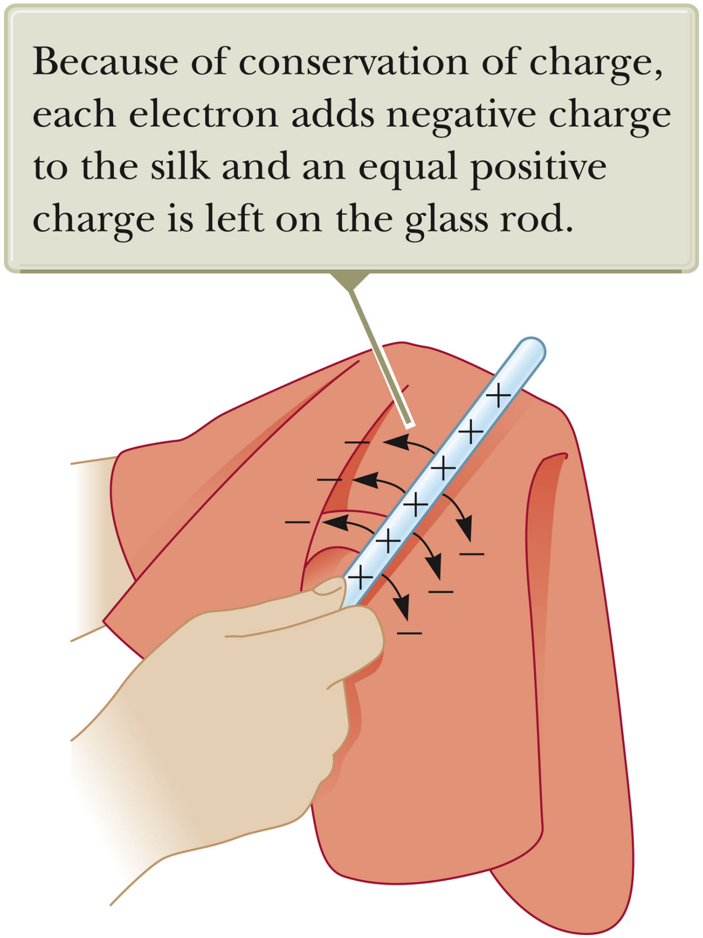
Conservation of Electric Charges

A glass rod is rubbed with silk.

Electrons are transferred from the glass to the silk.

Each electron adds a negative charge to the silk.

An equal positive charge is left on the rod.



Quantization of Electric Charges

The electric charge, q, is said to be quantized.

- q is the standard symbol used for charge as a variable.
- Electric charge exists as discrete packets.
- $q = \pm Ne$
  - N is an integer
  - e is the fundamental unit of charge
  - $|e| = 1.6 \times 10^{-19}\,\text{C}$
  - Electron: $q = -e$
  - Proton: $q = +e$

Conductors

Electrical conductors are materials in which some of the electrons are free electrons.

- Free electrons are not bound to the atoms.
- These electrons can move relatively freely through the material.
- Examples of good conductors include copper, aluminum and silver.
- When a good conductor is charged in a small region, the charge readily distributes itself over the entire surface of the material.

Insulators

Electrical insulators are materials in which all of the electrons are bound to atoms.

- These electrons can not move relatively freely through the material.
- Examples of good insulators include glass, rubber and wood.
- When a good insulator is charged in a small region, the charge is unable to move to other regions of the material.

Semiconductors

The electrical properties of semiconductors are somewhere between those of insulators and conductors.

Examples of semiconductor materials include silicon and germanium.

- Semiconductors made from these materials are commonly used in making electronic chips.

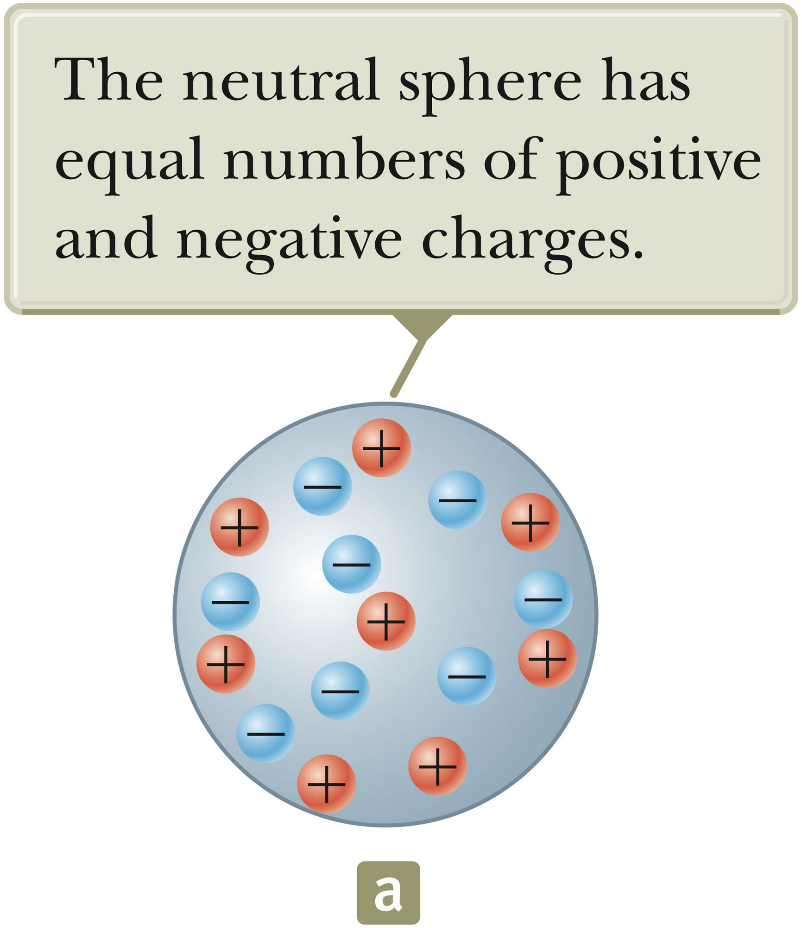
The electrical properties of semiconductors can be changed by the addition of controlled amounts of certain atoms to the material.

Charging by Induction

Charging by induction requires no contact with the object inducing the charge.

Assume we start with a neutral metallic sphere.

- The sphere has the same number of positive and negative charges.



Charging by Induction, 2

B:

A charged rubber rod is placed near the sphere.

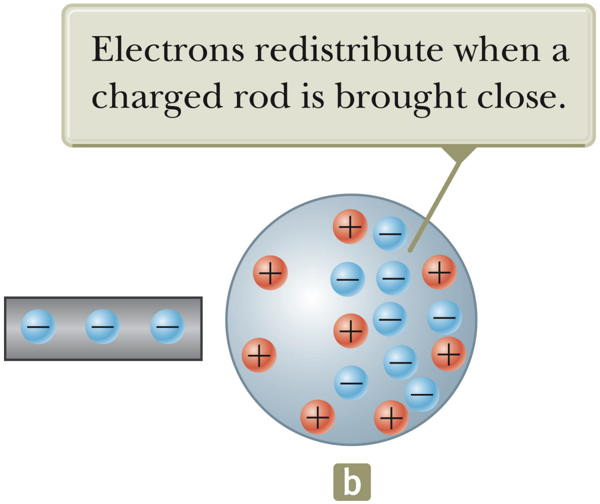
- It does **not** touch the sphere.

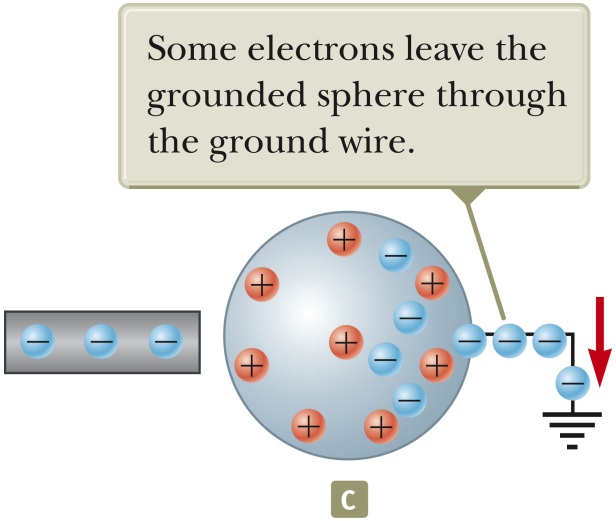
The electrons in the neutral sphere are redistributed.

C:

The sphere is grounded.

Some electrons can leave the sphere through the ground wire.





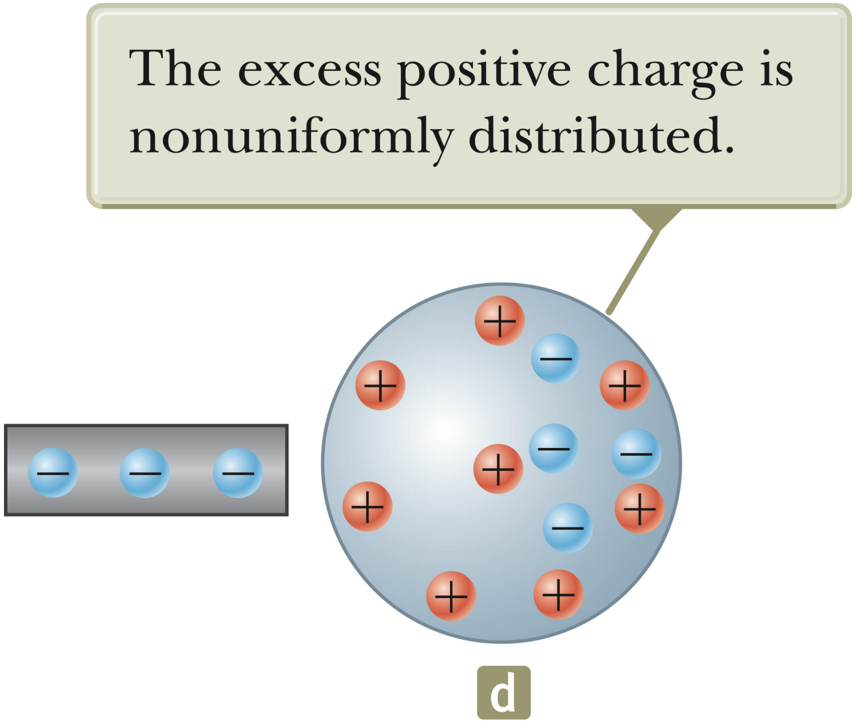
Charging by Induction, 3

The ground wire is removed.

There will now be more positive charges.

The charges are not uniformly distributed.

The positive charge has been ***induced*** in the sphere.



Charging by Induction, 4

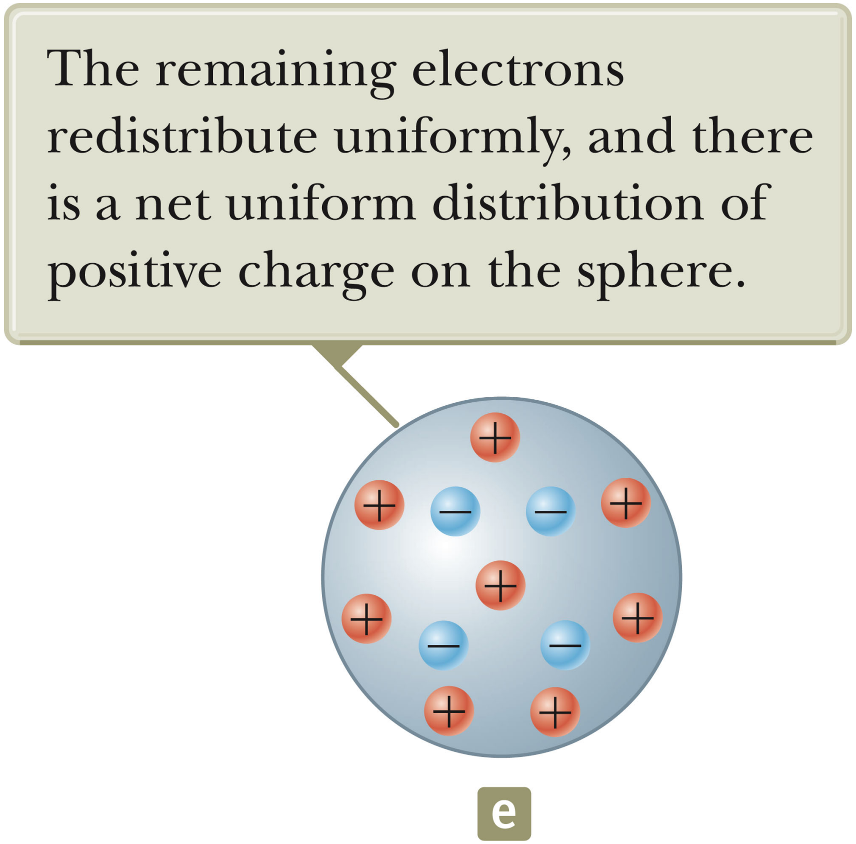
The rod is removed.

The electrons remaining on the sphere redistribute themselves.

There is still a net positive charge on the sphere.

The charge is now uniformly distributed.

Note the rod lost none of its negative charge during this process.

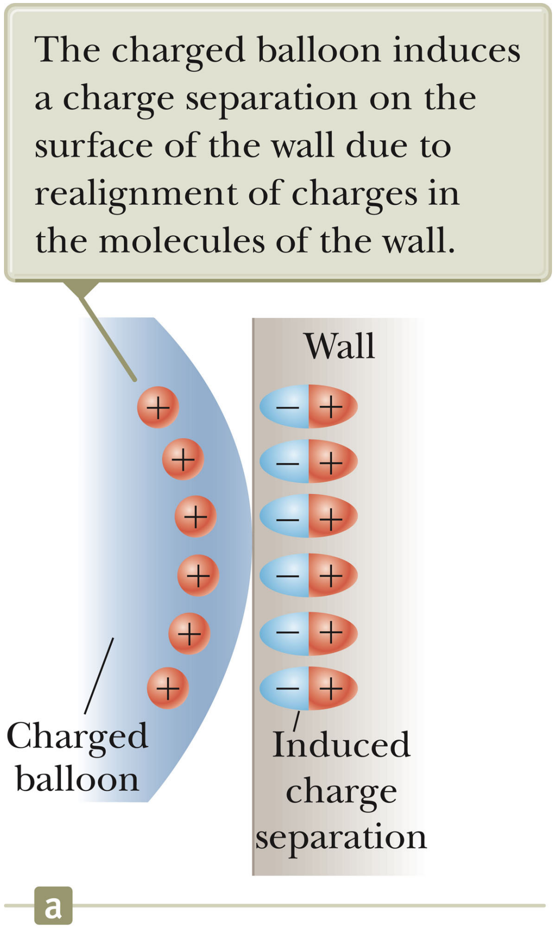


Charge Rearrangement in Insulators

A process similar to induction can take place in insulators.

The charges within the molecules of the material are rearranged.

The proximity of the positive charges on the surface of the object and the negative charges on the surface of the insulator results in an attractive force between the object and the insulator.



Charles Coulomb

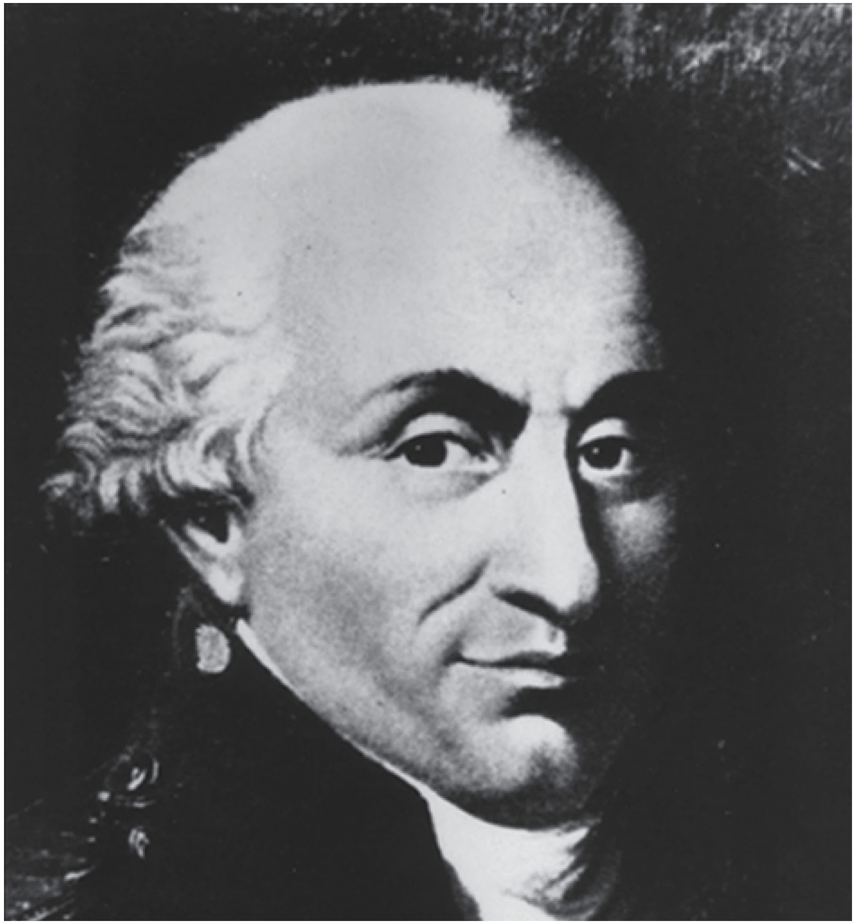
1736 – 1806

French physicist

Major contributions were in areas of electrostatics and magnetism

Also investigated in areas of

- Strengths of materials
- Structural mechanics
- Ergonomics



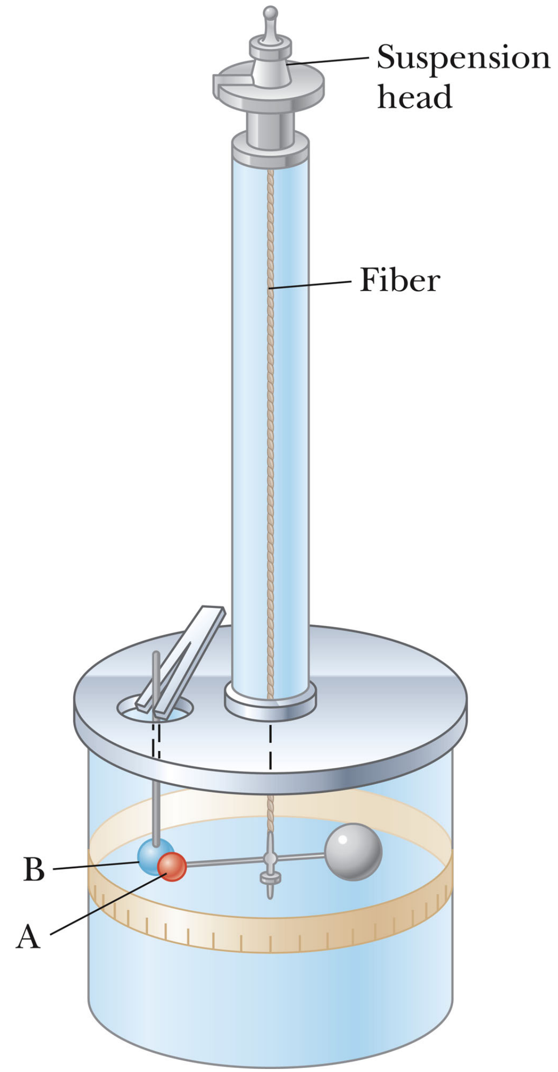
Coulomb's Law

Charles Coulomb measured the magnitudes of electric forces between two small charged spheres.

The force is inversely proportional to the square of the separation *r* between the charges and directed along the line joining them.

The force is proportional to the product of the charges, q_1 and q_2, on the two particles.

The electrical force between two stationary point charges is given by Coulomb's Law.



Point Charge

The term **point charge** refers to a particle of zero size that carries an electric charge.

- The electrical behavior of electrons and protons is well described by modeling them as point charges.

Coulomb’s Law, cont.

The force is attractive if the charges are of opposite sign.

The force is repulsive if the charges are of like sign.

The force is a conservative force.

Coulomb's Law, Equation

Mathematically,

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

The SI unit of charge is the **coulomb**.

k_e is called the **Coulomb constant.**

- $k_e = 8.9876 \times 10^9 \text{ N} \cdot \text{ m}^2/\text{C}^2 = 1/(4\pi\epsilon_o)$
- ϵ_o is the **permittivity of free space.**
- $\epsilon_0 = 8.8542 \times 10^{-12}\,\text{C}^2/\text{N} \cdot \text{m}^2$

Coulomb's Law, Notes

Remember the charges need to be in coulombs.

- e is the smallest unit of charge.
  - except quarks
- $e = 1.6 \times 10^{-19}\mathrm{C}$
- So $1\ \mathrm{C}$ needs 6.24×10^{18} electrons or protons

Typical charges can be in the $\mu\mathrm{C}$ range.

Remember that force is a *vector* quantity.

Particle Summary

Table 22.1 Charge and Mass of the Electron, Proton, and Neutron

Particle	**Charge (C)**	**Mass (kg)**
Electron (e)	$-1.602\,176\,6 \times 10^{-19}$	$9.109\,4 \times 10^{-31}$
Proton (p)	$+1.602\,176\,6 \times 10^{-19}$	$1.672\,62 \times 10^{-27}$
Neutron (n)	0	$1.674\,93 \times 10^{-27}$

The electron and proton are identical in the magnitude of their charge, but very different in mass.

The proton and the neutron are similar in mass, but very different in charge.

Example 22.01: The Hydrogen Atom

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

Use Coulomb's law to find the magnitude of the electric force

$$F_e = k_e \frac{|e||-e|}{r^2} = \left(8.988 \times 10^9 \text{ N} \cdot \text{ m}^2/\text{C}^2\right) \frac{\left(1.60 \times 10^{-19} \text{C}\right)^2}{\left(5.3 \times 10^{-11} \text{ m}\right)^2}$$

$$= 8.2 \times 10^{-8} \text{ N}$$

Use Newton's law of universal gravitation to find the magnitude of the gravitational force:

$$F_g = G \frac{m_e m_p}{r^2}$$

$$= \left(6.674 \times 10^{-11} \text{ N} \cdot \text{ m}^2/\text{kg}^2\right) \frac{\left(9.11 \times 10^{-31} \text{ kg}\right)\left(1.67 \times 10^{-27} \text{ kg}\right)}{\left(5.3 \times 10^{-11} \text{ m}\right)^2}$$

$$= 3.6 \times 10^{-47} \text{ N}$$

Problem 22.09:

In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is 5.29×10^{-11} m. (a) Find the magnitude of the electric force exerted on each particle. (b) If this force causes the centripetal acceleration of the electron, what is the speed of the electron?

(a) $F = \frac{k_e e^2}{r^2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{ m}^2/\text{C}^2\right) \frac{\left(1.60 \times 10^{-19} \text{C}\right)^2}{\left(0.529 \times 10^{-10} \text{ m}\right)^2} = 8.22 \times 10^{-8} \text{ N}$

toward the other particle.

(b) We have $F = \frac{mv^2}{r}$ from which

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{\left(8.22 \times 10^{-8} \text{ N}\right)\left(0.529 \times 10^{-10} \text{ m}\right)}{9.11 \times 10^{-31} \text{ kg}}}$$

$$= 2.19 \times 10^6 \text{ m/s}$$

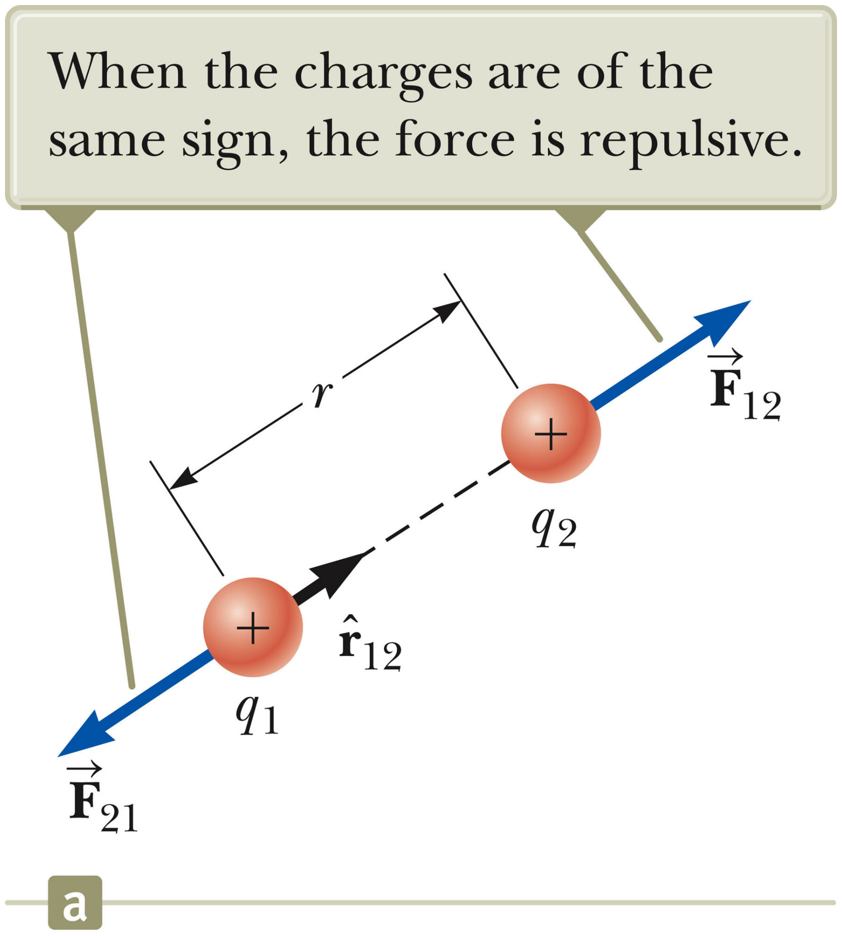
Vector Nature of Electric Forces

In vector form,

$$\vec{\mathbf{F}}_{12} = k_e \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}_{12}$$

$\hat{\mathbf{r}}_{12}$ is a unit vector directed from q_1 to q_2.

The like charges produce a repulsive force between them.



Vector Nature of Electrical Forces, cont.

Electrical forces obey Newton's Third Law.

The force on q_1 is equal in magnitude and opposite in direction to the force on q_2

- $\vec{\mathbf{F}}_{21} = -\vec{\mathbf{F}}_{12}$

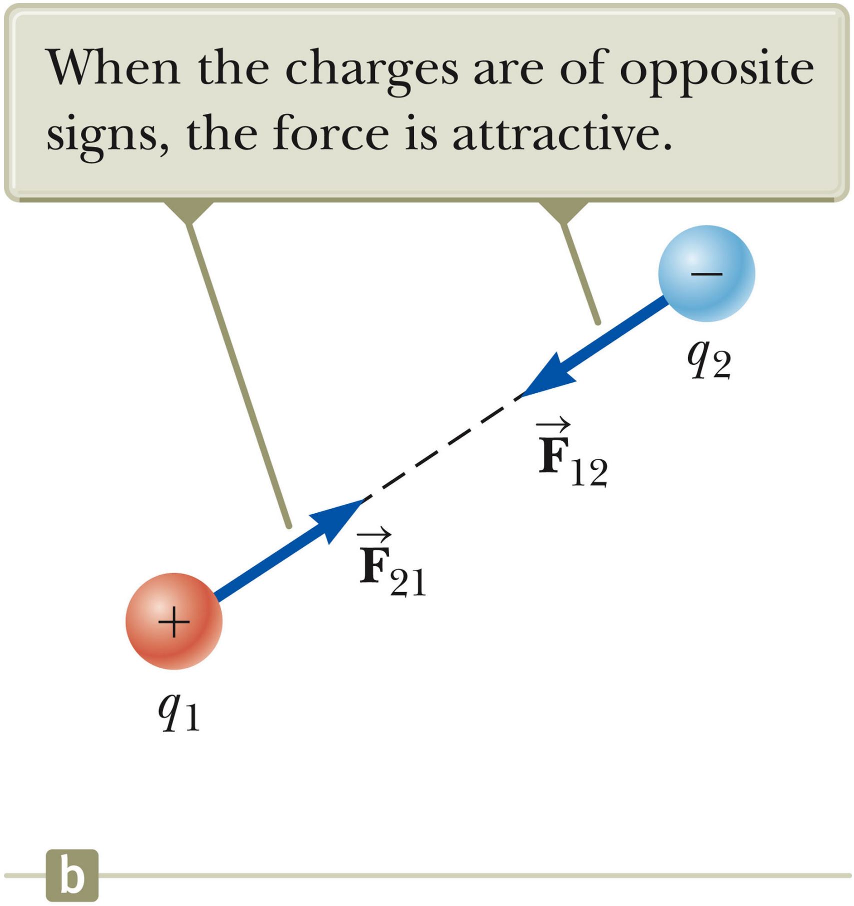
With like signs for the charges, the product $q_1 q_2$ is positive and the force is repulsive.

Vector Nature of Electrical Forces, 3

Two point charges are separated by a distance r.

The unlike charges produce an attractive force between them.

With unlike signs for the charges, the product q_1q_2 is negative and the force is attractive.



A Final Note about Directions

The sign of the product of q_1q_2 gives the *relative* direction of the force between q_1 and q_2.

The *absolute* direction is determined by the actual location of the charges.

Multiple Charges

The resultant force on any one charge equals the vector sum of the forces exerted by the other individual charges that are present.

- Remember to add the forces *as vectors.*

The resultant force on q_1 is the vector sum of all the forces exerted on it by other charges.

For example, if four charges are present, the resultant force on one of these equals the vector sum of the forces exerted on it by each of the other charges.

$$\sum \vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{41}$$

Example 22.02: Find the Resultant Force

Consider three point charges located at the corners of a right triangle as shown in the figure, where $q_1 = q_3 = 5.00\mu\text{C}, q_2 = -2.00\mu\text{C}$, and $a = 0.100\ \text{m}$. Find the resultant force exerted on q_3.

Find the magnitude of $\vec{\mathbf{F}}_{23}$

$$F_{23} = k_e \frac{|q_2||q_3|}{a^2}$$

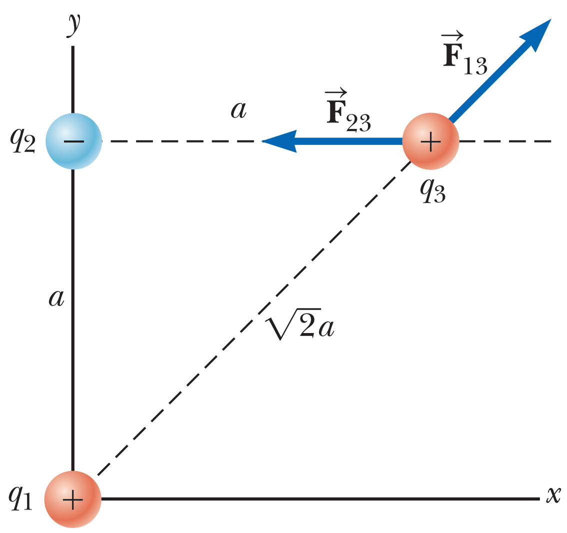
$$= \left(8.988 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right) \frac{\left(2.00 \times 10^{-6}\text{C}\right)\left(5.00 \times 10^{-6}\text{C}\right)}{(0.100\ \text{m})^2} = 8.99\ \text{N}$$

Find the magnitude of $\vec{\mathbf{F}}_{13}$

$$F_{13} = k_e \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

$$= \left(8.988 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right) \frac{\left(5.00 \times 10^{-6}\text{C}\right)\left(5.00 \times 10^{-6}\text{C}\right)}{2(0.100\ \text{m})^2} = 11.2\ \text{N}$$

$$F_{13x} = (11.2\ \text{N})\cos 45.0^\circ = 7.94\ \text{N}$$
$$F_{13y} = (11.2\ \text{N})\sin 45.0^\circ = 7.94\ \text{N}$$



Find the components of the resultant force acting on q_3

$$F_{3x} = F_{13x} + F_{23x} = 7.94\ \text{N} + (-8.99\ \text{N}) = -1.04\ \text{N}$$
$$F_{3y} = F_{13y} + F_{23y} = 7.94\ \text{N} + 0 = 7.94\ \text{N}$$
$$\vec{\mathbf{F}}_3 = (-1.04\hat{\mathbf{i}} + 7.94\hat{\mathbf{j}})\text{N}$$

Example 22.03: Where Is the Net Force Zero?

Three point charges lie along the x axis as shown in the figure. The positive charge $q_1 = 15.0\mu\text{C}$ is at $x = 2.00$ m, the positive charge $q_2 = 6.00\mu\text{C}$ is at the origin, and the net force acting on q_3 is zero. What is the x coordinate of q_3?

Write an expression for the net force on charge q_3 when it is in equilibrium:

$$\sum \vec{\mathbf{F}}_3 = \vec{\mathbf{F}}_{23} + \vec{\mathbf{F}}_{13} = -k_e \frac{|q_2|\,|q_3|}{x^2}\hat{\mathbf{i}} + k_e \frac{|q_1|\,|q_3|}{(2.00 - x)^2}\hat{\mathbf{i}} = 0$$

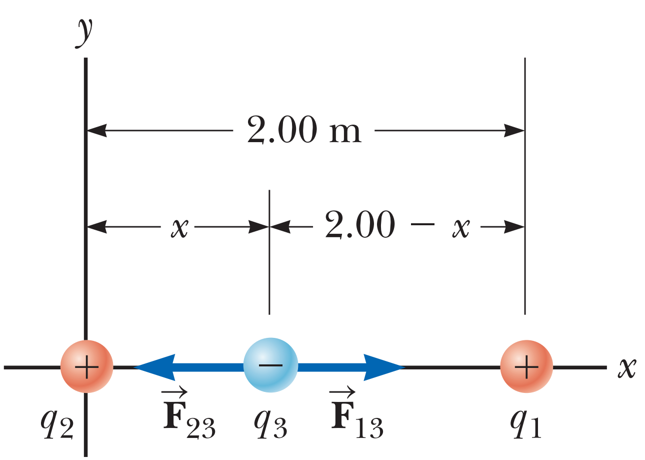
$$k_e \frac{|q_2|\,|q_3|}{x^2} = k_e \frac{|q_1|\,|q_3|}{(2.00 - x)^2}$$

$$(2.00 - x)^2 |q_2| = x^2 |q_1|$$

$$(2.00 - x)\sqrt{|q_2|} = \pm x\sqrt{|q_1|}$$

$$x = \frac{2.00\sqrt{|q_2|}}{\sqrt{|q_2|} \pm \sqrt{|q_1|}}$$

$$x = \frac{2.00\sqrt{6.00 \times 10^{-6}\text{C}}}{\sqrt{6.00 \times 10^{-6}\text{C}} + \sqrt{15.0 \times 10^{-6}\text{C}}} = 0.775 \text{ m}$$



Problem 22.12:

Particle A of charge $3.00 \times 10^{-4}\mathrm{C}$ is at the origin, particle B of charge $-6.00 \times 10^{-4}\mathrm{C}$ is at ($4.00\ \mathrm{m}, 0$), and particle C of charge $1.00 \times 10^{-4}\mathrm{C}$ is at $(0, 3.00\ \mathrm{m})$. We wish to find the net electric force on C .

(a) What is the x component of the electric force exerted by A on C?

(b) What is the y component of the force exerted by A on C ?

(c) Find the magnitude of the force exerted by B on C .

(d) Calculate the x component of the force exerted by B on C .

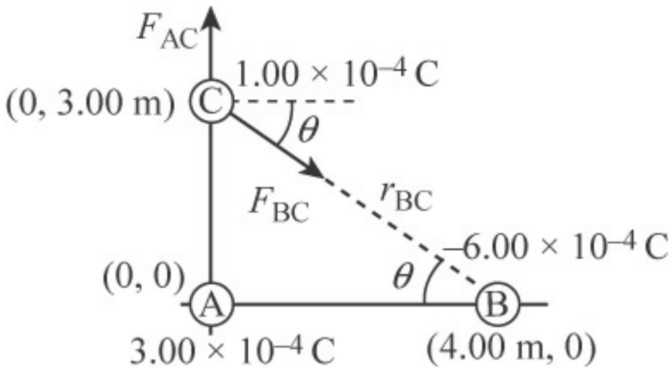
(e) Calculate the y component of the force exerted by B on C.

(f) Sum the two x components from parts (a) and (d) to obtain the resultant x component of the electric force acting on C.

(g) Similarly, find the y component of the resultant force vector acting on C.

(h) Find the magnitude and direction of the resultant electric force acting on C.

Problem 22.12:



$$\theta = \tan^{-1}\left(\frac{3.00\ \text{m}}{4.00\ \text{m}}\right) = 36.9^\circ$$

$$r_{BC} = \sqrt{(4.00\ \text{m})^2 + (3.00\ \text{m})^2} = 5.00\ \text{m}$$

(a) $\left(F_{AC}\right)_x = 0$

(b)
$$\left(F_{AC}\right)_y = \left|F_{AC}\right| = k_e \frac{\left|q_A\right|\left|q_C\right|}{r_{AC}^2}$$

$$\left(F_{AC}\right)_y = \left(8.99 \times 10^9\ \text{N}\cdot\ \text{m}^2/\text{C}^2\right) \frac{\left(3.00 \times 10^{-4}\text{C}\right)\left(1.00 \times 10^{-4}\text{C}\right)}{(3.00\ \text{m})^2}$$
$$= 30.0\ \text{N}$$

(c)
$$\left|F_{BC}\right| = k_e \frac{\left|q_B\right|\left|q_C\right|}{r_{BC}^2}$$
$$= \left(8.99 \times 10^9\ \text{N}\cdot\ \text{m}^2/\text{C}^2\right) \frac{\left(6.00 \times 10^{-4}\text{C}\right)\left(1.00 \times 10^{-4}\text{C}\right)}{(5.00\ \text{m})^2}$$
$$= 21.6\ \text{N}$$

(d) $\left(F_{BC}\right)_x = \left|F_{BC}\right| \cos\theta = (21.6\ \text{N})\cos\left(36.9^\circ\right) = 17.3\ \text{N}$

(e) $\left(F_{BC}\right)_y = -\left|F_{BC}\right| \sin\theta = -(21.6\ \text{N})\sin\left(36.9^\circ\right) = -13.0\ \text{N}$

(f) $\left(F_R\right)_x = \left(F_{AC}\right)_x + \left(F_{BC}\right)_x = 0 + 17.3\ \text{N} = 17.3\ \text{N}$

(g) $\left(F_R\right)_y = \left(F_{AC}\right)_y + \left(F_{BC}\right)_y = 30.0 - 13.0\ \text{N} = 17.0\ \text{N}$

(h) $F_R = \sqrt{\left(F_R\right)_x^2 + \left(F_R\right)_y^2} = \sqrt{(17.3\ \text{N})^2 + (17.0\ \text{N})^2} = 24.3\ \text{N}$

Both components are positive, placing the force in the first quadrant:

$$\varphi = \tan^{-1}\left[\frac{\left(F_R\right)_y}{\left(F_R\right)_x}\right] = \tan^{-1}\left(\frac{17.0\ \text{N}}{17.3\ \text{N}}\right) = 44.5^\circ$$

Therefore, $\vec{\mathbf{F}}_R = 24.3\ \text{N}$ at 44.5° above the $+x$ direction.

Problem 22.30:

A particle with charge $-3.00\,\mathrm{nC}$ is at the origin, and a particle with negative charge of magnitude Q is at $x = 50.0$ cm. A third particle with a positive charge is in equilibrium at $x = 20.9$ cm. What is Q ?

The positive charge, call it q, is $50.0\ \mathrm{cm} - 20.9\ \mathrm{cm} = 29.1\ \mathrm{cm}$ from charge Q. The force on q from the $-3.00\,\mathrm{nC}$ charge balances the force on q from the $-Q$ charge:

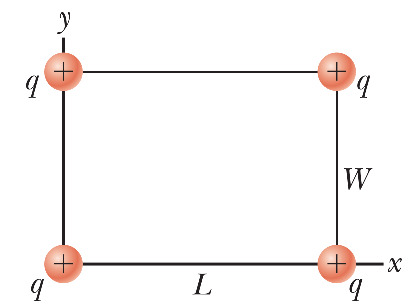
$$\frac{k_e(3.00\mathrm{nC})q}{(0.209\ \mathrm{m})^2} = \frac{k_e\mathrm{Q}q}{(0.291\ \mathrm{m})^2}$$

which then gives

$$Q = (3.00\mathrm{nC})\left(\frac{0.291\ \mathrm{m}}{0.209\ \mathrm{m}}\right)^2 = 5.82\mathrm{nC}$$

Problem 22.38:

Four identical charged particles ($q = +10.0\mu C$) are located on the corners of a rectangle as shown in the figure. The dimensions of the rectangle are $L = 60.0$ cm and $W = 15.0$ cm . Calculate (a) the magnitude and (b) the direction of the total electric force exerted on the charge at the lower left corner by the other three charges.



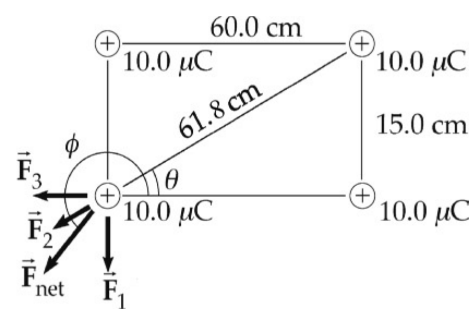
$$F_1 = \frac{(8.99\times 10^9\ \text{N}\cdot\ \text{m}^2/\text{C}^2)\left(10.0\times 10^{-6}\text{C}\right)^2}{(0.150\ \text{m})^2} = 40.0\ \text{N}$$

$$F_2 = \frac{(8.99\times 10^9\ \text{N}\cdot\ \text{m}^2/\text{C}^2)\left(10.0\times 10^{-6}\text{C}\right)^2}{(0.618\ \text{m})^2} = 2.35\ \text{N}$$

$$F_3 = \frac{(8.99\times 10^9\ \text{N}\cdot\ \text{m}^2/\text{C}^2)\left(10.0\times 10^{-6}\text{C}\right)^2}{(0.600\ \text{m})^2} = 2.50\ \text{N}$$

$$F_x = -F_3 - F_2\cos 14.0^\circ = -2.50 - 2.35\cos 14.0^\circ = -4.78\ \text{N}$$

$$F_y = -F_1 - F_2\sin 14.0^\circ = -40.0 - 2.35\sin 14.0^\circ = -40.5\ \text{N}$$



(a) $F_{net} = \sqrt{F_x^2 + F_y^2} = \sqrt{(-4.78\ \text{N})^2 + (-40.5\ \text{N})^2} = 40.8\ \text{N}$

$$\theta = \tan^{-1}\left(\frac{15.0}{60.0}\right) = 14.0^\circ$$

(b) $\tan\phi = \frac{F_y}{F_x} = \frac{-40.5\ \text{N}}{-4.78\ \text{N}} \rightarrow \phi = 263^\circ$

Electric Field – Introduction

The electric force is a field force.

Field forces can act through space.

- The effect is produced even with no physical contact between objects.

Faraday developed the concept of a field in terms of electric fields.

Electric Field – Definition

An **electric field** is said to exist in the region of space around a charged object.

- This charged object is the **source charge.**

When another charged object, the **test charge**, enters this electric field, an electric force acts on it.

Electric Field – Definition, cont

The electric field vector, $\vec{\mathbf{E}}$, at a point in space is defined as the electric force acting on a positive test charge, q_o, placed at that point divided by the test charge:

$$\vec{\mathbf{E}} \equiv \frac{\vec{\mathbf{F}}}{q_0}$$

Electric Field, Notes

$\vec{\mathbf{E}}$ is the field produced by some charge or charge distribution, separate from the test charge.

The existence of an electric field is a property of the source charge.

- The presence of the test charge is not necessary for the field to exist.

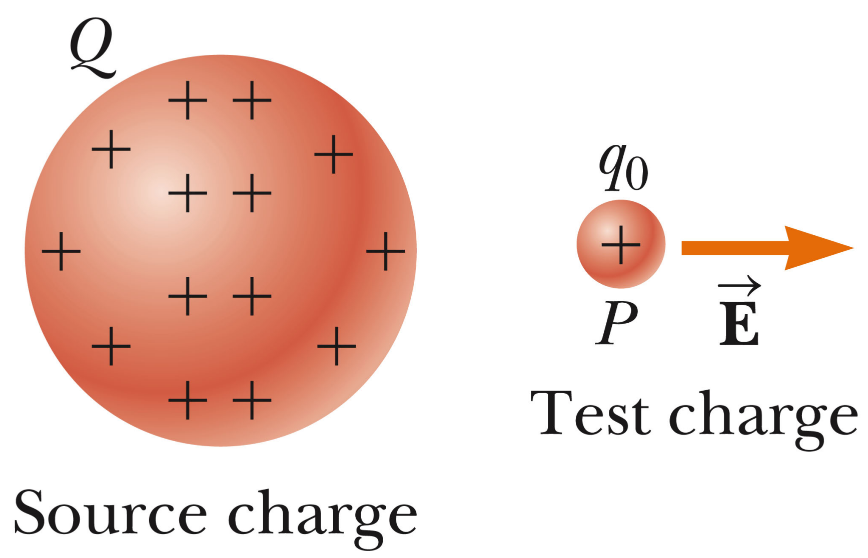
The test charge serves as a detector of the field.

Electric Field Notes, Final

The direction of $\vec{\mathbf{E}}$ is that of the force on a positive test charge.

The SI units of $\vec{\mathbf{E}}$ are N/C.

We can also say that an electric field exists at a point if a test charge at that point experiences an electric force.



Relationship Between F and E

$$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$$

- This is valid for a point charge only.
- One of zero size
- For larger objects, the field may vary over the size of the object.

If q is positive, the force and the field are in the same direction.

If q is negative, the force and the field are in opposite directions.

Electric Field, Vector Form

Remember Coulomb's law, between the source and test charges, can be expressed as

$$\vec{\mathbf{F}}_e = k_e \frac{q q_o}{r^2} \hat{\mathbf{r}}$$

Then, the electric field will be

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}_e}{q_0} = k_e \frac{q}{r^2} \hat{\mathbf{r}}$$

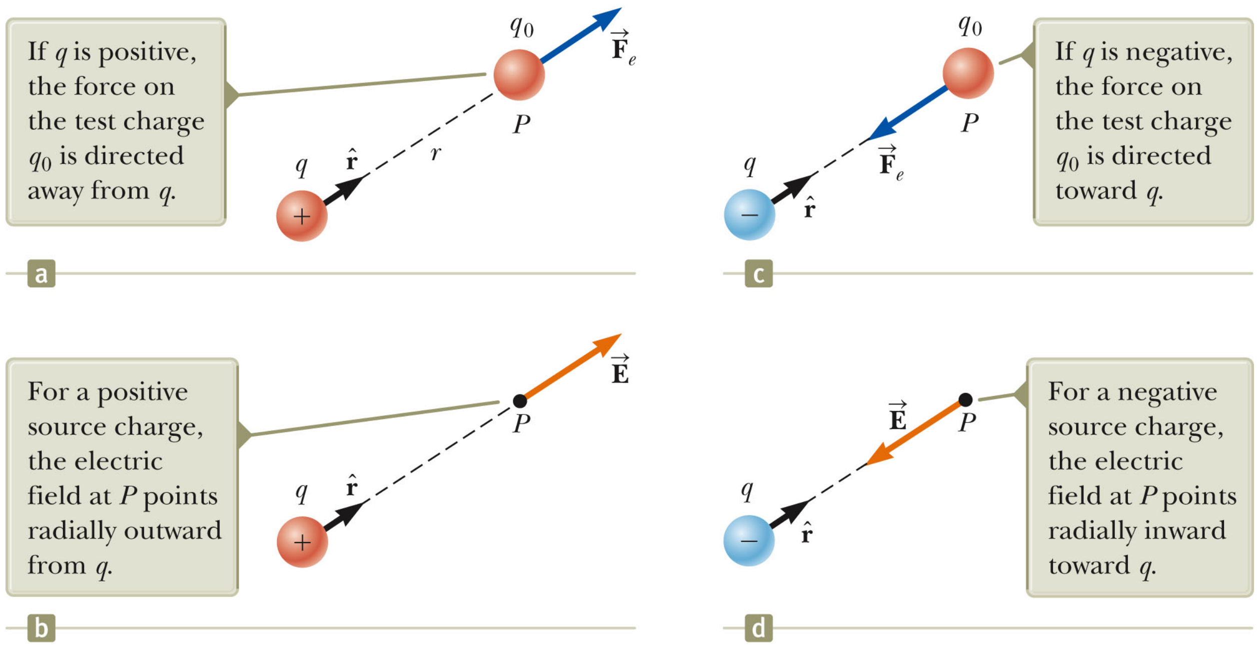
More About Electric Field Direction

a) q is positive, the force is directed away from q.

b) The direction of the field is also away from the positive source charge.

c) q is negative, the force is directed toward q.

d) The field is also toward the negative source charge.



Electric Fields from Multiple Charges

At any point P, the total electric field due to a group of source charges equals the vector sum of the electric fields of all the charges.

$$\vec{\mathbf{E}} = k_e \sum_i \frac{q_i}{r_i^2}\hat{\mathbf{r}}_i$$

Example 22.05: A Suspended Water Droplet

A water droplet of mass 3.00×10^{-12} kg is located in the air near the ground during a stormy day. An atmospheric electric field of magnitude 6.00×10^{3} N/C points vertically downward in the vicinity of the water droplet. The droplet remains suspended at rest in the air. What is the electric charge on the droplet?

Write Newton's second law from the particle in equilibrium model in the vertical direction

$$\sum F_y = 0 \rightarrow F_e - F_g = 0$$

$$q(-E) - mg = 0$$

$$q = -\frac{mg}{E}$$

$$q = -\frac{(3.00 \times 10^{-12}\ \text{kg})(9.80\ \text{m/s}^2)}{6.00 \times 10^{3}\ \text{N/C}} = -4.90 \times 10^{-15}\text{C}$$

Example 22.06: Electric Field Due to Two Charges

Charges q_1 and q_2 are located on the x axis, at distances a and b, respectively, from the origin as shown in the figure.

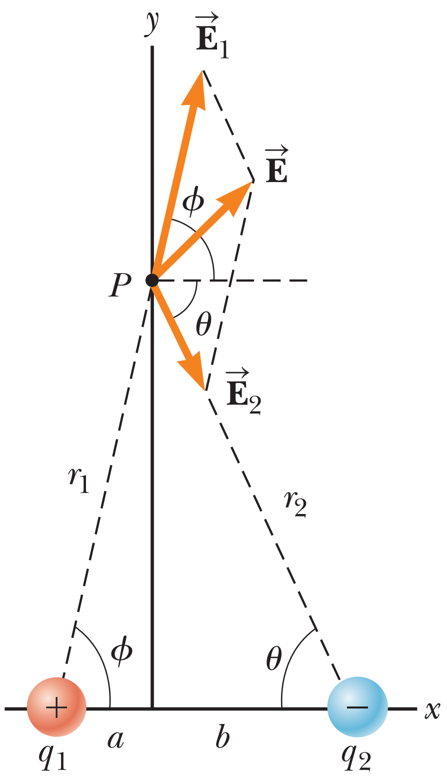
(A) Find the components of the net electric field at the point P, which is at position $(0,y)$.

$$E_1 = k_e \frac{|q_1|}{r_1^2} = k_e \frac{|q_1|}{a^2 + y^2}$$

$$E_2 = k_e \frac{|q_2|}{r_2^2} = k_e \frac{|q_2|}{b^2 + y^2}$$

$$\vec{\mathbf{E}}_1 = k_e \frac{|q_1|}{a^2 + y^2} \cos\phi \,\hat{\mathbf{i}} + k_e \frac{|q_1|}{a^2 + y^2} \sin\phi \,\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}}_2 = k_e \frac{|q_2|}{b^2 + y^2} \cos\theta \,\hat{\mathbf{i}} - k_e \frac{|q_2|}{b^2 + y^2} \sin\theta \,\hat{\mathbf{j}}$$



(1) $E_x = E_{1x} + E_{2x} = k_e \frac{|q_1|}{a^2 + y^2} \cos\phi + k_e \frac{|q_2|}{b^2 + y^2} \cos\theta$

(2) $E_y = E_{1y} + E_{2y} = k_e \frac{|q_1|}{a^2 + y^2} \sin\phi - k_e \frac{|q_2|}{b^2 + y^2} \sin\theta$

Example 22.06: Electric Field Due to Two Charges

(B) Evaluate the electric field at point P in the special case that $|q_1| = |q_2|$ and $a = b$.

(3)

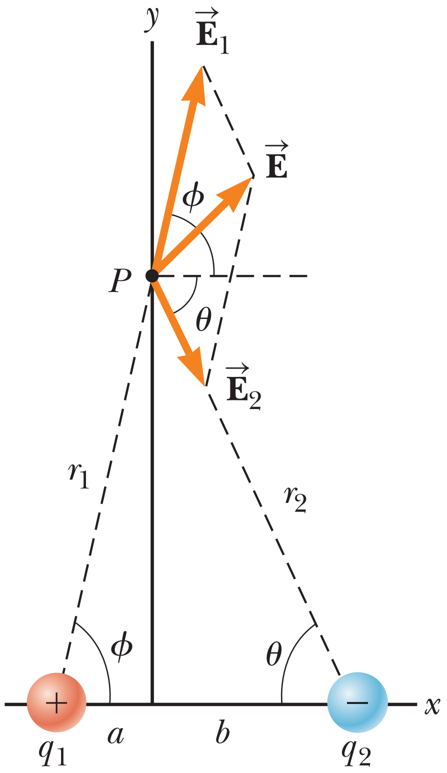
$$E_x = k_e \frac{q}{a^2 + y^2} \cos\theta + k_e \frac{q}{a^2 + y^2} \cos\theta = 2k_e \frac{q}{a^2 + y^2} \cos\theta$$

$$E_y = k_e \frac{q}{a^2 + y^2} \sin\theta - k_e \frac{q}{a^2 + y^2} \sin\theta = 0$$

$$\cos\theta = \frac{a}{r} = \frac{a}{\left(a^2 + y^2\right)^{1/2}}$$

(4)

$$E_x = 2k_e \frac{q}{a^2 + y^2} \left[\frac{a}{\left(a^2 + y^2\right)^{1/2}} \right] = k_e \frac{2aq}{\left(a^2 + y^2\right)^{3/2}}$$

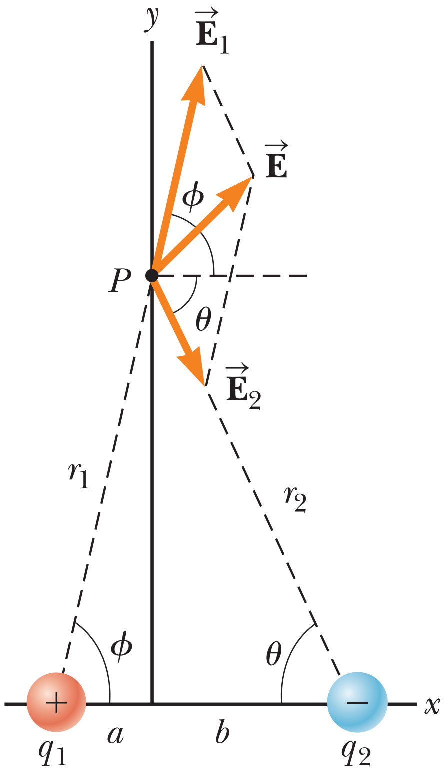


Example 22.06: Electric Field Due to Two Charges

(c) Find the electric field due to the electric dipole when point P is a distance $y \gg a$ from the origin.

In the solution to part (B), because $y \gg a$, neglect a^2 compared with y^2 and write the expression for E in this case:

(5) $E \approx k_e \dfrac{2aq}{y^3}$



Problem 22.15:

What are the magnitude and direction of the electric field that will balance the weight of (a) an electron and (b) a proton?

For equilibrium,$\vec{\mathbf{F}}_e = -\vec{\mathbf{F}}_g$ or $q\vec{\mathbf{E}} = -mg(-\hat{\mathbf{j}})$. Thus,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}}.$$

(a) For an electron,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(9.11\times10^{-31}\ \mathrm{kg}\right)\left(9.80\ \mathrm{m/s^2}\right)}{-1.60\times10^{-19}\mathrm{C}}\hat{\mathbf{j}}$$
$$= -\left(5.58\times10^{-11}\ \mathrm{N/C}\right)\hat{\mathbf{j}}$$

(b) For a proton, which is 1836 times more massive than an electron,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{\left(1.67\times10^{-27}\ \mathrm{kg}\right)\left(9.80\ \mathrm{m/s^2}\right)}{-1.60\times10^{-19}\mathrm{C}}\hat{\mathbf{j}}$$
$$= \left(1.02\times10^{-7}\ \mathrm{N/C}\right)\hat{\mathbf{j}}$$

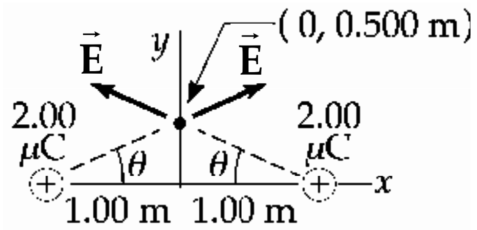
Problem 22.20:

Two $2.00 - \mu C$ point charges are located on the x axis. One is at $x = 1.00$ m, and the other is at $x = -1.00$ m.

(a) Determine the electric field on the y axis at $y = 0.500$ m.

(b) Calculate the electric force on a $-3.00 - \mu C$ charge placed on the y axis at $y = 0.500$ m.

(a) $E = \frac{k_e q}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{ m}^2/\text{C}^2)(2.00 \times 10^{-6}\text{C})}{(1.12 \text{ m})^2} = 14400 \text{ N/C}$



The x components of the two fields cancel and the y components add, giving

$E_x = 0$ and $E_y = 2(14400 \text{ N/C})\sin 26.6° = 1.29 \times 10^4 \text{ N/C}$

$d = \sqrt{(1.00 \text{ m})^2 + (0.500 \text{ m})^2} = 1.12 \text{ m}$

so $\vec{\mathbf{E}} = 1.29 \times 10^4 \hat{\mathbf{j}}$ N/C.

$\theta = \tan^{-1}\left(\frac{0.5}{1}\right) = 26.6°$

(b) The electric force at this point is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = (-3.00 \times 10^{-6}\text{C})\left(1.29 \times 10^4 \text{ N/C}\hat{\mathbf{j}}\right)$$

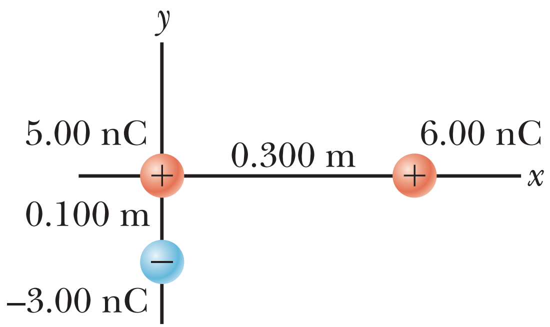
$$= -3.86 \times 10^{-2}\hat{\mathbf{j}} \text{ N}$$

Problem 22.21:

Three point charges are arranged as shown in the figure.

(a) Find the vector electric field that the $6.00 - \mathrm{nC}$ and $-3.00 - \mathrm{nC}$ charges together create at the origin.

(b) Find the vector force on the $5.00 - \mathrm{nC}$ charge.



(a) The electric field at the origin due to each of the charges is given by

$$\vec{\mathbf{E}}_1 = \frac{k_e |q_1|}{r_1^2}(-\hat{\mathbf{j}})$$
$$= \frac{(8.99 \times 10^9 \ \mathrm{N \cdot \ m^2/C^2})(3.00 \times 10^{-9}\mathrm{C})}{(0.100 \ \mathrm{m})^2}(-\hat{\mathbf{j}})$$
$$= -(2.70 \times 10^3 \ \mathrm{N/C})\,\hat{\mathbf{j}}$$
$$\vec{\mathbf{E}}_2 = \frac{k_e |q_2|}{r_2^2}(-\hat{\mathbf{i}})$$
$$= \frac{(8.99 \times 10^9 \ \mathrm{N \cdot \ m^2/C^2})(6.00 \times 10^{-9}\mathrm{C})}{(0.300 \ \mathrm{m})^2}(-\hat{\mathbf{i}})$$
$$= -(5.99 \times 10^2 \ \mathrm{N/C})\,\hat{\mathbf{i}}$$

and their sum is

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_1 = -(5.99 \times 10^2 \ \mathrm{N/C})\,\hat{\mathbf{i}} - (2.70 \times 10^3 \ \mathrm{N/C})\,\hat{\mathbf{j}}$$

(b) The vector electric force is

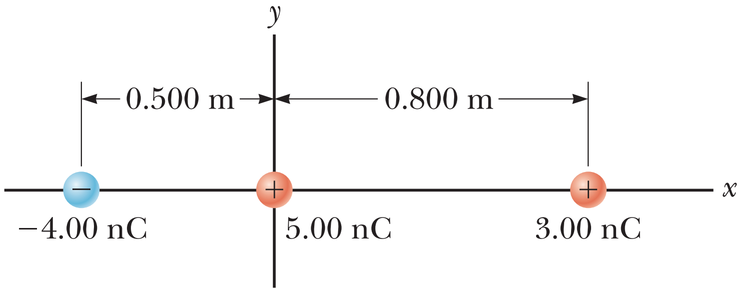
$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = (5.00 \times 10^{-9}\mathrm{C})(-599\hat{\mathbf{i}} - 2700\hat{\mathbf{j}})\mathrm{N/C}$$
$$\vec{\mathbf{F}} = \left(-3.00 \times 10^{-6}\hat{\mathbf{i}} - 13.5 \times 10^{-6}\hat{\mathbf{j}}\right)\mathrm{N} = (-3.00\hat{\mathbf{i}} - 13.5\hat{\mathbf{j}})\mu\mathrm{N}$$

Problem 22.35:

Three charged particles are aligned along the x axis as shown in the figure. Find the electric field at

(a) the position $(2.00\ \text{m},0)$

(b) the position $(0,2.00\ \text{m})$.



(a)

$$\vec{\mathbf{E}}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{(8.99\times10^9\ \text{N}\cdot\ \text{m}^2/\text{C}^2)(-4.00\times10^{-9}\text{C})}{(2.50\ \text{m})^2}\hat{\mathbf{i}}$$

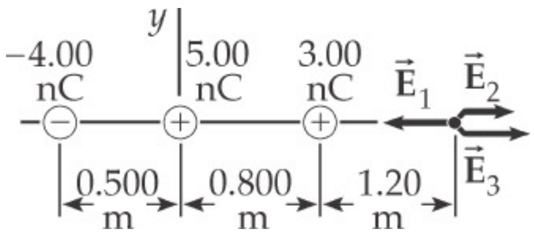
$$= -5.75\hat{\mathbf{i}}\ \text{N/C}$$

$$\vec{\mathbf{E}}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{(8.99\times10^9\ \text{N}\cdot\ \text{m}^2/\text{C}^2)(5.00\times10^{-9}\text{C})}{(2.00\ \text{m})^2}\hat{\mathbf{i}}$$

$$= 11.2\ \text{N/C}\hat{\mathbf{i}}$$

$$\vec{\mathbf{E}}_3 = \frac{(8.99\times10^9\ \text{N}\cdot\ \text{m}^2/\text{C}^2)(3.00\times10^{-9}\text{C})}{(1.20\ \text{m})^2}\hat{\mathbf{i}} = 18.7\ \text{N/C}\hat{\mathbf{i}}$$

$\vec{\mathbf{E}}_R = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 = 24.2\ \text{N/C}$ in $+x$ direction

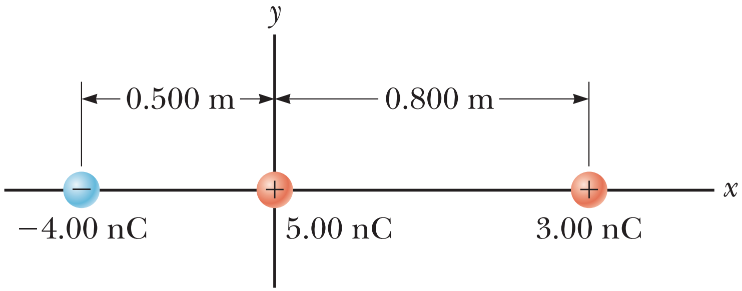


Problem 22.35:

Three charged particles are aligned along the x axis as shown in the figure. Find the electric field at

(a) the position $(2.00\ \text{m},0)$

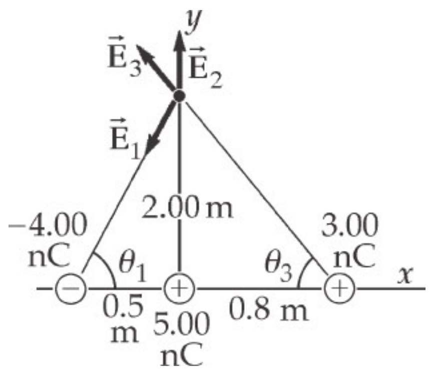
(b) the position $(0,2.00\ \text{m})$.



$$\vec{\mathbf{E}}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (-8.46\ \text{N/C})(0.243\hat{\mathbf{i}} + 0.970\hat{\mathbf{j}})$$

$$\vec{\mathbf{E}}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (11.2\ \text{N/C})(+\hat{\mathbf{j}})$$

$$\vec{\mathbf{E}}_3 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = (5.81\ \text{N/C})(-0.371\hat{\mathbf{i}} + 0.928\hat{\mathbf{j}})$$



The components of the resultant electric field are

$$E_x = E_{1x} + E_{3x} = -4.21\hat{\mathbf{i}}\ \text{N/C} \quad E_y = E_{1y} + E_{2y} + E_{3y} = 8.43\hat{\mathbf{j}}\ \text{N/C}$$

$$\theta_1 = \tan^{-1}\left(\frac{2}{0.5}\right) = 75.96°$$

then, the magnitude of the resultant electric field is

$$E_R = 9.42\ \text{N/C}$$

$$\theta_3 = \tan^{-1}\left(\frac{2}{0.8}\right) = 68.20°$$

and is directed at

$$\theta = \tan^{-1}\left(\frac{|E_y|}{|E_x|}\right) = \tan^{-1}\left(\frac{8.43\ \text{N/C}}{4.21\ \text{N/C}}\right) = 63.4° \text{ above } -x \text{ axis}$$

Electric Field Lines

Field lines give us a means of representing the electric field pictorially.

The electric field vector is tangent to the electric field line at each point.

- The line has a direction that is the same as that of the electric field vector.

The number of lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of the electric field in that region.

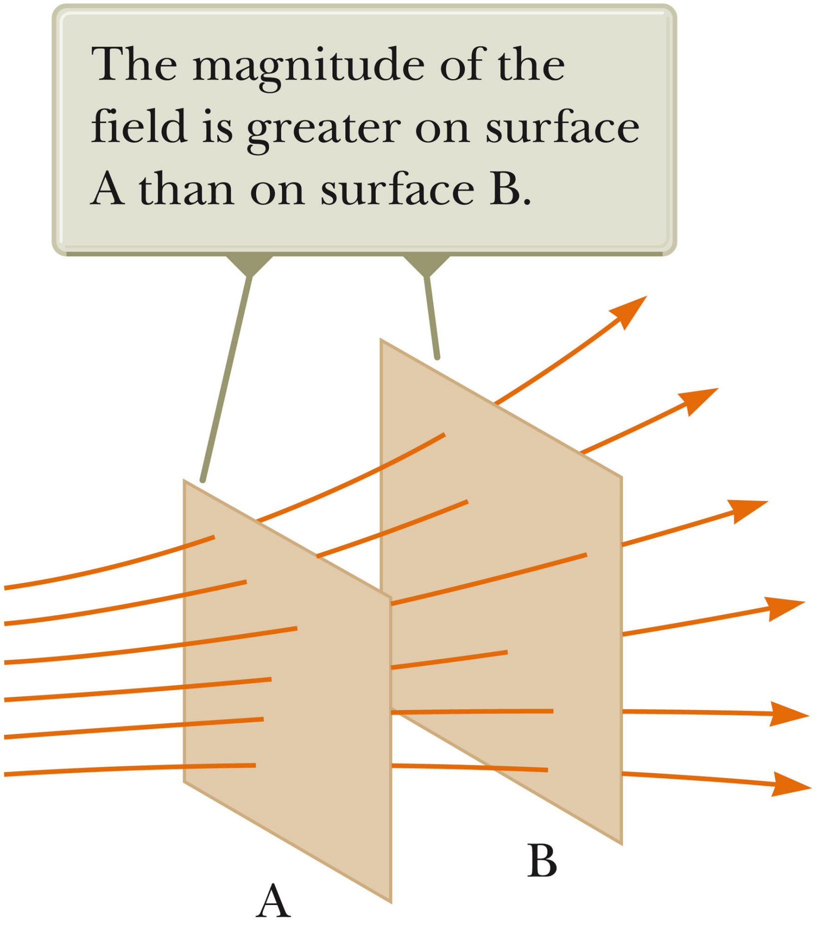
Electric Field Lines, General

The density of lines through surface A is greater than through surface B.

The magnitude of the electric field is greater on surface A than B.

The lines at different locations point in different directions.

- This indicates the field is nonuniform.



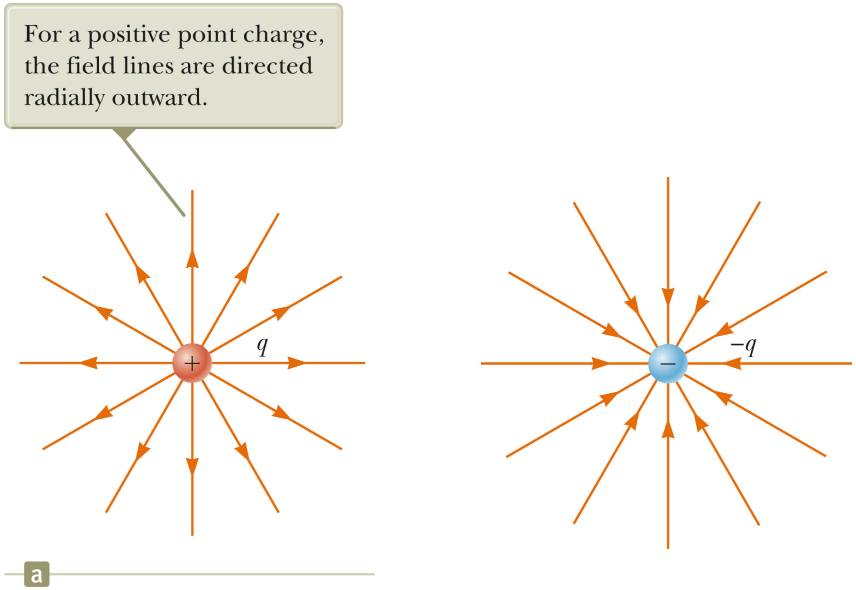
Electric Field Lines, Positive Point Charge

The field lines radiate outward in all directions.

- In three dimensions, the distribution is spherical.

The lines are directed away from the source charge.

- A positive test charge would be repelled away from the positive source charge.

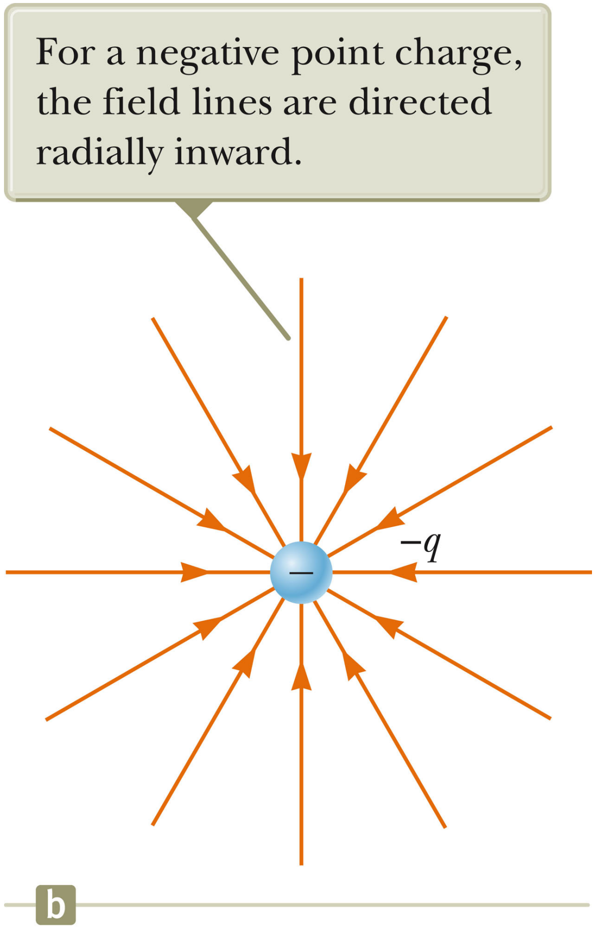


Electric Field Lines, Negative Point Charge

The field lines radiate inward in all directions.

The lines are directed toward the source charge.

- A positive test charge would be attracted toward the negative source charge.



Electric Field Lines – Rules for Drawing

The lines must begin on a positive charge and terminate on a negative charge.

- In the case of an excess of one type of charge, some lines will begin or end infinitely far away.

The number of lines drawn leaving a positive charge or approaching a negative charge is proportional to the magnitude of the charge.

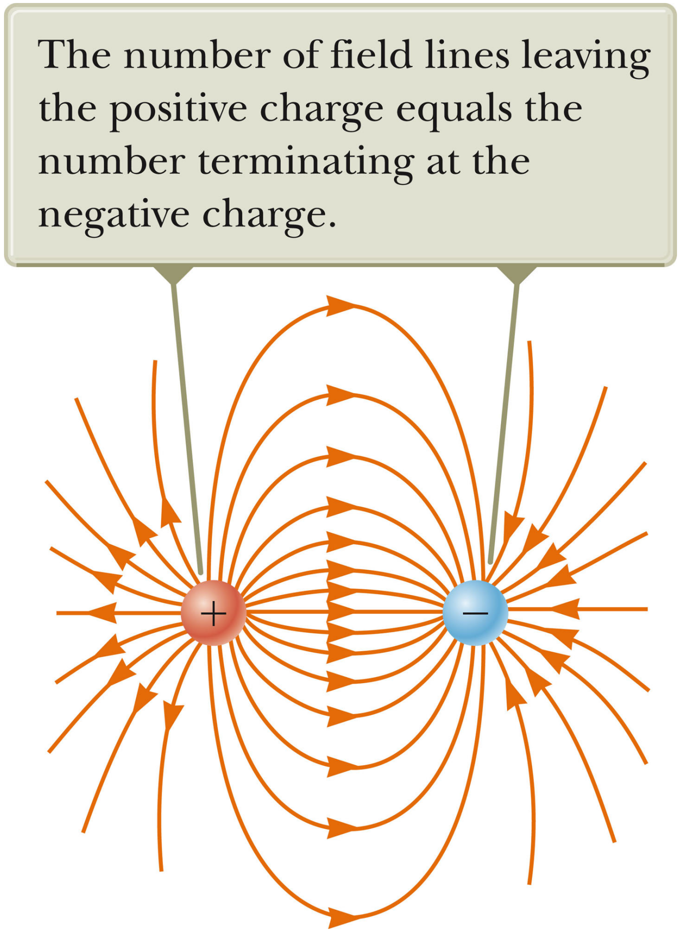
No two field lines can cross.

Remember field lines are **not** material objects, they are a pictorial representation used to qualitatively describe the electric field.

Electric Field Lines – Dipole

The charges are equal and opposite.

The number of field lines leaving the positive charge equals the number of lines terminating on the negative charge.



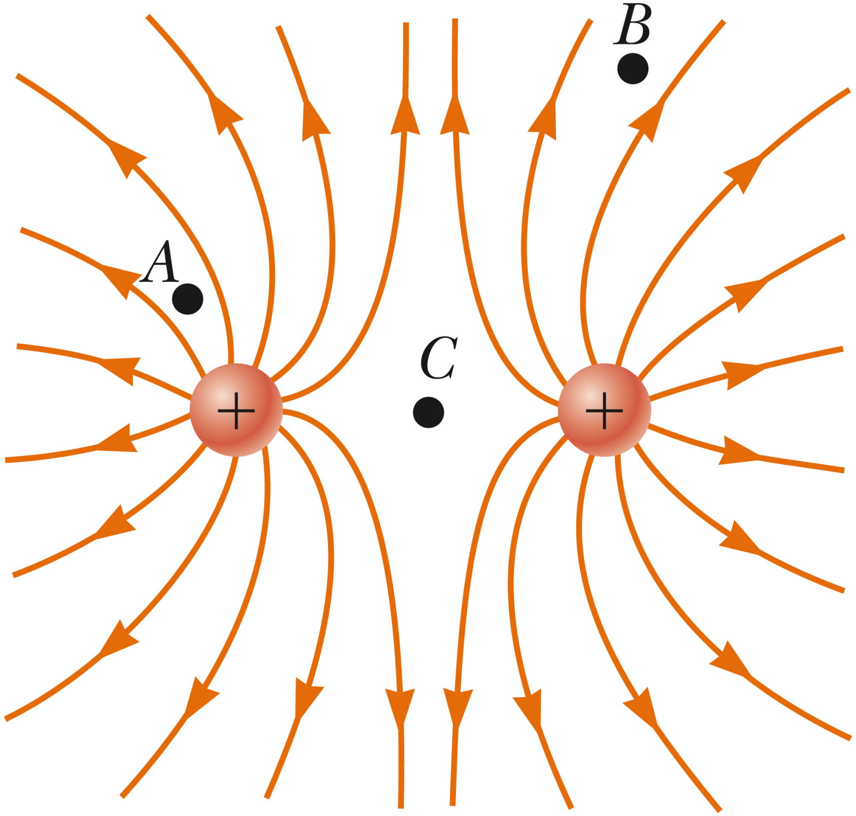
Electric Field Lines – Like Charges

The charges are equal and positive.

The same number of lines leave each charge since they are equal in magnitude.

At a great distance, the field is approximately equal to that of a single charge of 2*q.*

Since there are no negative charges available, the field lines end infinitely far away.

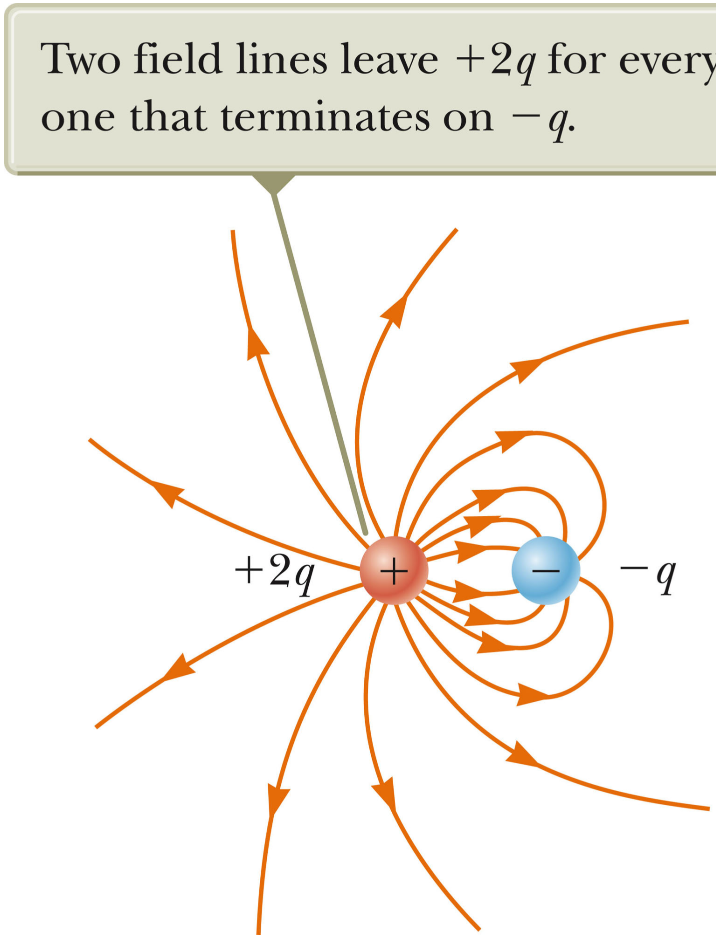


Electric Field Lines, Unequal Charges

The positive charge is twice the magnitude of the negative charge.

Two lines leave the positive charge for each line that terminates on the negative charge.

At a great distance, the field would be approximately the same as that due to a single charge of $+q$.



Motion of Charged Particles

When a charged particle is placed in an electric field, it experiences an electrical force.

If this is the only force on the particle, it must be the net force.

The net force will cause the particle to accelerate according to Newton's second law.

Motion of Particles, cont

$$\vec{\mathbf{F}}_e = q\vec{\mathbf{E}} = m\vec{\mathbf{a}}$$

If the field is uniform, then the acceleration is constant.

The particle under constant acceleration model can be applied to the motion of the particle.

- The electric force causes a particle to move according to the models of forces and motion.

If the particle has a positive charge, its acceleration is in the direction of the field.

If the particle has a negative charge, its acceleration is in the direction opposite the electric field.

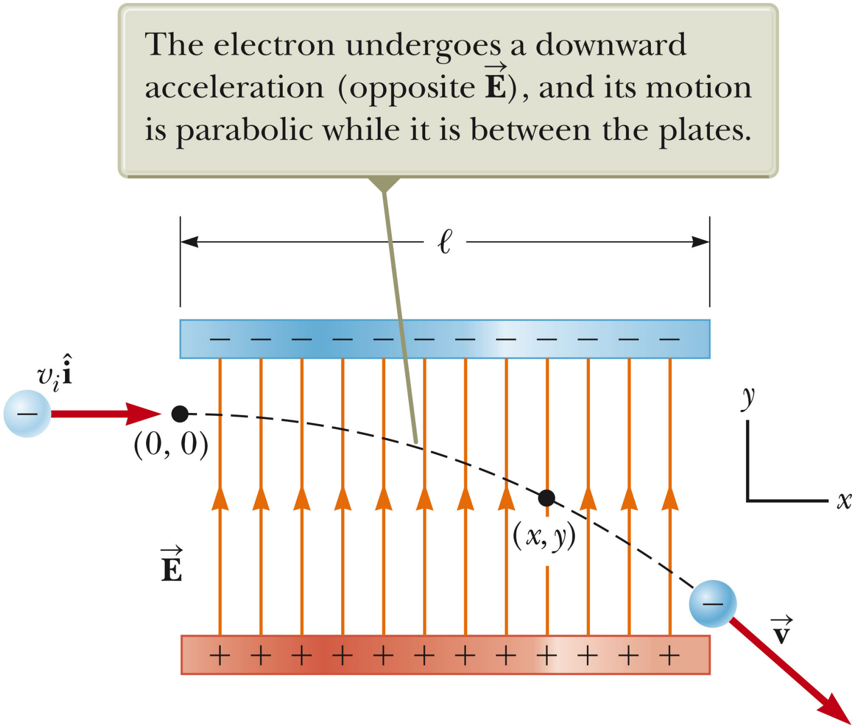
Electron in a Uniform Field, Example

The electron is projected horizontally into a uniform electric field.

The electron undergoes a downward acceleration.

- It is negative, so the acceleration is opposite the direction of the field.

Its motion is parabolic while between the plates.



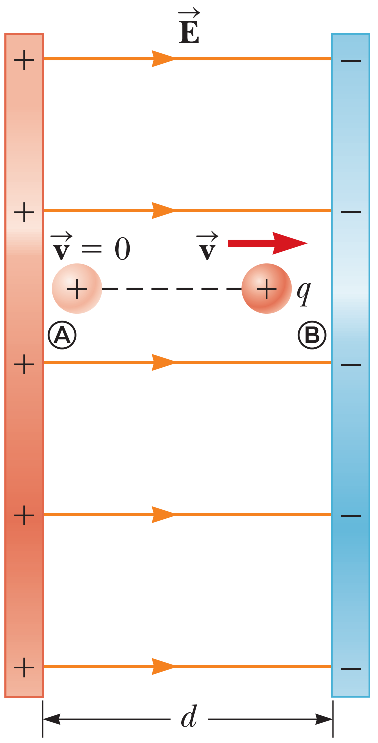
Example 22.07: An Accelerating Positive Charge: Two Models

A uniform electric field $\vec{\mathbf{E}}$ is directed along the x axis between parallel plates of charge separated by a distance d as shown in the figure. A positive point charge q of mass m is released from rest at a point (A) next to the positive plate and accelerates to a point (B) next to the negative plate.

(A) Find the speed of the particle at (B) by modeling it as a particle under constant acceleration.

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 0 + 2a(d - 0) = 2ad$$

$$v_f = \sqrt{2ad} = \sqrt{2\left(\frac{qE}{m}\right)d} = \sqrt{\frac{2qEd}{m}}$$



(B) Find the speed of the particle at (B) by modeling it as a nonisolated system in terms of energy.

$$W = \Delta K$$

$$F_e \Delta x = K_B - K_A = \frac{1}{2}mv_f^2 - 0 \rightarrow v_f = \sqrt{\frac{2F_e\Delta x}{m}}$$

$$v_f = \sqrt{\frac{2(qE)(d)}{m}} = \sqrt{\frac{2qEd}{m}}$$

Example 22.08: An Accelerated Electron

An electron enters the region of a uniform electric field as shown in the figure, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m.

(A) Find the acceleration of the electron while it is in the electric field.

$$a_y = -\frac{eE}{m_e}$$

$$a_y = -\frac{(1.60 \times 10^{-19}\,\text{C})(200\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}} = -3.51 \times 10^{13}\ \text{m/s}^2$$

(B) Assuming the electron enters the field at time $t = 0$, find the time at which it leaves the field.

$$x_f = x_i + v_x t \rightarrow t = \frac{x_f - x_i}{v_x}$$

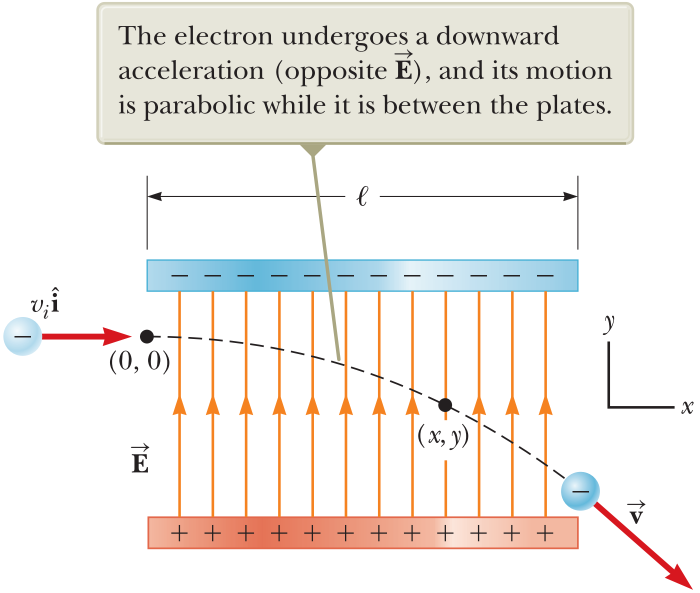
$$t = \frac{\ell - 0}{v_x} = \frac{0.100\ \text{m}}{3.00 \times 10^6\ \text{m/s}} = 3.33 \times 10^{-8}\ \text{s}$$

(C) Assuming the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_f = 0 + 0 + \frac{1}{2}\left(-3.51 \times 10^{13}\ \text{m/s}^2\right)\left(3.33 \times 10^{-8}\ \text{s}\right)^2$$

$$= -0.0195\ \text{m} = -1.95\ \text{cm}$$



Problem 22.24:

A proton accelerates from rest in a uniform electric field of $640\ \mathrm{N/C}$. At one later moment, its speed is $1.20\mathrm{Mm/s}$ (nonrelativistic because v is much less than the speed of light).

(a) Find the acceleration of the proton. (b) Over what time interval does the proton reach this speed? (c) How far does it move in this time interval? (d) What is its kinetic energy at the end of this interval?

(a) We obtain the acceleration of the proton from the particle under a net force model, with $F = qE$ representing the electric force:

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{\left(1.602 \times 10^{-19}\mathrm{C}\right)(640\ \mathrm{N/C})}{1.67 \times 10^{-27}\ \mathrm{kg}} = 6.14 \times 10^{10}\ \mathrm{m/s^2}$$

(b) The particle under constant acceleration model gives us $v_f = v_i + at$, from which we obtain

$$t = \frac{v_f - 0}{a} = \frac{1.20 \times 10^6\ \mathrm{m/s}}{6.14 \times 10^{10}\ \mathrm{m/s^2}} = 19.5\mu\ \mathrm{s}$$

(c) Again, from the particle under constant acceleration model,

$$\Delta x = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(6.14 \times 10^{10}\ \mathrm{m/s^2}\right)\left(19.5 \times 10^{-6}\ \mathrm{s}\right)^2$$
$$= 11.7\ \mathrm{m}$$

(d) The final kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67 \times 10^{-27}\ \mathrm{kg}\right)\left(1.20 \times 10^6\ \mathrm{m/s}\right)^2 = 1.20 \times 10^{-15}\ \mathrm{J}$$

Problem 22.25:

A proton moves at 4.50×10^5 m/s in the horizontal direction. It enters a uniform vertical electric field with a magnitude of 9.60×10^3 N/C. Ignoring any gravitational effects, find (a) the time interval required for the proton to travel 5.00 cm horizontally, (b) its vertical displacement during the time interval in which it travels 5.00 cm horizontally, and (c) the horizontal and vertical components of its velocity after it has traveled 5.00 cm horizontally.

$\vec{\mathbf{E}}$ is directed along the y direction; therefore, $a_x = 0$ and $x = v_{xi}t$.

(a) $t = \dfrac{x}{v_{xi}} = \dfrac{0.0500 \text{ m}}{4.50 \times 10^5 \text{ s}} = 1.11 \times 10^{-7} \text{ s} = 111 \text{ ns}$

(b) $a_y = \dfrac{qE}{m} = \dfrac{\left(1.602 \times 10^{-19}\text{C}\right)\left(9.60 \times 10^3 \text{ N/C}\right)}{1.67 \times 10^{-27} \text{ kg}} = 9.21 \times 10^{11} \text{ m/s}^2$

$y_f - y_i = v_{yi}t + \dfrac{1}{2}a_y t^2$:

$y_f = \dfrac{1}{2}\left(9.21 \times 10^{11} \text{ m/s}^2\right)\left(1.11 \times 10^{-7} \text{ s}\right)^2$

$= 5.68 \times 10^{-3} \text{ m} = 5.67 \text{ mm}$

(c) $v_x = 4.50 \times 10^5 \text{ m/s}$

$v_{yf} = v_{yi} + a_y t = \left(9.21 \times 10^{11} \text{ m/s}^2\right)\left(1.11 \times 10^{-7} \text{ s}\right) = 1.02 \times 10^5 \text{ m/s}$

$\vec{\mathbf{v}} = (450\hat{\mathbf{i}} + 102\hat{\mathbf{j}})\text{km/s}$