Chapter 12

Equilibrium and Elasticity

12.4: Elastic Properties of Solids

Elasticity

We can discuss how objects deform under load conditions.

An elastic object returns to its original shape when the deforming forces are removed.

Various elastic constants will be defined, each corresponding to a different type of deformation.

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Definitions Associated With Deformation

So far we have assumed that objects remain rigid when external forces act on them.

- Except springs

Actually, all objects are deformable to some extent.

- It is possible to change the size and/or shape of the object by applying external forces.

Internal forces resist the deformation.

Elastic Modulus

The elastic modulus is the constant of proportionality between the stress and the strain.

- For sufficiently small stresses, the stress is directly proportional to the stress.
- It depends on the material being deformed.
- It also depends on the nature of the deformation.

The elastic modulus, in general, relates what is done to a solid object to how that object responds.

$$\textit{elastic modulus} \equiv \frac{\textit{stress}}{\textit{strain}}$$

Various types of deformation have unique elastic moduli.

Three Types of Moduli

Young's Modulus

- Measures the resistance of a solid to a change in its length

Shear Modulus

- Measures the resistance of motion of the planes within a solid parallel to each other

Bulk Modulus

- Measures the resistance of solids or liquids to changes in their volume

Young's Modulus

The bar is stretched by an amount ΔL under the action of the force F.

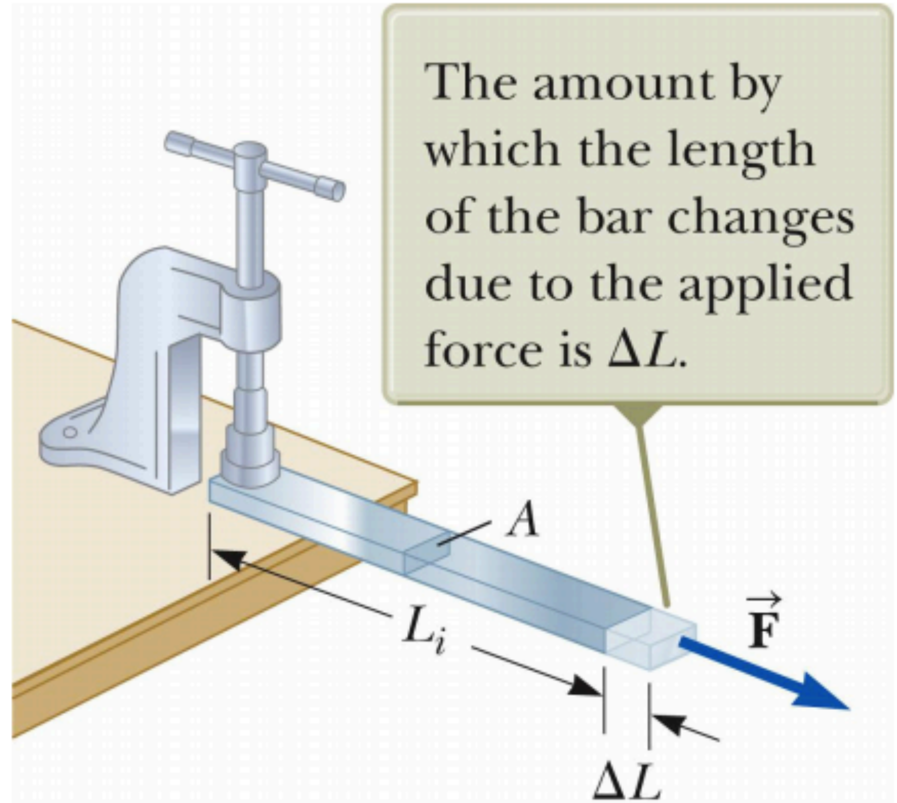
The **tensile stress** is the ratio of the magnitude of the external force to the cross-sectional area A.

The **tension strain** is the ratio of the change in length to the original length.

Young's modulus, Y, is the ratio of those two ratios:

$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i}$$

Units are N / m^2

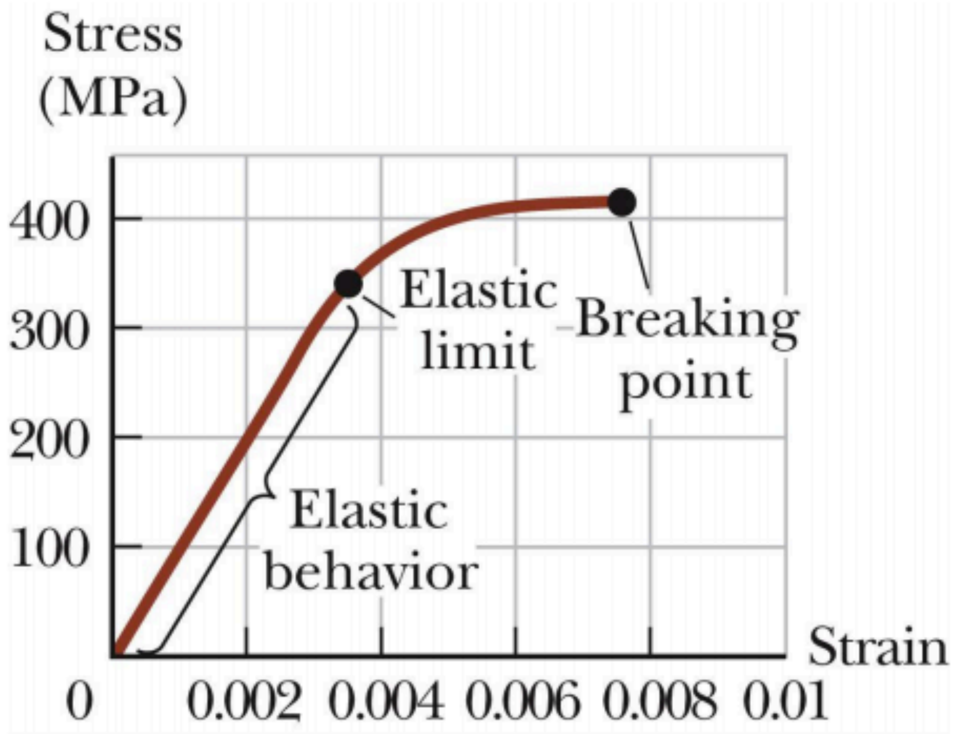


Stress vs. Strain Curve

Experiments show that for certain stresses, the stress is directly proportional to the strain.

This is the elastic behavior part of the curve.

The **elastic limit** is the maximum stress that can be applied to the substance before it becomes permanently deformed.



Stress vs. Strain Curve, cor

When the stress exceeds the elastic limit, the substance will be permanently deformed.

- The curve is no longer a straight line.

With additional stress, the material ultimately breaks.

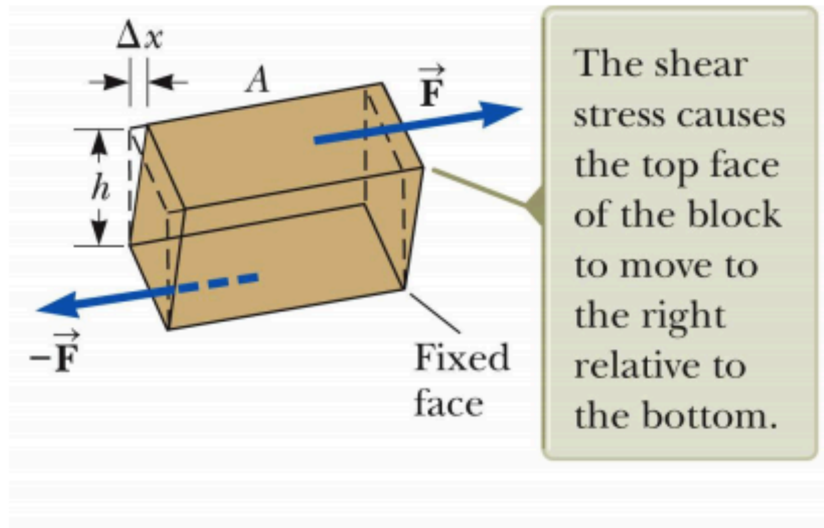
Shear Modulus

Another type of deformation occurs when a force acts parallel to one of its faces while the opposite face is held fixed by another force.

This is called a ***shear stress.***

For small deformations, no change in volume occurs with this deformation.

- A good first approximation



Shear Modulus, cont.

The shear strain is Δx / h.

- Δx is the horizontal distance the sheared face moves.
- h is the height of the object.

The shear stress is F / A.

- F is the tangential force.
- A is the area of the face being sheared.

The shear modulus is the ratio of the shear stress to the shear strain.

$$S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$

Units are N / m^2

Bulk Modulus

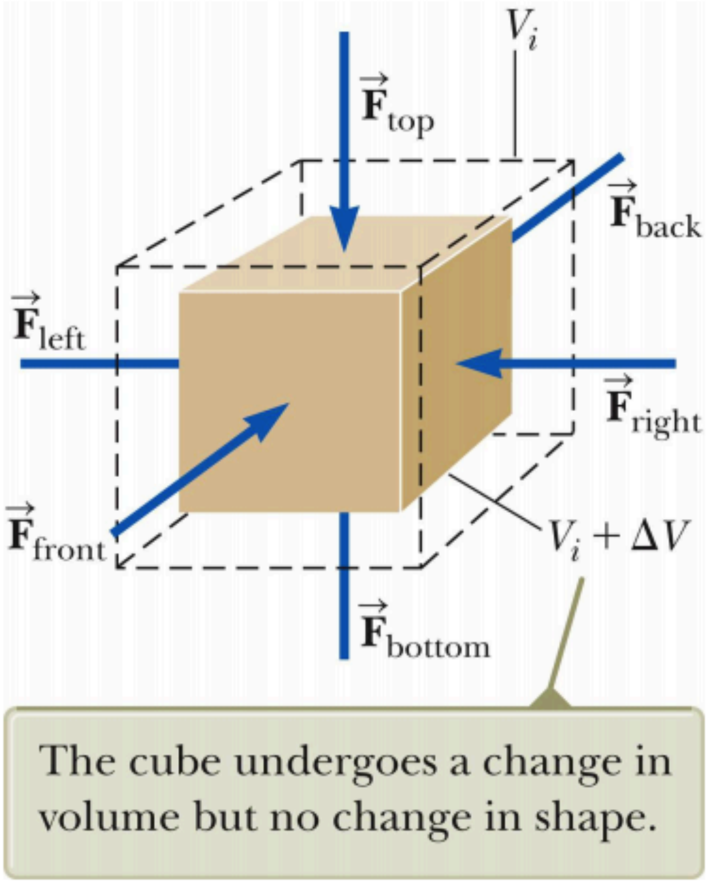
Another type of deformation occurs when a force of uniform magnitude is applied perpendicularly over the entire surface of the object.

The object will undergo a change in volume, but not in shape.

The volume stress is defined as the ratio of the magnitude of the total force, F, exerted on the surface to the area, A, of the surface.

- This is also called the ***pressure.***

The volume strain is the ratio of the change in volume to the original volume.



Bulk Modulus, cont.

The bulk modulus is the ratio of the volume stress to the volume strain.

$$B = \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F / A}{\Delta V / V_i} = -\frac{\Delta P}{\Delta V / V_i}$$

The negative indicates that an increase in pressure will result in a decrease in volume.

Moduli Values

TABLE 12.1 *Typical Values for Elastic Moduli*

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Example 12.5 Stage Design

In Example 8.2, we analyzed a cable used to support an actor as he swings onto the stage. Now suppose the tension in the cable is 940 N as the actor reaches the lowest point. What diameter should a 10-m-long steel cable have if we do not want it to stretch more than 0.50 cm under these conditions?

SOLUTION

Conceptualize Look back at Example 8.2 to recall what is happening in this situation. We ignored any stretching of the cable there, but we wish to address this phenomenon in this example.

Categorize We perform a simple calculation involving Equation 12.6, so we categorize this example as a substitution problem.

Solve Equation 12.6 for the cross-sectional area of the cable:

$$A = \frac{FL_i}{Y\,\Delta L}$$

Assuming the cross section is circular, find the diameter of the cable from $d = 2r$ and $A = \pi r^2$:

$$d = 2r = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{FL_i}{\pi Y\,\Delta L}}$$

Substitute numerical values:

$$d = 2\sqrt{\frac{(940\text{ N})(10\text{ m})}{\pi(20 \times 10^{10}\text{ N/m}^2)(0.005\,0\text{ m})}} = 3.5 \times 10^{-3}\text{ m} = 3.5\text{ mm}$$

To provide a large margin of safety, you would probably use a flexible cable made up of many smaller wires having a total cross-sectional area substantially greater than our calculated value.

Example 12.6 Squeezing a Brass Sphere

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is 1.0×10^5 N/m^2 (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is 2.0×10^7 N/m^2. The volume of the sphere in air is 0.50 m^3. By how much does this volume change once the sphere is submerged?

SOLUTION

Conceptualize Think about movies or television shows you have seen in which divers go to great depths in the water in submersible vessels. These vessels must be very strong to withstand the large pressure under water. This pressure squeezes the vessel and reduces its volume.

Categorize We perform a simple calculation involving Equation 12.8, so we categorize this example as a substitution problem.

Solve Equation 12.8 for the volume change of the sphere:

$$\Delta V = -\frac{V_i \Delta P}{B}$$

Substitute numerical values:

$$\Delta V = -\frac{(0.50\ \text{m}^3)(2.0 \times 10^7\ \text{N/m}^2 - 1.0 \times 10^5\ \text{N/m}^2)}{6.1 \times 10^{10}\ \text{N/m}^2}$$

$$= -1.6 \times 10^{-4}\ \text{m}^3$$

The negative sign indicates that the volume of the sphere decreases.

(Elastic Modulus)(معامل المرونة)	Strain(الانفعال)	Stress(الاجهاد)	
النسبة بين الاجهاد والانفعال The elastic modulus is the constant of proportionality between the stress and the strain.	إستجابة المادة للقوة المؤثرة عليها • Is the result of a stress • Is a measure of the degree of deformation	القوة المؤثرة على وحدة المساحات • It is the external force acting on the object per unit cross-sectional area • Is proportional to the force causing the deformation	
معامل يونج:يقيس مقاومة الجسم الصلب للتغير في الطول Young's Modulus:Measures the resistance of a solid to a change in its length	النسبة بين التغير في الطول أو الاستطالة إلى الطول الأصلي	القوة المؤثرة عاموديا على وحدة المساحات The tensile stress is the ratio of the magnitude of the external force to the cross-sectional area A	الطولي lengh
Y= TensileStress/Tensile Strain=$(FL_i)/(\Delta LA)$ N/m^2	$e=\Delta L/L_i$	S=F/A	
يقيس مقاومة حركة المستويات المنزلقة فوق بعضها البعض Shear Modulus: Measures the resistance of motion of the planes within a solid parallel to each other	النسبة بين الازاحة الحاصلة والارتفاع	القوة المماسية التي تؤثر على وحدة المساحة	السطحي (القصي) shear
S= ShearStress/ShearStrain=$Fh/XA=F/A\theta$	$e_s=X/h=\theta$	$S_s=F/A$	
يقيس مقاومة الجسم الصلب أو السائل للتغير في الحجم Bulk Modulus: Measures the resistance of solids or liquids to changes in their volume	النسبة بين التغير في الحجم إلى الحجم الأصلي	الزيادة في القوة التي تؤثر على وحدة المساحات من السطح الكلي للجسم أو يعرف بأنه الزيادة في الضغط ΔP.	الحجمي valume
B= ValumeStress/ValumeStrain=$-\Delta P/(\Delta V/V_0)$	$e=\Delta V/V_0$	$S_s=F/A=\Delta P$	

Homework

1-A 200-kg load is hung on a wire having a length of 4.00 m, cross-sectional area 0.200 $\times 10^{-4}$m^2, and Young's modulus 8.00 - 10^{10} N/^{m2}. What is its increase in length?

$$\frac{F}{A} = Y\frac{\Delta L}{L_i}$$

$$\Delta L = \frac{FL_i}{AY} = \frac{(200)(9.80)(4.00)}{(0.200\times 10^{-4})(8.00\times 10^{10})} = \boxed{4.90\text{ mm}}$$

2-Assume that Young's modulus is 1.50 - 10^{10} N/m^2 for bone and that the bone will fracture if stress greater than 1.50×10-8 N/m2 is imposed on it. (a) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50 cm? (b) If this much force is applied compressively, by how much does the 25.0-cm-long bone shorten?

(a) $$\text{stress} = \frac{F}{A} = \frac{F}{\pi r^2}$$

$$F = (\text{stress})\pi\left(\frac{d}{2}\right)^2$$

$$F = (1.50\times 10^8\text{ N/m}^2)\pi\left(\frac{2.50\times 10^{-2}\text{ m}}{2}\right)^2$$

$$F = \boxed{73.6\text{ kN}}$$

(b) $$\text{stress} = Y(\text{strain}) = \frac{Y\Delta L}{L_i}$$

$$\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{(1.50\times 10^8\text{ N/m}^2)(0.250\text{ m})}{1.50\times 10^{10}\text{ N/m}^2} = \boxed{2.50\text{ mm}}$$