Chapter 8

Conservation of Energy

8.1: Analysis Model : Nonisolated System (Energy)

8.2: Analysis Model: Isolated System (Energy)

8.3: Situations Involving Kinetic Friction

8.4: Changes in Mechanical Energy for Nonconservative Forces

8.5: Power

Energy Review

Kinetic Energy

- Associated with movement of members of a system

Potential Energy

- Determined by the configuration of the system
- Gravitational and Elastic Potential Energies have been studied

Internal Energy

- Related to the temperature of the system

Types of Systems

Non-isolated systems

- Energy can cross the system boundary in a variety of ways.
- Total energy of the system changes

Isolated systems

- Energy does not cross the boundary of the system
- Total energy of the system is constant

Conservation of energy

- Can be used if no non-conservative forces act within the isolated system
- Applies to biological organisms, technological systems, engineering situations, etc

Ways to Transfer Energy Into or Out of A System

In non-isolated systems, energy crosses the boundary of the system during some time interval due to an interaction with the environment.

Work – transfers energy by applying a force and causing a displacement of the point of application of the force.

Mechanical Wave – transfers energy by allowing a disturbance to propagate through a medium.

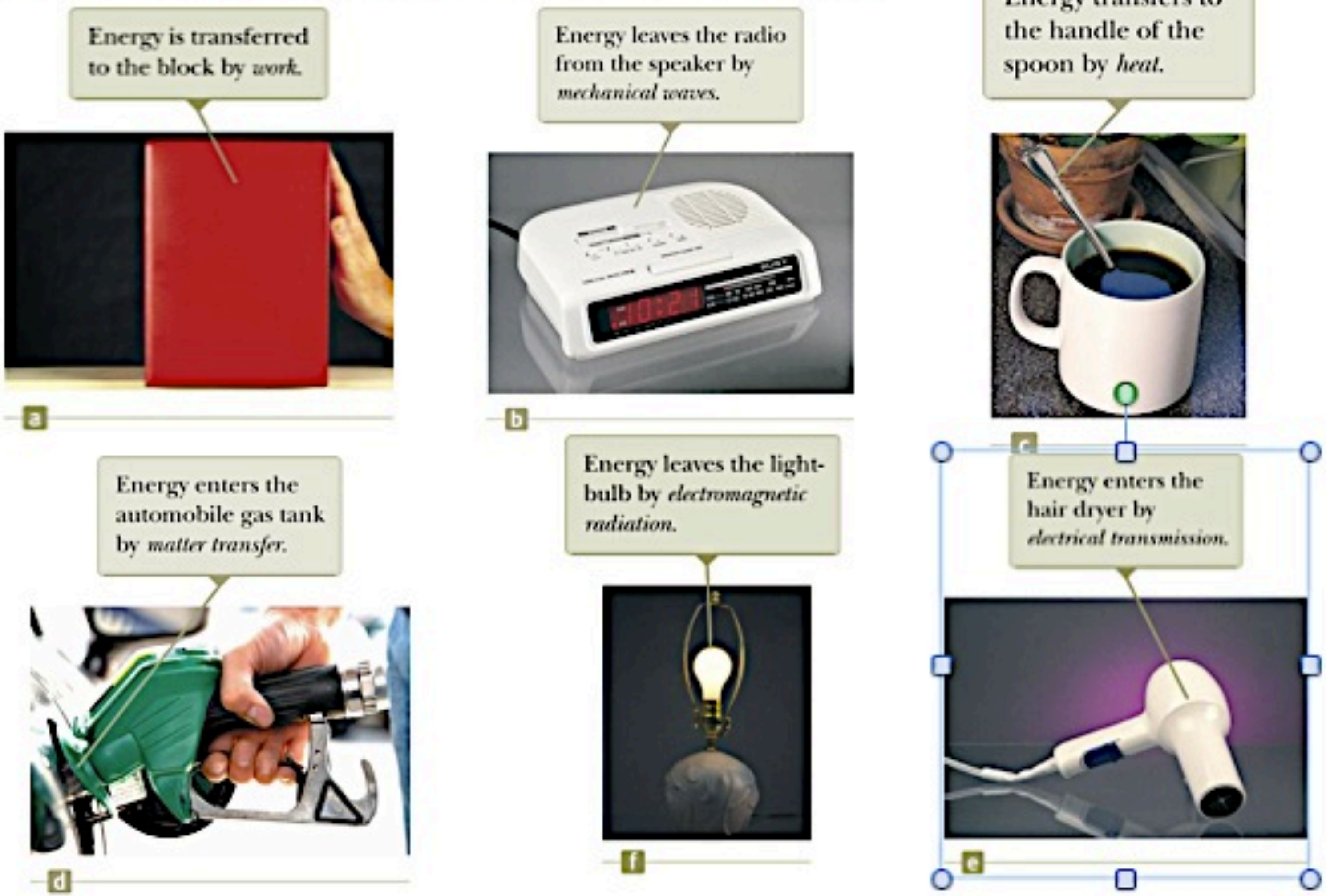
Heat – the mechanism of energy transfer that is driven by a temperature difference between two regions in space.

Matter Transfer – matter physically crosses the boundary of the system, carrying energy with it.

Electrical Transmission – energy transfer into or out of a system by electric current.

Electromagnetic Radiation – energy is transferred by electromagnetic waves.

Examples of Ways to Transfer Energy



Conservation of Energy

Energy is conserved

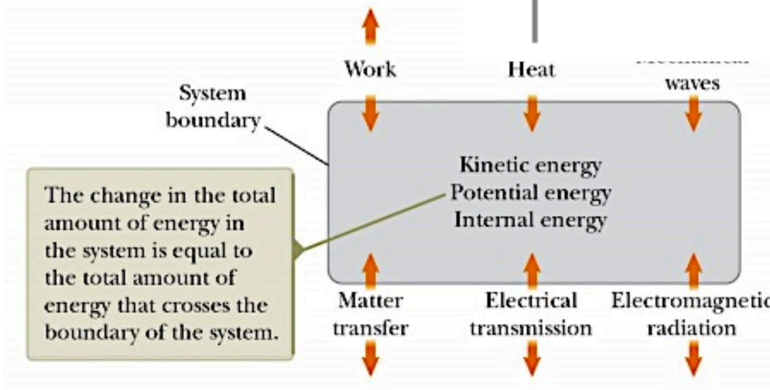
- This means that energy cannot be created nor destroyed.

- If the total amount of energy in a system changes, it can only be due to the fact that energy has crossed the boundary of the system by some method of energy transfer.

Conservation of Energy, cont.

Mathematically, $\Delta E_{system} = \Sigma T$

- E_{system} is the total energy of the system
- T is the energy transferred across the system boundary by some mechanism



- Established symbols: $T_{work} = W$ and $T_{heat} = Q$
- Others just use subscripts The primarily mathematical representation of the energy version of the analysis model of the non-isolated system is given by the full expansion of the above equation.
- $\Delta K + \Delta U + \Delta E_{int} = W + Q + T_{MW} + T_{MT} + T_{ET} + T_{ER}$
- T_{MW} – transfer by mechanical waves
- T_{MT} – by matter transfer
- T_{ET} – by electrical transmission
- T_{ER} – by electromagnetic transmission

Isolated System

For an isolated system, $\Delta E_{mech} = 0$

- Remember $E_{mech} = K + U$
- This is ***conservation of energy*** for an isolated system with no non-conservative forces acting.

If non-conservative forces are acting, some energy is transformed into internal energy.

Conservation of Energy becomes $\Delta E_{system} = 0$

- E_{system} is all kinetic, potential, and internal energies
- This is the most general statement of the isolated system model.

Isolated System, cont.

The changes in energy can be written out and rearranged.

$K_f + U_f = K_i + U_i$

- Remember, this applies only to a system in which conservative forces act.

Problem Solving Strategy – Conservation of Mechanical Energy for an Isolated System with No Non-conservative Forces

Conceptualize

- Form a mental representation
- Imagine what types of energy are changing in the system

Categorize

- Define the system
- It may consist of more than one object and may or may not include springs or other sources of storing potential energy.
- Determine if any energy transfers occur across the boundary of your system.
  - If there are transfers, use $\Delta E_{system} = \Sigma T$
  - If there are no transfers, use $\Delta E_{system} = 0$
- Determine if there are any non-conservative forces acting.
  - If not, use the principle of conservation of mechanical energy.

Example 8.1 Ball in Free Fall AM

A ball of mass m is dropped from a height h above the ground as shown in Figure 8.4.

(A) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground. Choose the system as the ball and the Earth.

SOLUTION

Conceptualize Figure 8.4 and our everyday experience with falling objects allow us to conceptualize the situation. Although we can readily solve this problem with the techniques of Chapter 2, let us practice an energy approach.

Categorize As suggested in the problem, we identify the system as the ball and the Earth. Because there is neither air resistance nor any other interaction between the system and the environment, the system is isolated and we use the *isolated system* model. The only force between members of the system is the gravitational force, which is conservative.

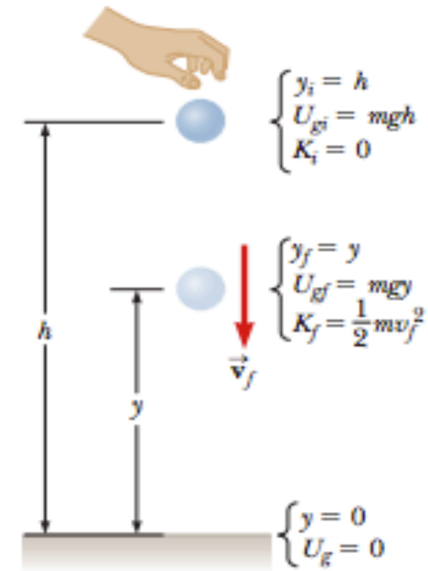


Figure 8.4 (Example 8.1) A ball is dropped from a height h above the ground. Initially, the total energy of the ball–Earth system is gravitational potential energy, equal to mgh relative to the ground. At the position y, the total energy is the sum of the kinetic and potential energies.

Analyze Because the system is isolated and there are no nonconservative forces acting within the system, we apply the principle of conservation of mechanical energy to the ball–Earth system. At the instant the ball is released, its kinetic energy is $K_i = 0$ and the gravitational potential energy of the system is $U_{gi} = mgh$. When the ball is at a position y above the ground, its kinetic energy is $K_f = \frac{1}{2}mv_f^2$ and the potential energy relative to the ground is $U_{gf} = mgy$.

Write the appropriate reduction of Equation 8.2, noting that the only types of energy in the system that change are kinetic energy and gravitational potential energy:

$$\Delta K + \Delta U_g = 0$$

Substitute for the energies:

$$(\tfrac{1}{2}mv_f^2 - 0) + (mgy - mgh) = 0$$

Solve for v_f:

$$v_f^2 = 2g(h - y) \quad \rightarrow \quad v_f = \boxed{\sqrt{2g(h - y)}}$$

The speed is always positive. If you had been asked to find the ball's velocity, you would use the negative value of the square root as the y component to indicate the downward motion.

(B) Find the speed of the ball again at height y by choosing the ball as the system.

SOLUTION

Categorize In this case, the only type of energy in the system that changes is kinetic energy. A single object that can be modeled as a particle cannot possess potential energy. The effect of gravity is to do work on the ball across the boundary of the system. We use the *nonisolated system* model.

Analyze Write the appropriate reduction of Equation 8.2:

$$\Delta K = W$$

Substitute for the initial and final kinetic energies and the work:

$$\left(\tfrac{1}{2}mv_f^2 - 0\right) = \vec{\mathbf{F}}_g \cdot \Delta\vec{\mathbf{r}} = -mg\hat{\mathbf{j}} \cdot \Delta y\hat{\mathbf{j}}$$
$$= -mg\Delta y = -mg(y - h) = mg(h - y)$$

Solve for v_f:

$$v_f^2 = 2g(h - y) \quad \rightarrow \quad v_f = \sqrt{2g(h - y)}$$

Example 8.3 The Spring-Loaded Popgun **AM**

The launching mechanism of a popgun consists of a trigger-released spring (Fig. 8.6a). The spring is compressed to a position $y_{\text{Ⓐ}}$, and the trigger is fired. The projectile of mass m rises to a position $y_{\text{Ⓒ}}$ above the position at which it leaves the spring, indicated in Figure 8.6b as position $y_{\text{Ⓑ}} = 0$. Consider a firing of the gun for which $m = 35.0$ g, $y_{\text{Ⓐ}} = -0.120$ m, and $y_{\text{Ⓒ}} = 20.0$ m.

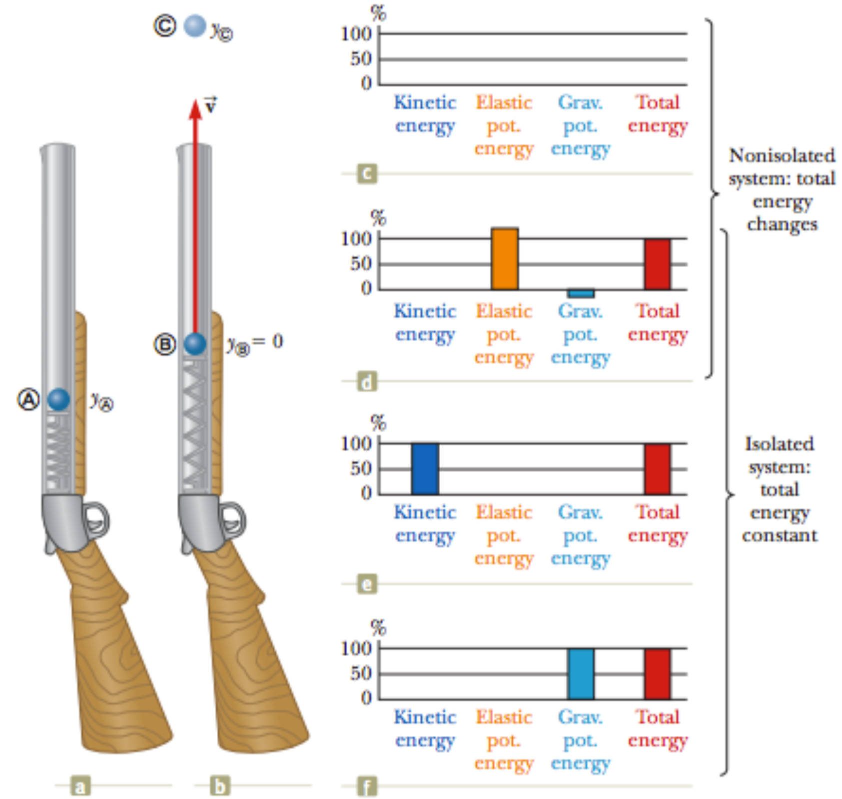
(A) Neglecting all resistive forces, determine the spring constant.

$$(1) \quad \Delta K + \Delta U_g + \Delta U_s = 0$$

$$(0 - 0) + (mgy_{\text{Ⓒ}} - mgy_{\text{Ⓐ}}) + (0 - \tfrac{1}{2}kx^2) = 0$$

$$k = \frac{2mg(y_{\text{Ⓒ}} - y_{\text{Ⓐ}})}{x^2}$$

$$k = \frac{2(0.035\,0 \text{ kg})(9.80 \text{ m/s}^2)[20.0 \text{ m} - (-0.120 \text{ m})]}{(0.120 \text{ m})^2} = 958 \text{ N/m}$$



(B) Find the speed of the projectile as it moves through the equilibrium position Ⓑ of the spring as shown in Figure 8.6b.

▸ 8.3 continued

Write Equation (1) again for the system between points Ⓐ and Ⓑ:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

Substitute for the initial and final energies:

$$(\tfrac{1}{2}mv_{Ⓑ}^2 - 0) + (0 - mgy_{Ⓐ}) + (0 - \tfrac{1}{2}kx^2) = 0$$

Solve for $v_{Ⓑ}$:

$$v_{Ⓑ} = \sqrt{\frac{kx^2}{m} + 2gy_{Ⓐ}}$$

Substitute numerical values:

$$v_{Ⓑ} = \sqrt{\frac{(958\ \mathrm{N/m})(0.120\ \mathrm{m})^2}{(0.035\,0\ \mathrm{kg})} + 2(9.80\ \mathrm{m/s^2})(-0.120\ \mathrm{m})} = 19.8\ \mathrm{m/s}$$

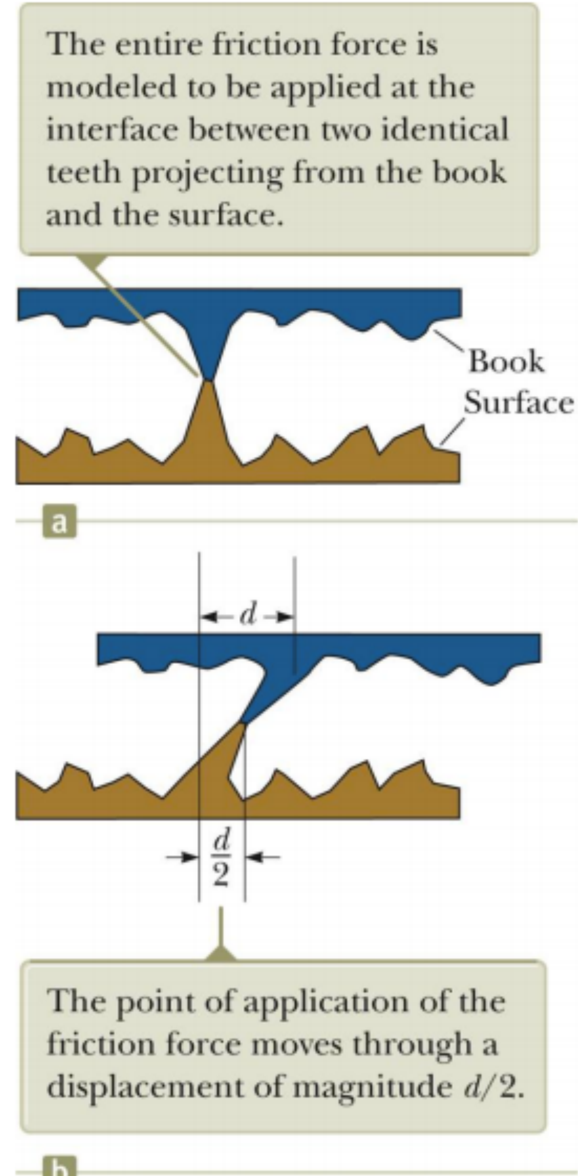
Kinetic Friction

Kinetic friction can be modeled as the interaction between identical teeth.

The frictional force is spread out over the entire contact surface.

The displacement of the point of application of the frictional force is not calculable.

Therefore, the work done by the frictional force is not calculable.



Work and Energy With Friction

In general, if friction is acting in a system:

- $\Delta K = \Sigma W_{other\ forces} - f_k d$
- This is a modified form of the work – kinetic energy theorem.
  - Use this form when friction acts on an object.
- If friction is zero, this equation becomes the same as Conservation of Mechanical Energy.

A friction force transforms kinetic energy in a system to internal energy.

The increase in internal energy of the system is equal to its decrease in kinetic energy.

- $\Delta E_{int} = f_k d$

In general, this equation can be written as $\Sigma W_{other\ forces} = W = \Delta K + \Delta E_{int}$

This represents the non-isolated system model for a system within which a non-conservative force acts.

Example 8.4 A Block Pulled on a Rough Surface AM

A 6.0-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.

(A) Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15.

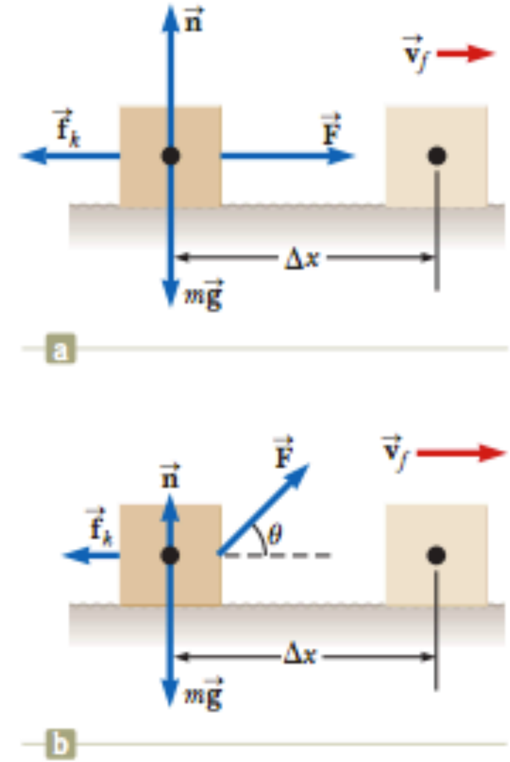


Figure 8.8 (Example 8.4) (a) A block pulled to the right on a rough surface by a constant horizontal force. (b) The applied force is at an angle θ to the horizontal.

SOLUTION

Conceptualize This example is similar to Example 7.6 (page 190), but modified so that the surface is no longer frictionless. The rough surface applies a friction force on the block opposite to the applied force. As a result, we expect the speed to be lower than that found in Example 7.6.

Categorize The block is pulled by a force and the surface is rough, so the block and the surface are modeled as a *nonisolated system* with a nonconservative force acting.

Analyze Figure 8.8a illustrates this situation. Neither the normal force nor the gravitational force does work on the system because their points of application are displaced horizontally.

Find the work done on the system by the applied force just as in Example 7.6:

$$\sum W_{\text{other forces}} = W_F = F\,\Delta x$$

Apply the *particle in equilibrium* model to the block in the vertical direction:

$$\sum F_y = 0 \rightarrow n - mg = 0 \rightarrow n = mg$$

Find the magnitude of the friction force:

$$f_k = \mu_k n = \mu_k mg = (0.15)(6.0\text{ kg})(9.80\text{ m/s}^2) = 8.82\text{ N}$$

Substitute the energies into Equation 8.15 and solve for the final speed of the block:

$$F\Delta x = \Delta K + \Delta E_{\text{int}} = (\tfrac{1}{2}mv_f^2 - 0) + f_k d$$

$$v_f = \sqrt{\frac{2}{m}(-f_k d + F\Delta x)}$$

Substitute numerical values:

$$v_f = \sqrt{\frac{2}{6.0\text{ kg}}[-(8.82\text{ N})(3.0\text{ m}) + (12\text{ N})(3.0\text{ m})]} = \boxed{1.8\text{ m/s}}$$

Finalize As expected, this value is less than the 3.5 m/s found in the case of the block sliding on a frictionless surface (see Example 7.6). The difference in kinetic energies between the block in Example 7.6 and the block in this example is equal to the increase in internal energy of the block–surface system in this example.

(B) Suppose the force $\vec{F}$ is applied at an angle θ as shown in Figure 8.8b. At what angle should the force be applied to achieve the largest possible speed after the block has moved 3.0 m to the right?

SOLUTION

Conceptualize You might guess that $\theta = 0$ would give the largest speed because the force would have the largest component possible in the direction parallel to the surface. Think about $\vec{F}$ applied at an arbitrary nonzero angle, however. Although the horizontal component of the force would be reduced, the vertical component of the force would reduce the normal force, in turn reducing the force of friction, which suggests that the speed could be maximized by pulling at an angle other than $\theta = 0$.

Categorize As in part (A), we model the block and the surface as a *nonisolated system* with a nonconservative force acting.

Analyze Find the work done by the applied force, noting that $\Delta x = d$ because the path followed by the block is a straight line:

$$(1)\quad \sum W_{\text{other forces}} = W_F = F\Delta x\cos\theta = Fd\cos\theta$$

Apply the particle in equilibrium model to the block in the vertical direction:

$$\sum F_y = n + F\sin\theta - mg = 0$$

Solve for n:

$$(2)\quad n = mg - F\sin\theta$$

Use Equation 8.15 to find the final kinetic energy for this situation:

$$W_F = \Delta K + \Delta E_{\text{int}} = (K_f - 0) + f_k d \rightarrow K_f = W_F - f_k d$$

Substitute the results in Equations (1) and (2):

$$K_f = Fd\cos\theta - \mu_k nd = Fd\cos\theta - \mu_k(mg - F\sin\theta)d$$

Maximizing the speed is equivalent to maximizing the final kinetic energy. Consequently, differentiate K_f with respect to θ and set the result equal to zero:

$$\frac{dK_f}{d\theta} = -Fd\sin\theta - \mu_k(0 - F\cos\theta)d = 0$$

$$-\sin\theta + \mu_k\cos\theta = 0$$

$$\tan\theta = \mu_k$$

Evaluate θ for $\mu_k = 0.15$:

$$\theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.15) = \boxed{8.5°}$$

Conceptual Example 8.5 Useful Physics for Safer Driving

A car traveling at an initial speed v slides a distance d to a halt after its brakes lock. If the car's initial speed is instead $2v$ at the moment the brakes lock, estimate the distance it slides.

SOLUTION

Let us assume the force of kinetic friction between the car and the road surface is constant and speeds. According to Equation 8.13, the friction force multiplied by the distance d is equal to the in of the car (because $K_f = 0$ and there is no work done by other forces). If the speed is doubled, as it the kinetic energy is quadrupled. For a given friction force, the distance traveled is four times as gre speed is doubled, and so the estimated distance the car slides is $4d$.

Example 8.6 A Block–Spring System AM

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1 000 N/m as shown in Figure 8.9a. The spring is compressed 2.0 cm and is then released from rest as in Figure 8.9b.

(A) Calculate the speed of the block as it passes through the equilibrium position $x = 0$ if the surface is frictionless.

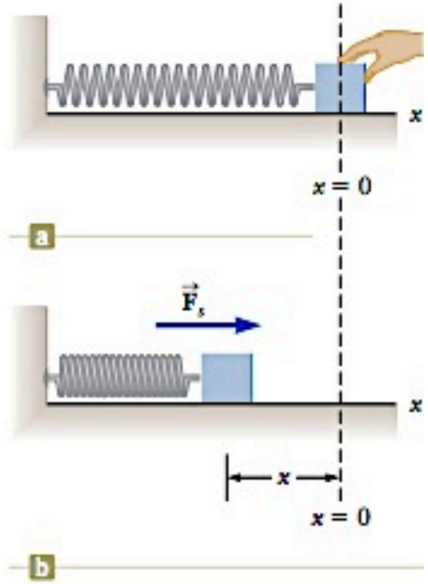


Figure 8.9 (Example 8.6) (a) A block attached to a spring is pushed inward from an initial position $x = 0$ by an external agent. (b) At position x, the block is released from rest and the spring pushes it to the right.

SOLUTION

Conceptualize This situation has been discussed before, and it is easy to visualize the block being pushed to the right by the spring and moving with some speed at $x = 0$.

Categorize We identify the system as the block and model the block as a *nonisolated system.*

Analyze In this situation, the block starts with $v_i = 0$ at $x_i = -2.0$ cm, and we want to find v_f at $x_f = 0$.

Use Equation 7.11 to find the work done by the spring on the system with $x_{max} = x_i$:

$$W_s = \tfrac{1}{2}kx_{max}^2$$

Work is done on the block, and its speed changes. The conservation of energy equation, Equation 8.2, reduces to the work–kinetic energy theorem. Use that theorem to find the speed at $x = 0$:

$$W_s = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$v_f = \sqrt{v_i^2 + \frac{2}{m}W_s} = \sqrt{v_i^2 + \frac{2}{m}\left(\tfrac{1}{2}kx_{max}^2\right)}$$

Substitute numerical values:

$$v_f = \sqrt{0 + \frac{2}{1.6\text{ kg}}\left[\tfrac{1}{2}(1\,000\text{ N/m})(0.020\text{ m})^2\right]} = \boxed{0.50\text{ m/s}}$$

Finalize Although this problem could have been solved in Chapter 7, it is presented here to provide contrast with the following part (B), which requires the techniques of this chapter.

(B) Calculate the speed of the block as it passes through the equilibrium position if a constant friction force of 4.0 N retards its motion from the moment it is released.

Analyze Write Equation 8.15: $W_s = \Delta K + \Delta E_{int} = (\frac{1}{2}mv_f^2 - 0) + f_k d$

Solve for v_f: $v_f = \sqrt{\frac{2}{m}(W_s - f_k d)}$

Substitute for the work done by the spring: $v_f = \sqrt{\frac{2}{m}(\frac{1}{2}kx_{max}^2 - f_k d)}$

Substitute numerical values: $v_f = \sqrt{\frac{2}{1.6\text{ kg}}[\frac{1}{2}(1\,000\text{ N/m})(0.020\text{ m})^2 - (4.0\text{ N})(0.020\text{ m})]} = 0.39\text{ m/s}$

Finalize As expected, this value is less than the 0.50 m/s found in part (A).

WHAT IF? What if the friction force were increased to 10.0 N? What is the block's speed at $x = 0$?

Answer In this case, the value of $f_k d$ as the block moves to $x = 0$ is

$$f_k d = (10.0\text{ N})(0.020\text{ m}) = 0.20\text{ J}$$

which is equal in magnitude to the kinetic energy at $x = 0$ for the frictionless case. (Verify it!). Therefore, all the kinetic energy has been transformed to internal energy by friction when the block arrives at $x = 0$, and its speed at this point is $v = 0$.

In this situation as well as that in part (B), the speed of the block reaches a maximum at some position other than $x = 0$. Problem 53 asks you to locate these positions.

Adding Changes in Potential Energy

If friction acts within an isolated system

$$\Delta E_{mech} = \Delta K + \Delta U = -f_k\, d$$

- ΔU is the change in all forms of potential energy

If non-conservative forces act within a non-isolated system and the external influence on the system is by means of work.

$$\Delta E_{mech} = -f_k\, d + \Sigma W_{\text{other forces}}$$

This equation represents the non-isolated system model for a system that possesses potential energy and within which a non-conservative force acts and can be rewritten as

$$\Sigma W_{\text{other forces}} = W = \Delta K + \Delta U + \Delta E_{int}$$

Example 8.7 Crate Sliding Down a Ramp AM

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0° as shown in Figure 8.10. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp.

(A) Use energy methods to determine the speed of the crate at the bottom of the ramp.

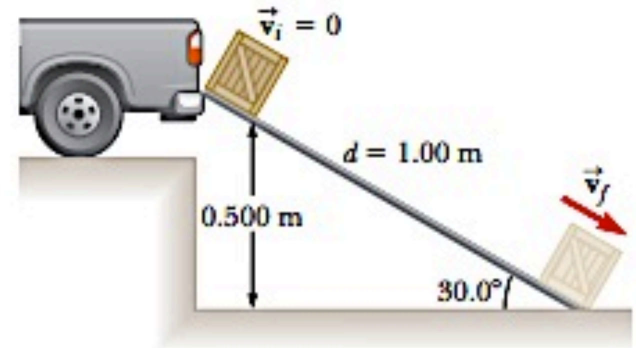


Figure 8.10 (Example 8.7) A crate slides down a ramp under the influence of gravity. The potential energy of the system decreases, whereas the kinetic energy increases.

SOLUTION

Conceptualize Imagine the crate sliding down the ramp in Figure 8.10. The larger the friction force, the more slowly the crate will slide.

Categorize We identify the crate, the surface, and the Earth as an *isolated system* with a nonconservative force acting.

Analyze Because $v_i = 0$, the initial kinetic energy of the system when the crate is at the top of the ramp is zero. If the y coordinate is measured from the bottom of the ramp (the final position of the crate, for which we choose the gravitational potential energy of the system to be zero) with the upward direction being positive, then $y_i = 0.500$ m.

Write the conservation of energy equation (Eq. 8.2) for this system:

$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

Substitute for the energies:

$$(\tfrac{1}{2}mv_f^2 - 0) + (0 - mgy_i) + f_k d = 0$$

Solve for v_f:

$$(1) \quad v_f = \sqrt{\frac{2}{m}(mgy_i - f_k d)}$$

Substitute numerical values:

$$v_f = \sqrt{\frac{2}{3.00\text{ kg}}[(3.00\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m}) - (5.00\text{ N})(1.00\text{ m})]} = 2.54\text{ m/s}$$

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(B) How far does the crate slide on the horizontal floor if it continues to experience a friction force of magnitude 5.00 N?

SOLUTION

Analyze This part of the problem is handled in exactly the same way as part (A), but in this case we can consider the mechanical energy of the system to consist only of kinetic energy because the potential energy of the system remains fixed.

Write the conservation of energy equation for this situation:

$$\Delta K + \Delta E_{\text{int}} = 0$$

Substitute for the energies:

$$(0 - \tfrac{1}{2}mv_i^2) + f_k d = 0$$

Solve for the distance d and substitute numerical values:

$$d = \frac{mv_i^2}{2f_k} = \frac{(3.00\text{ kg})(2.54\text{ m/s})^2}{2(5.00\text{ N})} = 1.94\text{ m}$$

Finalize For comparison, you may want to calculate the speed of the crate at the bottom of the ramp in the case in which the ramp is frictionless. Also notice that the increase in internal energy of the system as the crate slides down the ramp is $f_k d = (5.00\text{ N})(1.00\text{ m}) = 5.00\text{ J}$. This energy is shared between the crate and the surface, each of which is a bit warmer than before.

Also notice that the distance d the object slides on the horizontal surface is infinite if the surface is frictionless. Is that consistent with your conceptualization of the situation?

WHAT IF? A cautious worker decides that the speed of the crate when it arrives at the bottom of the ramp may be so large that its contents may be damaged. Therefore, he replaces the ramp with a longer one such that the new

Find the length d of the new ramp:

$$\sin 25.0° = \frac{0.500\text{ m}}{d} \rightarrow d = \frac{0.500\text{ m}}{\sin 25.0°} = 1.18\text{ m}$$

Find v_f from Equation (1) in part (A):

$$v_f = \sqrt{\frac{2}{3.00\text{ kg}}[(3.00\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m}) - (5.00\text{ N})(1.18\text{ m})]} = 2.42\text{ m/s}$$

The final speed is indeed lower than in the higher-angle case.

Example 8.8 Block–Spring Collision AM

A block having a mass of 0.80 kg is given an initial velocity $v_{Ⓐ} = 1.2$ m/s to the right and collides with a spring whose mass is negligible and whose force constant is $k = 50$ N/m as shown in Figure 8.11.

(A) Assuming the surface to be frictionless, calculate the maximum compression of the spring after the collision.

SOLUTION

Conceptualize The various parts of Figure 8.11 help us imagine what the block will do in this situation. All motion takes place in a horizontal plane, so we do not need to consider changes in gravitational potential energy.

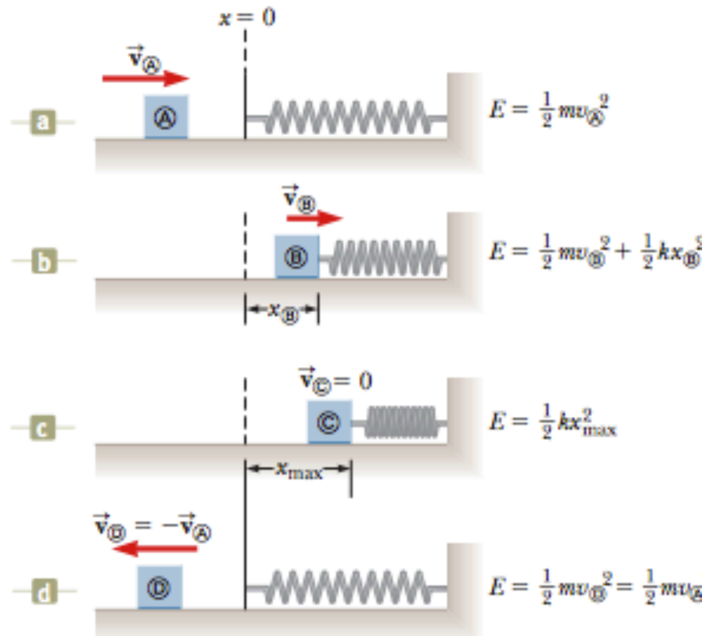


Figure 8.11 (Example 8.8) A block sliding on a frictionless, horizontal surface collides with a light spring. (a) Initially, the mechanical energy is all kinetic energy. (b) The mechanical energy is the sum of the kinetic energy of the block and the elastic potential energy in the spring. (c) The energy is entirely potential energy. (d) The energy is transformed back to the kinetic energy of the block. The total energy of the system remains constant throughout the motion.

Categorize We identify the system to be the block and the spring and model it as an *isolated system* with no nonconservative forces acting.

Analyze Before the collision, when the block is at Ⓐ, it has kinetic energy and the spring is uncompressed, so the elastic potential energy stored in the system is zero. Therefore, the total mechanical energy of the system before the collision is just $\frac{1}{2}mv_{Ⓐ}^2$. After the collision, when the block is at Ⓒ, the spring is fully compressed; now the block is at rest and so has zero kinetic energy. The elastic potential energy stored in the system, however, has its maximum value $\frac{1}{2}kx^2 = \frac{1}{2}kx_{max}^2$, where the origin of coordinates $x = 0$ is chosen to be the equilibrium position of the spring and x_{max} is the maximum compression of the spring, which in this case happens to be $x_{Ⓒ}$. The total mechanical energy of the system is conserved because no nonconservative forces act on objects within the isolated system.

Write the conservation of energy equation for this situation: $\Delta K + \Delta U = 0$

Substitute for the energies: $(0 - \frac{1}{2}mv_{Ⓐ}^2) + (\frac{1}{2}kx_{max}^2 - 0) = 0$

Solve for x_{max} and evaluate: $x_{max} = \sqrt{\frac{m}{k}}\,v_{Ⓐ} = \sqrt{\frac{0.80\text{ kg}}{50\text{ N/m}}}(1.2\text{ m/s}) = 0.15\text{ m}$

(B) Suppose a constant force of kinetic friction acts between the block and the surface, with $\mu_k = 0.50$. If the speed of the block at the moment it collides with the spring is $v_Ⓐ = 1.2$ m/s, what is the maximum compression $x_Ⓒ$ in the spring?

SOLUTION

Conceptualize Because of the friction force, we expect the compression of the spring to be smaller than in part (A) because some of the block's kinetic energy is transformed to internal energy in the block and the surface.

Categorize We identify the system as the block, the surface, and the spring. This is an *isolated system* but now involves a nonconservative force.

Analyze In this case, the mechanical energy $E_{mech} = K + U_s$ of the system is *not* conserved because a friction force acts on the block. From the *particle in equilibrium* model in the vertical direction, we see that $n = mg$.

Evaluate the magnitude of the friction force: $f_k = \mu_k n = \mu_k mg$

Write the conservation of energy equation for this situation: $\Delta K + \Delta U + \Delta E_{int} = 0$

Substitute the initial and final energies: $(0 - \frac{1}{2}mv_Ⓐ^2) + (\frac{1}{2}kx_Ⓒ^2 - 0) + \mu_k mgx_Ⓒ = 0$

Rearrange the terms into a qaudratic equation: $kx_Ⓒ^2 + 2\mu_k mgx_Ⓒ - mv_Ⓐ^2 = 0$

Substitute numerical values: $50x_Ⓒ^2 + 2(0.50)(0.80)(9.80)x_Ⓒ - (0.80)(1.2)^2 = 0$

$50x_Ⓒ^2 + 7.84x_Ⓒ - 1.15 = 0$

Solving the quadratic equation for $x_Ⓒ$ gives $x_Ⓒ = 0.092$ m and $x_Ⓒ = -0.25$ m. The physically meaningful root is $x_Ⓒ = 0.092$ m.

Finalize The negative root does not apply to this situation because the block must be to the right of the origin (positive value of x) when it comes to rest. Notice that the value of 0.092 m is less than the distance obtained in the frictionless case of part (A) as we expected.

Example 8.9 Connected Blocks in Motion AM

Two blocks are connected by a light string that passes over a frictionless pulley as shown in Figure 8.12. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface.

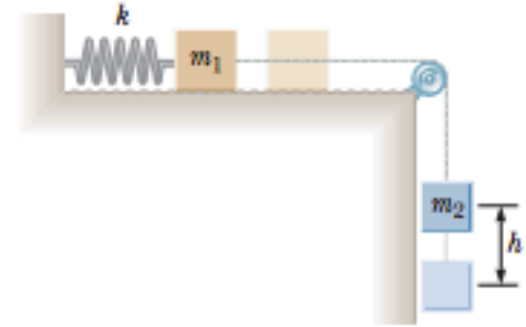


Figure 8.12 (Example 8.9) As the hanging block moves from its highest elevation to its lowest, the system loses gravitational potential energy but gains elastic potential energy in the spring. Some mechanical energy is transformed to internal energy because of friction between the sliding block and the surface.

SOLUTION

Conceptualize The key word *rest* appears twice in the problem statement. This word suggests that the configurations of the system associated with rest are good candidates for the initial and final configurations because the kinetic energy of the system is zero for these configurations.

Categorize In this situation, the system consists of the two blocks, the spring, the surface, and the Earth. This is an *isolated system* with a nonconservative force acting. We also model the sliding block as a *particle in equilibrium* in the vertical direction, leading to $n = m_1 g$.

Analyze We need to consider two forms of potential energy for the system, gravitational and elastic: $\Delta U_g = U_{gf} - U_{gi}$ is the change in the system's gravitational potential energy, and $\Delta U_s = U_{sf} - U_{si}$ is the change in the system's elastic potential energy. The change in the gravitational potential energy of the system is associated with only the falling block

8.9 continued

because the vertical coordinate of the horizontally sliding block does not change. The initial and final kinetic energies of the system are zero, so $\Delta K = 0$.

Write the appropriate reduction of Equation 8.2:

$$(1)\quad \Delta U_g + \Delta U_s + \Delta E_{\text{int}} = 0$$

Substitute for the energies, noting that as the hanging block falls a distance h, the horizontally moving block moves the same distance h to the right, and the spring stretches by a distance h:

$$(0 - m_2gh) + (\tfrac{1}{2}kh^2 - 0) + f_kh = 0$$

Substitute for the friction force:

$$-m_2gh + \tfrac{1}{2}kh^2 + \mu_k m_1gh = 0$$

Solve for μ_k:

$$\mu_k = \frac{m_2g - \frac{1}{2}kh}{m_1g}$$

Finalize This setup represents a method of measuring the coefficient of kinetic friction between an object and some surface. Notice how we have solved the examples in this chapter using the energy approach. We begin with Equation 8.2 and then tailor it to the physical situation. This process may include deleting terms, such as the kinetic energy term and all terms on the right-hand side of Equation 8.2 in this example. It can also include expanding terms, such as rewriting ΔU due to two types of potential energy in this example.

Conceptual Example 8.10 Interpreting the Energy Bars

The energy bar charts in Figure 8.13 show three instants in the motion of the system in Figure 8.12 and described in Example 8.9. For each bar chart, identify the configuration of the system that corresponds to the chart.

SOLUTION

In Figure 8.13a, there is no kinetic energy in the system. Therefore, nothing in the system is moving. The bar chart shows that the system contains only gravitational potential energy and no internal energy yet, which corresponds to the configuration with the darker blocks in Figure 8.12 and represents the instant just after the system is released.

In Figure 8.13b, the system contains four types of energy. The height of the gravitational potential energy bar is at 50%, which tells us that the hanging block has moved halfway between its position corresponding to Figure 8.13a and the position defined as $y = 0$. Therefore, in this configuration, the hanging block is between the dark and light images of the hanging block in Figure 8.12. The system has gained kinetic energy because the blocks are moving, elastic potential energy because the spring is stretching, and internal energy because of friction between the block of mass m_1 and the surface.

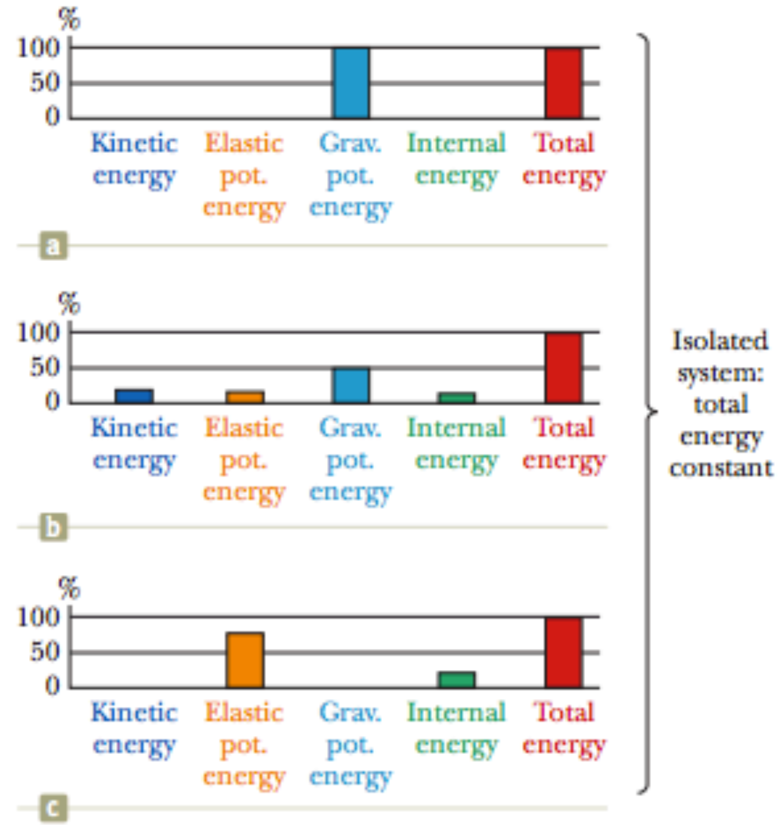


Figure 8.13 (Conceptual Example 8.10) Three energy bar charts are shown for the system in Figure 8.12.

In Figure 8.13c, the height of the gravitational potential energy bar is zero, telling us that the hanging block is at $y = 0$. In addition, the height of the kinetic energy bar is zero, indicating that the blocks have stopped moving momentarily. Therefore, the configuration of the system is that shown by the light images of the blocks in Figure 8.12. The height of the elastic potential energy bar is high because the spring is stretched its maximum amount. The height of the internal energy bar is higher than in Figure 8.13b because the block of mass m_1 has continued to slide over the surface after the configuration shown in Figure 8.13b.

Power

Power is the time rate of energy transfer.

The ***instantaneous power*** is defined as

$$P \equiv \frac{dE}{dt}$$

Using work as the energy transfer method, this can also be written as

$$P_{avg} = \frac{W}{\Delta t}$$

Instantaneous Power and Average Power

The instantaneous power is the limiting value of the average power as Δt approaches zero.

$$P = \lim_{\Delta t \to 0} \frac{W}{\Delta t} = \frac{dW}{dt} = \vec{\mathbf{F}} \cdot \frac{d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

This expression for power is valid for any means of energy transfer.

Units of Power

The SI unit of power is called the watt.

- 1 watt = 1 joule / second = 1 kg · m^2 / s^3

A unit of power in the US Customary system is horsepower.

- 1 hp = 746 W

Units of power can also be used to express units of work or energy.

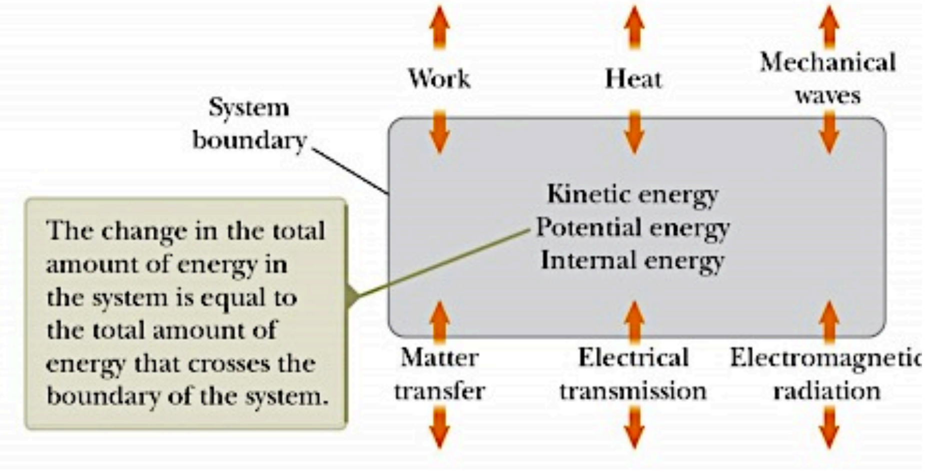
- 1 kWh = (1000 W)(3600 s) = 3.6×10^6 J

Problem Solving Summary – Non-isolated System

The most general statement describing the behavior of a non-isolated system is the conservation of energy equation.

$$\Delta E_{system} = \Sigma T$$

This equation can be expanded or have terms deleted depending upon the specific situation.



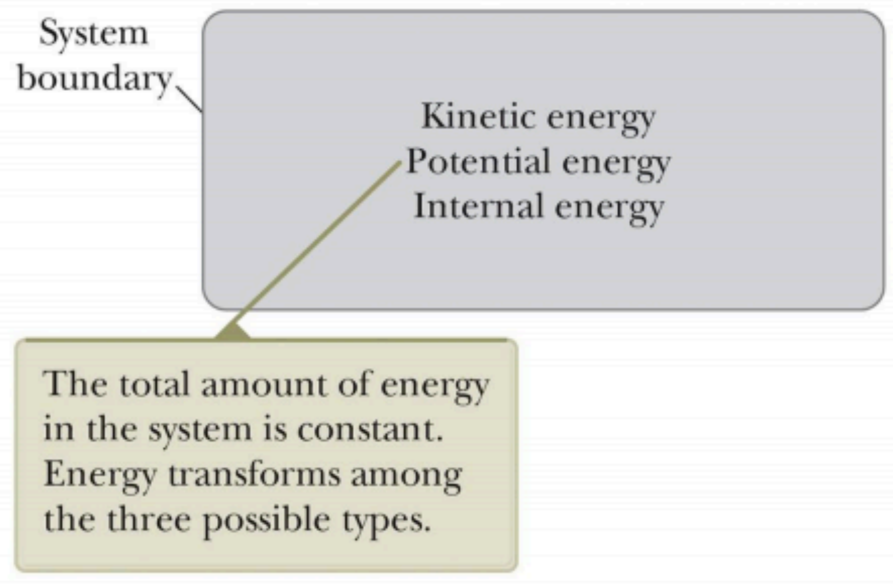
Problem Solving Summary – Isolated System

The total energy of an isolated system is conserved

$$\Delta E_{system} = 0$$

If no non-conservative forces act within the isolated system, the mechanical energy of the system is conserved.

$$\Delta E_{mech} = 0$$



Example 8.11 Power Delivered by an Elevator Motor AM

An elevator car (Fig. 8.14a) has a mass of 1 600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4 000 N retards its motion.

(A) How much power must a motor deliver to lift the elevator car and its passengers at a constant speed of 3.00 m/s?

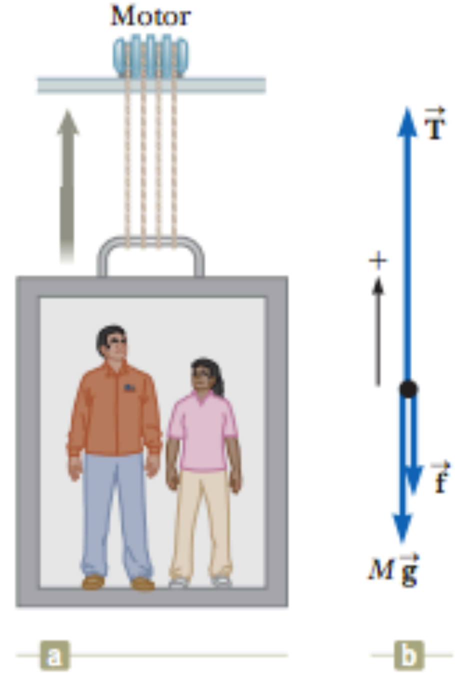


Figure 8.14 (Example 8.11) (a) The motor exerts an upward force $\vec{\mathbf{T}}$ on the elevator car. The magnitude of this force is the total tension T in the cables connecting the car and motor. The downward forces acting on the car are a friction force $\vec{\mathbf{f}}$ and the gravitational force $\vec{\mathbf{F}}_g = M\vec{\mathbf{g}}$. (b) The free-body diagram for the elevator car.

SOLUTION

Conceptualize The motor must supply the force of magnitude T that pulls the elevator car upward.

Categorize The friction force increases the power necessary to lift the elevator. The problem states that the speed of the elevator is constant, which tells us that $a = 0$. We model the elevator as a *particle in equilibrium.*

Analyze The free-body diagram in Figure 8.14b specifies the upward direction as positive. The *total* mass M of the system (car plus passengers) is equal to 1 800 kg.

Using the particle in equilibrium model, apply Newton's second law to the car:

$$\sum F_y = T - f - Mg = 0$$

Solve for T:

$$T = Mg + f$$

Use Equation 8.19 and that $\vec{\mathbf{T}}$ is in the same direction as $\vec{\mathbf{v}}$ to find the power:

$$P = \vec{\mathbf{T}} \cdot \vec{\mathbf{v}} = Tv = (Mg + f)v$$

Substitute numerical values:

$$P = [(1\,800 \text{ kg})(9.80 \text{ m/s}^2) + (4\,000 \text{ N})](3.00 \text{ m/s}) = 6.49 \times 10^4 \text{ W}$$

(B) What power must the motor deliver at the instant the speed of the elevator is v if the motor is designed to provide the elevator car with an upward acceleration of 1.00 m/s^2?

SOLUTION

Conceptualize In this case, the motor must supply the force of magnitude T that pulls the elevator car upward with an increasing speed. We expect that more power will be required to do that than in part (A) because the motor must now perform the additional task of accelerating the car.

Categorize In this case, we model the elevator car as a *particle under a net force* because it is accelerating.

Analyze Using the particle under a net force model, apply Newton's second law to the car:

$$\sum F_y = T - f - Mg = Ma$$

Solve for T:

$$T = M(a + g) + f$$

Use Equation 8.19 to obtain the required power:

$$P = Tv = [M(a + g) + f]v$$

Substitute numerical values:

$$P = [(1\,800 \text{ kg})(1.00 \text{ m/s}^2 + 9.80 \text{ m/s}^2) + 4\,000 \text{ N}]v$$

$$= (2.34 \times 10^4)v$$

where v is the instantaneous speed of the car in meters per second and P is in watts.

Finalize To compare with part (A), let $v = 3.00$ m/s, giving a power of

$$P = (2.34 \times 10^4 \text{ N})(3.00 \text{ m/s}) = 7.02 \times 10^4 \text{ W}$$

which is larger than the power found in part (A), as expected.

	Isolated System	Non-Isolated System
Forces	conservative Forces	Non-conservative Forces
conservation of energy	$\Delta E_{system}=0$	$\Delta E_{system}=\Sigma T$
Conservation of Mechanical Energy	$\Delta E_{mech}=0$ $\Delta K+\Delta U=0$ $\Delta U_g=mgh$ $\Delta U_s=1/2kx^2$	$\Delta E_{mech}=\Sigma W$ $\Delta K+\Delta U=\Sigma W=\Sigma W-F_sd$ $\Delta U_g=mgh$ $\Delta U_s=1/2kx^2$
Cases	1- $\Delta K+\Delta U_g=0$ Or $\Delta K=\Sigma W$ أذا تم رفع الجسم مسافة عن الارض 2- $\Delta K+\Delta U_s=0$ جسم معلق بنابض يتحرك على المحول الافقي 3- $\Delta K+\Delta U_g+\Delta U_{s+}=0$ إذا تم رفع الجسم بنابض مسافة عن الارض 4-$\Delta U_s=0$ تكون الطاقة الحركية مساوية لصفر عند اقصى قيمة للازاحة($X_{max}Or,X_{min}$) حيث تكون الطاقة الكامنة أعلى ما يمكن. تكون الطاقة الكامنة مساوية لصفر عند لحظة الاتزان($x=0$) في النابض	1- $\Delta K+\Delta U_g=\Sigma W-F_sd$ أذا تم رفع الجسم مسافة عن الارض 2- $\Delta K+\Delta U_s=\Sigma W-F_sd$ جسم معلق بنابض يتحرك على المحول الافقي 3- $\Delta K+\Delta U_g+\Delta U_{s+}=\Sigma W-F_sd$ إذا تم رفع الجسم بنابض مسافة عن الارض 4-$\Delta U_s=\Sigma W-F_sd$ تكون الطاقة الحركية مساوية لصفر عند اقصى قيمة للازاحة($X_{max}Or,X_{min}$) حيث تكون الطاقة الكامنة أعلى ما يمكن. تكون الطاقة الكامنة مساوية لصفر عند لحظة الاتزان($x=0$) في النابض

Homework

Section 7.2 Work Done by a Constant Force

1- A 2.00-kg block is attached to a spring of force constant 500 N/m. The block is pulled 5.00 cm to the right of equilibrium and released from rest. Find the speed of the block as it passes through equilibrium if (a) the horizontal surface is frictionless and (b) the coefficient of friction between block and surface is 0.350.

2- . Explain why the total energy of a system can be either positive or negative, whereas the kinetic energy is always positive.3- Can a normal force do work? If not, why not? If so, give an example.

3-A ball is thrown straight up into the air. At what position is its kinetic energy a maximum? At what position is the gravitational potential energy of the ball–Earth system a maximum?**.**

4- A pile driver is a device used to drive objects into the Earth by repeatedly dropping a heavy weight on them. By how much does the energy of the pile driver–Earth system increase when the weight it drops is doubled? Assume the weight is dropped from the same height each time.
5- A ball is thrown straight up into the air. At what position is its kinetic energy a maximum? At what position is the gravitational potential energy of the ball–Earth system a maximum?

5- A 1 000-kg roller coaster train is initially at the top of a rise, at point A. It then moves 135 ft, at an angle of 40.0° below the horizontal, to a lower point B. (a) Choose point B to be the zero level for gravitational potential energy. Find the potential energy of the roller coaster–Earth system at points A and B, and the change in potential energy as the coaster moves. (b) Repeat part (a), setting the zero reference level at point A.

6- A glider of mass 0.150 kg moves on a horizontal frictionless air track. It is permanently attached to one end of a massless horizontal spring, which has a force constant of 10.0 N/m both for extension and for compression. The other end of the spring is fixed. The glider is moved to compress the spring by 0.180 m and then released from rest. Calculate the speed of the glider (a) at the point where it has moved 0.180 m from its starting point, so that the spring is momentarily exerting no force and (b) at the point where it has moved 0.250 m from its starting point.

7- A particle of mass m = 5.00 kg is released from point A and slides on the frictionless track shown in Figure P8.24. Determine (a) the particle's speed at points B and C and (b) the net work done by the gravitational force in moving the particle from A to C.

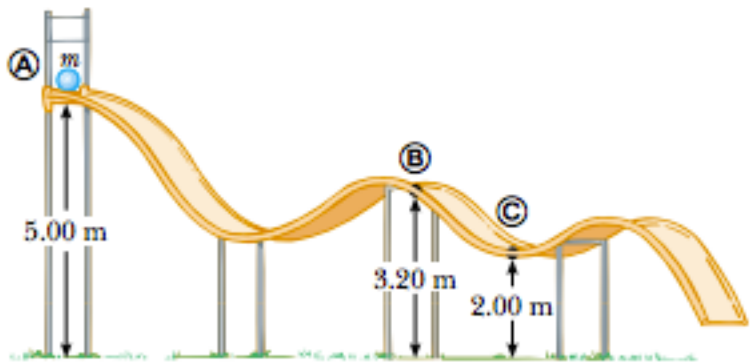


Figure P8.24

8-A block of mass 0.250 kg is placed on top of a light vertical spring of force constant 5 000 N/m and pushed downward so that the spring is compressed by 0.100 m. After the block is released from rest, it travels upward and then leaves the spring. To what maximum height above the point of release does it rise?

9-A 5.00-kg block is set into motion up an inclined plane with an initial speed of 8.00 m/s (Fig. P8.33). The block comes to rest after traveling 3.00 m along the plane, which is inclined at an angle of 30.0° to the horizontal. For this motion determine

(a) the change in the block's kinetic energy, (b) the change in the potential energy of the block–Earth system, and (c) the friction force exerted on the block (assumed to

be constant). (d) What is the coefficient of kinetic friction?

9- A 700-N Marine in basic training climbs a 10.0-m vertical rope at a constant speed in 8.00 s. What is his power output?

10-The electric motor of a model train accelerates the train from rest to 0.620 m/s in 21.0 m/s. The total mass of the train is 875 g. Find the average power delivered to the train during the acceleration.