Exercices on Chapter 2

1 The Natural Logarithm and the Exponential Functions

Exercise 1 :
Find $f'(x)$ if

1. $f(x) = \ln(x^2 + 2x + 4)$,
2. $f(x) = \ln(|2 - 3x|^5)$,
3. $f(x) = \ln(\sqrt{\dfrac{4+x^2}{4-x^2}})$,
4. $f(x) = \dfrac{(x^2+1)^3(x^2+4)^{10}}{(x^2+2)^5(x^2+3)^4}$,
5. $f(x) = \dfrac{(x+1)^3(2x-3)^{\frac{3}{4}}}{(1+7x)^{\frac{1}{3}}(2x+3)^{\frac{3}{2}}}$,
6. $f(x) = \ln\left(\dfrac{1-x}{1+x}\right)$, $-1 < x < 1$,
7. $f(x) = e^{1-x^2}$,
8. $f(x) = e^{x\ln(x)}$,
9. $f(x) = x^2 e^{-x^3}$,

Exercise 2 :
Use implicit differentiation to find y' if

1. $y^2 + \ln(\dfrac{x}{y}) - 4x = -3$,
2. $y(x) = \sqrt{(3x^2+2)\sqrt{6x-7}}$
3. $y(x) = (x+1)^2(x+2)^3(x-5)^7$.
4. $xe^y + 2x - \ln(y+1) = 3$.

Exercise 3 :
Prove that

$$\begin{aligned}\int \sec(x)dx &= \ln|\sec(x) + \tan(x)| + c \\ \int \csc(x)dx &= \ln|\csc(x) - \cot(x)| + c \\ &= -\ln|\csc(x) + \cot(x)| + c.\end{aligned}$$

Exercise 4 :
Find the equation of the tangent line to the graph of the function $f(x) = x - e^{-x}$ that is parallel to the line (D) of equation $6x - 2y = 7$.

Exercise 5 :
Give the value of the following integrals with the indicate change of variable:

1. Evaluate $\int xe^{-x^2}dx, \quad (t = x^2)$,
2. Evaluate $\int \frac{\sin(\ln x)}{x}dx, \quad (t = \ln x)$,
3. $\int_0^1 \frac{dx}{e^x+1}, \quad (t = e^x)$,
4. $\int_1^e \sin(\pi \ln x)dx, \quad (t = \pi \ln x)$.

Exercise 6 :
Give the value of the following integrals:

1. $\int \frac{dx}{2x+7}$,
2. $\int x^2 e^{3x^3} dx$,
3. $\int \frac{x-2}{x^2-4x+9}dx$,
4. $\int \frac{(2+\ln(x))^{10}}{x}dx$,
5. $\int \frac{1}{x(\ln(x))^2}dx$,
6. $\int \frac{e^x}{(e^x+1)^2}dx$,
7. $\int \frac{\tan(e^{-3x})}{e^{3x}}dx$,

2 The General Logarithm and Exponential Functions

Exercise 7 :
Solve the following equations for x:

1. $\log_3(x^4) + \log_3(x^3) - 2\log_3(x^{\frac{1}{2}}) = 5$.
2. $\frac{e^x}{1+e^x} = \frac{1}{3}$.

Exercise 8 :
Find $f'(x)$ if

1. $f(x) = 4\text{csch}^2(2x-1)$,
2. $f(x) = \sinh(2x)\text{csch}(3x)$,
3. $f(x) = \log_2(x)$,
4. $f(x) = \log_5(x^3+1)$,
5. $f(x) = \log_{10}(3x+1)$,
6. $f(x) = \log_{10}(x^2+4)$,
7. $f(x) = 10^{x^2}$,
8. $f(x) = 2^{(x^3+1)}$,
9. $f(x) = 5^{(x^4+x^2)}$,
10. $f(x) = (5^x)$,
11. $f(x) = (6^{\sqrt{x}})$
12. $f(x) = (x^2+1)^{\sin(2x)}$,
13. $f(x) = 2^{x^2}$,

14. $f(x) = \log_2(\sec x + \tan x)$,
15. $f(x) = \ln(x + \sqrt{x^2 - 4})$,
16. $f(x) = \ln(x + \sqrt{4 + x^2})$,
17. $f(x) = (x^2 + 4)^{(x^3+1)}$,
18. $f(x) = (\sin x + 3)^{(4\cos x + 7)}$,
19. $f(x) = (3\sinh x + \cos x + 5)^{(x^3+1)}$,
20. $f(x) = (e^{x^2} + 1)^{(2x+1)}$,
21. $f(x) = x^2(x^2 + 1)^{(x^3+1)}$.

Exercise 9 :
Give the value of the following integrals:

1. $\int 3^x dx$,
2. $\int_{-1}^{0} 3^x dx$,
3. $\int \frac{(2^x + 1)^2}{2^x} dx$,
4. $\int \frac{5^{\tan(x)}}{\cos^2(x)} dx$.
5. $\int \frac{e^x}{1 + e^{2x}} dx$,
6. $\int_0^1 \frac{e^x}{1 + e^{2x}} dx$,
7. $\int \frac{x}{\sqrt{1 - x^4}} dx$,
8. $\int_0^{2^{-\frac{1}{4}}} \frac{x}{\sqrt{1 - x^4}} dx$,
9. $\int \frac{dx}{x\sqrt{x^6 - 1}}$,
10. $\int \frac{e^x}{\sqrt{e^{2x} - 1}} dx$,
11. $\int e^{3x} \sec^2(2 + e^{3x}) dx$,
12. $\int 10^{\cos x} \sin x dx$,
13. $\int x 10^{x^2+3} dx$.

3 The Trigonometric Functions and their Inverse Functions

Exercise 10 :
Compute $\frac{dy}{dx}$ for each of the following:

1. $y = \sin^{-1}(\frac{x}{2})$,
2. $y = \cos^{-1}(\frac{x}{3})$,
3. $y = \tan^{-1}(\frac{x}{5})$,
4. $y = \cot^{-1}(\frac{x}{7})$,
5. $y = \sec^{-1}(\frac{x}{2})$,
6. $y = \csc^{-1}(\frac{x}{3})$,

Exercise 11 :
Simplify each of the following expressions by eliminating the radical by using an appropriate trigonometric substitution.

1. $\dfrac{x}{\sqrt{9-x^2}}$,
2. $\dfrac{3+x}{\sqrt{16+x^2}}$,
3. $\dfrac{x-2}{x\sqrt{x^2-25}}$,
4. $\dfrac{1+x}{\sqrt{x^2+2x+2}}$,
5. $\dfrac{2-2x}{\sqrt{x^2-2x-3}}$.

Exercise 12 :
Find the exact value of y in each of the following

1. $y = 3\sin^{-1}(\dfrac{1}{2}) + 2\cos^{-1}(\dfrac{\sqrt{3}}{2})$,
2. $y = 4\tan^{-1}(\dfrac{1}{\sqrt{3}}) + 5\cot^{-1}(\dfrac{1}{\sqrt{3}})$,
3. $y = 2\sec^{-1}(-2) + 3\cos^{-1}(-\dfrac{\sqrt{3}}{2})$,
4. $y = \cos(2\cos^{-1}(x))$,
5. $y = \sin(2\cos^{-1}(x))$.
6. $y = \cos^{-1}(-\frac{1}{2})$,
7. $y = \sin^{-1}\frac{\sqrt{3}}{2}$,
8. $y = \tan^{-1}(-\sqrt{3})$,
9. $y = \cot^{-1}(-\frac{\sqrt{3}}{3})$,
10. $y = \sec^{-1}(-\sqrt{2})$,
11. $y = \csc^{-1}(-\sqrt{2})$
12. $y = \sec^{-1}(-\frac{2}{\sqrt{3}})$,
13. $y = \csc^{-1}(-\frac{2}{\sqrt{3}})$,
14. $y = \sec^{-1}(-2)$,
15. $y = \csc^{-1}(-2)$,
16. $y = \tan^{-1}(\frac{-1}{\sqrt{3}}$,
17. $y = \cot^{-1}(-\sqrt{3})$.

Exercise 13 :
Prove the following identities

1. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}, \forall x \in [-1,1]$.
2. $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}, \forall x \in \mathbb{R}$
3. $\tan^{-1}x + \tan^{-1}(\frac{1}{x}) = \frac{\pi}{2}, \quad x > 0$.
4. $\tan^{-1}x + \tan^{-1}(\frac{1}{x}) = -\frac{\pi}{2}, \quad x < 0$.
5. $\sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}, \forall x \in \mathbb{R} \setminus [-1,1]$

Exercise 14 :
Evaluate the following integrals.

1. $\int \frac{e^{\tan^{-1} x}}{1+x^2} dx$,

2. $\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}} dx$,

3. $\int e^{\sin(2x)} \cos(2x) dx$,

4. $\int \frac{e^{2x}}{1+e^{2x}} dx$,

5. $\int e^x \cos(1+2e^x) dx$,

6. $\int \frac{4^{\sec^{-1} x}}{x\sqrt{x^2-1}} dx$.