Chapter 7

Energy of a System

7.1: Systems and Environments

7.2: Work Done by a Constant Force

7.4: Work Done by a Varying Force

7.5: Kinetic Energy and the Work-Kinetic Energy Theorem

7.6: Potential Energy of a System

7.7: Conservative and Neoconservative Forces

7.8: Relationship Between Conservative Forces and Potential Energy

Introduction to Energy

A variety of problems can be solved with Newton's Laws and associated principles.
Some problems that could theoretically be solved with Newton's Laws are very difficult in practice.

- These problems can be made easier with other techniques.

The concept of energy is one of the most important topics in science and engineering.
Every physical process that occurs in the Universe involves energy and energy transfers or transformations.
Energy is not easily defined.

Analysis Model

The new approach will involve changing from a particle model to a system model.

These analysis models will be formally introduced in the next chapter.

In this chapter, systems are introduced along with three ways to store energy in a system.

Systems

A *system* is a small portion of the Universe.

- We will ignore the details of the rest of the Universe.

A critical skill is to identify the system.

- The first step to take in solving a problem

A valid system:

- May be a single object or particle
- May be a collection of objects or particles
- May be a region of space

Problem Solving Notes

The general problem solving approach may be used with an addition to the categorize step.
Categorize step of general strategy

- Identify the need for a system approach
- Identify the particular system
- Also identify a system boundary
- The environment surrounds the system

System Example

A force applied to an object in empty space

- System is the object
- Its surface is the system boundary
- The force is an influence on the system
- from its environment that acts across the system boundary.

Work

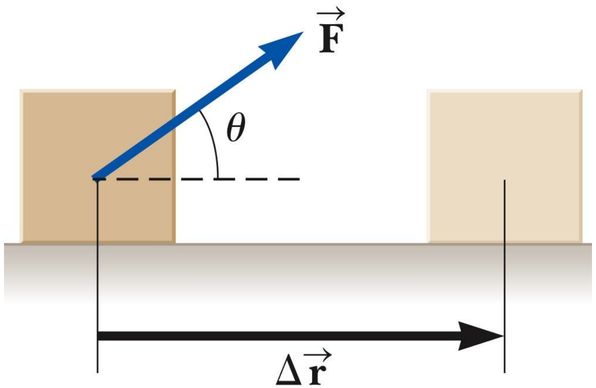
The **work**, *W*, done on a system by an agent exerting a constant force on the system is the product of the magnitude F of the force, the magnitude Δr of the displacement of the point of application of the force, and $\cos\theta$, where θ is the angle between the force and the displacement vectors.

- The meaning of the term *work* is distinctly different in physics than in everyday meaning.
- Work is done *by* some part of the environment that is interacting directly with the system.
- Work is done *on* the system.

Work, cont

$$W = F\,\Delta r \cos\theta$$

- Δr The displacement is that of the point of application of the force.



① A force does no work on the object if the force does not move through a displacement.

② The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application.

Displacement in the Work Equation

The displacement is that of the point of application of the force.

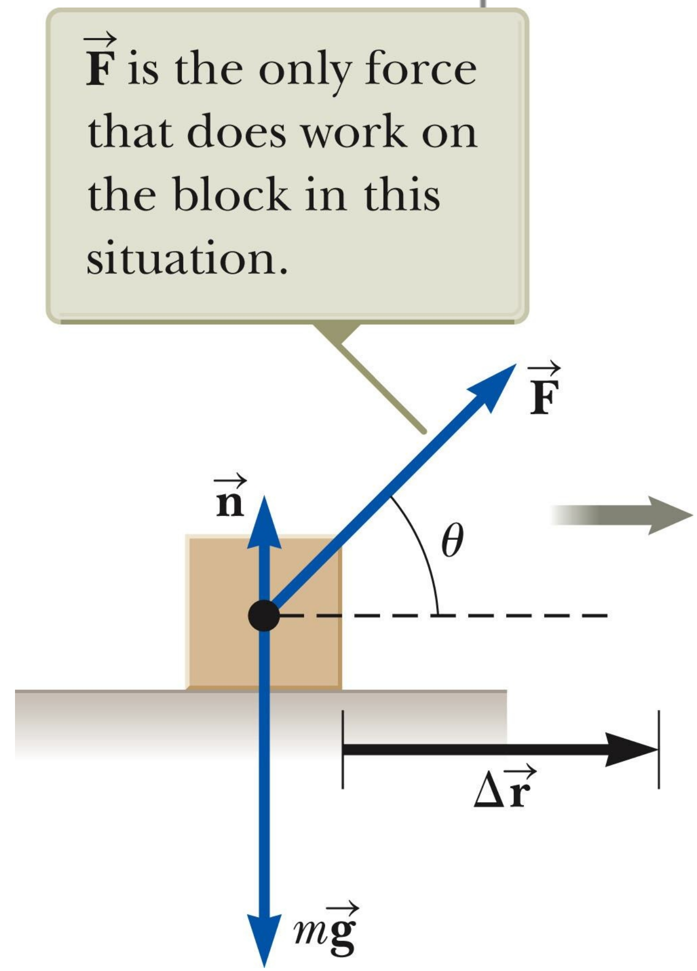
- If the force is applied to a rigid object that can be modeled as a particle, the displacement is the same as that of the particle.
- For a deformable system, the displacement of the object generally is not the same as the displacement associated with the forces applied.

Work Example

The normal force and the gravitational force do no work on the object.

- $\cos\theta = \cos 90° = 0$

The force $\mathbf{F}$ is the only force that does work on the object.



More About Work

The sign of the work depends on the direction of the force relative to the displacement.

- Work is positive when projection of $\mathbf{F}$ onto $\Delta\,\mathbf{r}$ is in the same direction as the displacement.
- Work is negative when the projection is in the opposite direction.

The work done by a force can be calculated, but that force is not necessarily
the cause of the displacement.

Work is a scalar quantity.

The unit of work is a joule (J)

- $1 \text{ joule} = 1 \text{ newton} \quad 1 \text{ meter} = \text{kg} \cdot \text{m}^2 / \text{s}^2$
- $\text{J} = \text{N} \cdot \text{m}$

Work Is An Energy Transfer

This is important for a system approach to solving a problem.

If the work is done on a system and it is positive, energy is transferred to the system.

If the work done on the system is negative, energy is transferred from the system.

- This will result in a change in the amount of energy stored in the system.

Scalar Product of Two Vectors

The scalar product of two vectors is written as $\mathbf{A} \cdot \mathbf{B}$

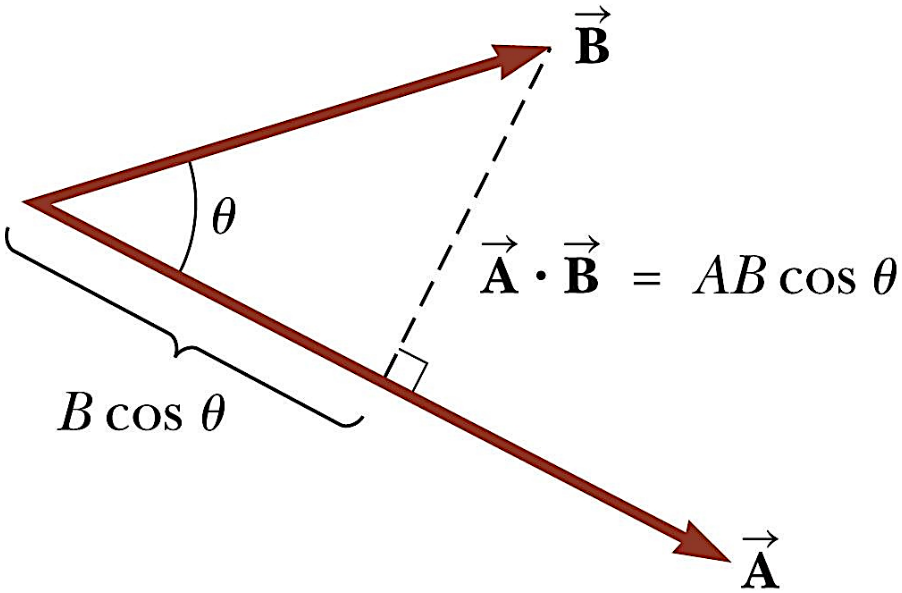
It is also called the dot product.

$$\mathbf{A} \cdot \mathbf{B} \equiv A\,B \cos\theta$$

θ is the angle *between* A and B

Applied to work, this means

$$\mathbf{W = F\Delta r \cos\theta = F \cdot \Delta r}$$



Scalar Product, cont

The scalar product is commutative.

- $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$

The scalar product obeys the distributive law of multiplication.

- $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$

Dot Products of Unit Vectors

$$\mathbf{i}\cdot\mathbf{i}=\mathbf{j}\cdot j=k\cdot k=1$$

$$\mathbf{i}\cdot\mathbf{j}=\mathbf{i}\cdot k=j\cdot k=1$$

Using component form with vectors:

- $A=A_x i+A_y j+A_z k$
- $B=B_x i+B_y j+B_z k$
- $A\cdot B=A_x B_x+A_y B_y+A_z B_z$

- In the special case where
- $A=B$
  $A\cdot A=A_x A_x+A_y A_y+A_z A_z=A_x{}^2+A_y{}^2+A_z{}^2=A^2$

Example 7.1 Mr. Clean

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50.0$ N at an angle of 30.0° with the horizontal (Fig. 7.5). Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

7.1 continued

SOLUTION

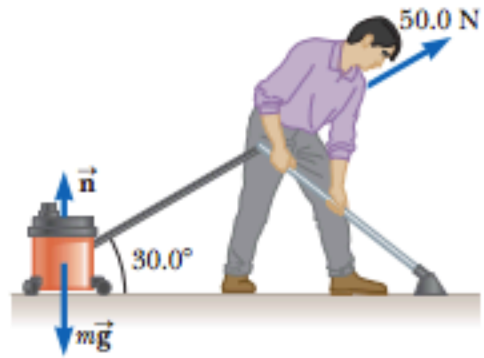


Figure 7.5 (Example 7.1) A vacuum cleaner being pulled at an angle of 30.0° from the horizontal.

Conceptualize Figure 7.5 helps conceptualize the situation. Think about an experience in your life in which you pulled an object across the floor with a rope or cord.

Categorize We are asked for the work done on an object by a force and are given the force on the object, the displacement of the object, and the angle between the two vectors, so we categorize this example as a substitution problem. We identify the vacuum cleaner as the system.

Use the definition of work (Eq. 7.1):

$$W = F\Delta r\cos\theta = (50.0\text{ N})(3.00\text{ m})(\cos 30.0°)$$
$$= \boxed{130\text{ J}}$$

Notice in this situation that the normal force $\vec{n}$ and the gravitational $\vec{F}_g = m\vec{g}$ do no work on the vacuum cleaner because these forces are perpendicular to the displacements of their points of application. Furthermore, there was no mention of whether there was friction between the vacuum cleaner and the floor. The presence or absence of friction is not important when calculating the work done by the applied force. In addition, this work does not depend on whether the vacuum moved at constant velocity or if it accelerated.

Example 7.2 The Scalar Product

The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are given by $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$.

(A) Determine the scalar product $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$.

SOLUTION

Conceptualize There is no physical system to imagine here. Rather, it is purely a mathematical exercise involving two vectors.

Categorize Because we have a definition for the scalar product, we categorize this example as a substitution problem.

Substitute the specific vector expressions for $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$:

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \cdot (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})$$

$$= -2\hat{\mathbf{i}} \cdot \hat{\mathbf{i}} + 2\hat{\mathbf{i}} \cdot 2\hat{\mathbf{j}} - 3\hat{\mathbf{j}} \cdot \hat{\mathbf{i}} + 3\hat{\mathbf{j}} \cdot 2\hat{\mathbf{j}}$$

$$= -2(1) + 4(0) - 3(0) + 6(1) = -2 + 6 = \boxed{4}$$

The same result is obtained when we use Equation 7.6 directly, where $A_x = 2$, $A_y = 3$, $B_x = -1$, and $B_y = 2$.

7.2 continued

(B) Find the angle θ between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

SOLUTION

Evaluate the magnitudes of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ using the Pythagorean theorem:

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(2)^2 + (3)^2} = \sqrt{13}$$

$$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + (2)^2} = \sqrt{5}$$

Use Equation 7.2 and the result from part (A) to find the angle:

$$\cos\theta = \frac{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{65}} = \boxed{60.3^\circ}$$

Example 7.3 Work Done by a Constant Force

A particle moving in the xy plane undergoes a displacement given by $\Delta\vec{\mathbf{r}} = (2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})$ m as a constant force $\vec{\mathbf{F}} = (5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})$ N acts on the particle. Calculate the work done by $\vec{\mathbf{F}}$ on the particle.

SOLUTION

Conceptualize Although this example is a little more physical than the previous one in that it identifies a force and a displacement, it is similar in terms of its mathematical structure.

Categorize Because we are given force and displacement vectors and asked to find the work done by this force on the particle, we categorize this example as a substitution problem.

Substitute the expressions for $\vec{\mathbf{F}}$ and $\Delta\vec{\mathbf{r}}$ into Equation 7.3 and use Equations 7.4 and 7.5:

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = [(5.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}})\text{ N}] \cdot [(2.0\hat{\mathbf{i}} + 3.0\hat{\mathbf{j}})\text{ m}]$$

$$= (5.0\hat{\mathbf{i}} \cdot 2.0\hat{\mathbf{i}} + 5.0\hat{\mathbf{i}} \cdot 3.0\hat{\mathbf{j}} + 2.0\hat{\mathbf{j}} \cdot 2.0\hat{\mathbf{i}} + 2.0\hat{\mathbf{j}} \cdot 3.0\hat{\mathbf{j}})\text{ N} \cdot \text{m}$$

$$= [10 + 0 + 0 + 6]\text{ N} \cdot \text{m} = \boxed{16\text{ J}}$$

Work Done by a Varying Force

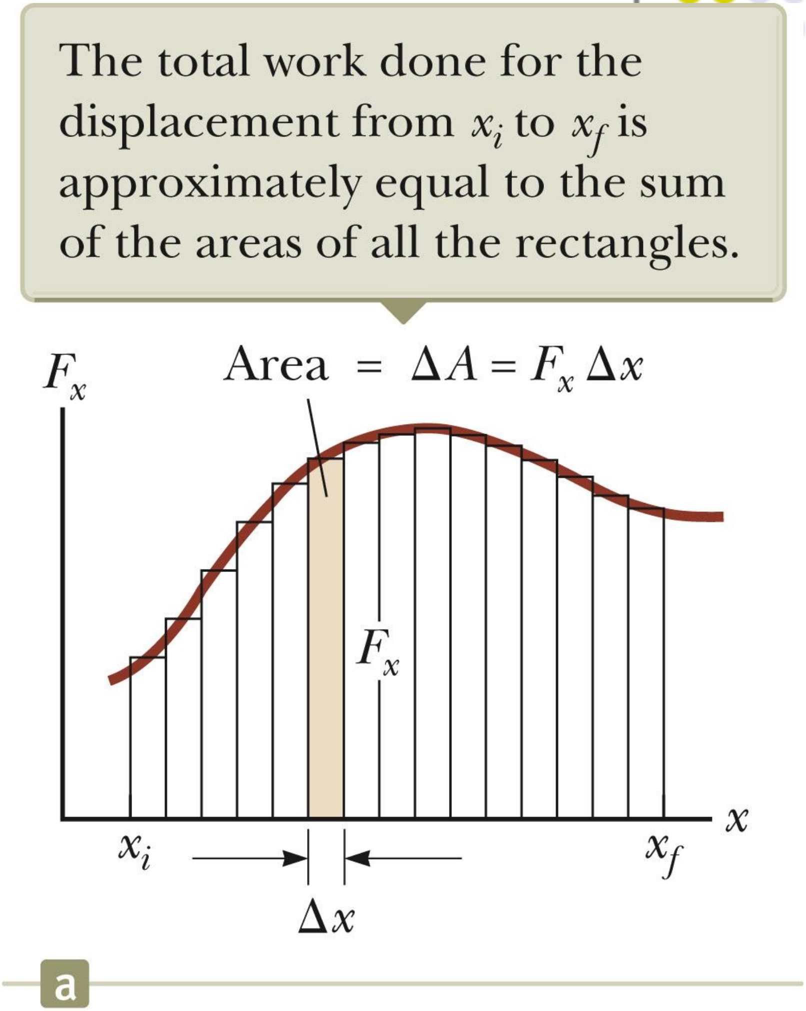
To use W = F Δ r cos θ, the force must be constant, so the equation cannot be used to calculate the work done by a varying force.

Assume that during a very small displacement, Δ*x*, *F* is constant.

For that displacement, $W \sim F \Delta x$

For all of the intervals,

$$W \approx \sum_{x_i}^{x_f} F_x \Delta x$$



Work Done by a Varying Force, cont.

Let the size of the small displacements approach zero.

Since

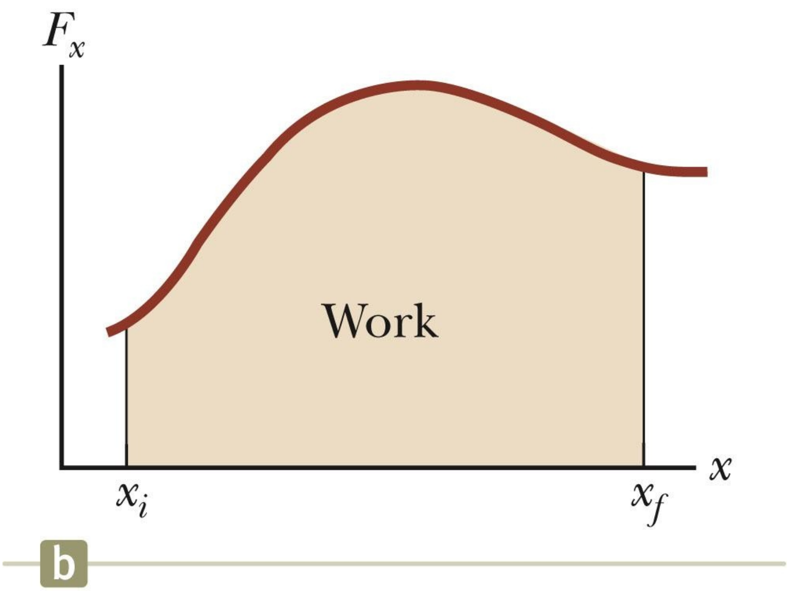
$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

Therefore

$$W = \int_{x_i}^{x_f} F_x dx$$

- The work done is equal to the area under the curve between x_i and x_f

The work done by the component F_x of the varying force as the particle moves from x_i to x_f is *exactly* equal to the area under the curve.



Work Done By Multiple Forces

If more than one force acts on a system and the system can be modeled as a particle, the total work done on the system is the work done by the net force.

$$\sum W = W_{ext} = \int_{x_i}^{x_f} \left(\sum F_x\right) dx$$

In the general case of a net force whose magnitude and direction may vary.

$$\sum W = W_{ext} = \int_{x_i}^{x_f} \left(\sum F\right) dx$$

The subscript "ext" indicates the work is done by an external agent on the system.

Work Done by Multiple Forces, cont.

If the system cannot be modeled as a particle, then the total work is equal to the algebraic sum of the work done by the individual forces.

$$\sum W = W_{ext} = \sum_{Force} \int F.dr \qquad (deformable\ system)$$

- Remember work is a scalar, so this is the algebraic sum.

Example 7.4 Calculating Total Work Done from a Graph

A force acting on a particle varies with x as shown in Figure 7.8. Calculate the work done by the force on the particle as it moves from $x = 0$ to $x = 6.0$ m.

SOLUTION

Conceptualize Imagine a particle subject to the force in Figure 7.8. The force remains constant as the particle moves through the first 4.0 m and then decreases linearly to zero at 6.0 m. In terms of earlier discussions of motion, the particle could be modeled as a particle under constant acceleration for the first 4.0 m because the force is constant. Between 4.0 m and 6.0 m, however, the motion does not fit into one of our earlier analysis models because the acceleration of the particle is changing. If the particle starts from rest, its speed increases throughout the motion, and the particle is always moving in the positive x direction. These details about its speed and direction are not necessary for the calculation of the work done, however.

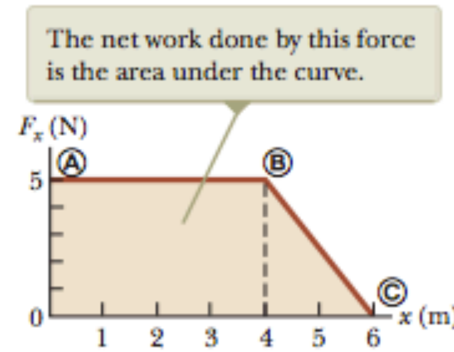


Figure 7.8 (Example 7.4) The force acting on a particle is constant for the first 4.0 m of motion and then decreases linearly with x from $x_{Ⓑ} = 4.0$ m to $x_{Ⓒ} = 6.0$ m.

Categorize Because the force varies during the motion of the particle, we must use the techniques for work done by varying forces. In this case, the graphical representation in Figure 7.8 can be used to evaluate the work done.

7.4 continued

Analyze The work done by the force is equal to the area under the curve from $x_{Ⓐ} = 0$ to $x_{Ⓒ} = 6.0$ m. This area is equal to the area of the rectangular section from Ⓐ to Ⓑ plus the area of the triangular section from Ⓑ to Ⓒ.

Evaluate the area of the rectangle: $W_{Ⓐ \text{ to } Ⓑ} = (5.0 \text{ N})(4.0 \text{ m}) = 20 \text{ J}$

Evaluate the area of the triangle: $W_{Ⓑ \text{ to } Ⓒ} = \frac{1}{2}(5.0 \text{ N})(2.0 \text{ m}) = 5.0 \text{ J}$

Find the total work done by the force on the particle: $W_{Ⓐ \text{ to } Ⓒ} = W_{Ⓐ \text{ to } Ⓑ} + W_{Ⓑ \text{ to } Ⓒ} = 20 \text{ J} + 5.0 \text{ J} = 25 \text{ J}$

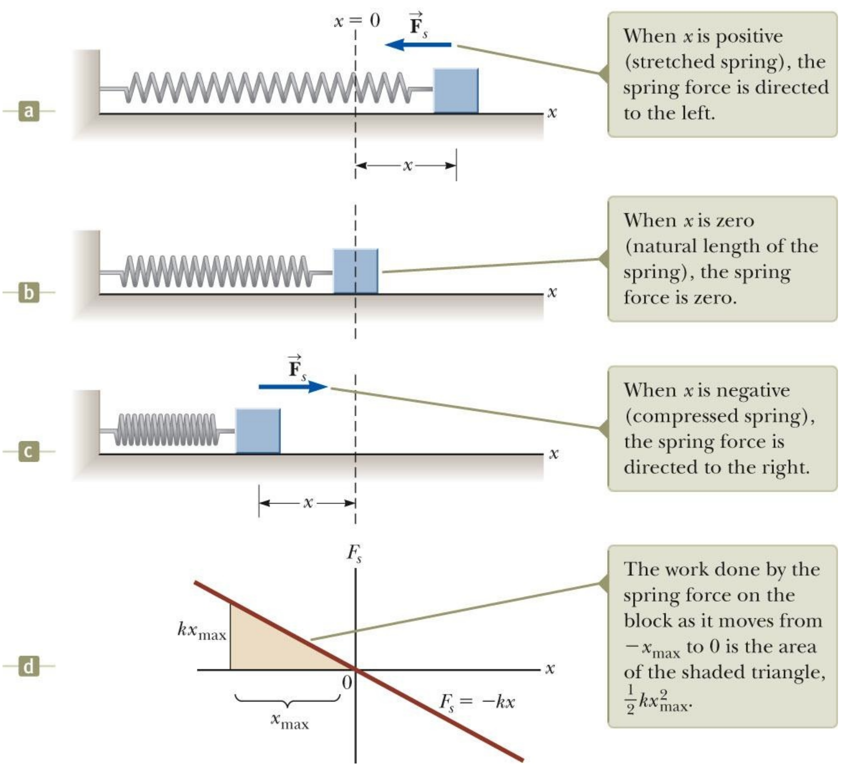
Finalize Because the graph of the force consists of straight lines, we can use rules for finding the areas of simple geometric models to evaluate the total work done in this example. If a force does not vary linearly as in Figure 7.7, such rules cannot be used and the force function must be integrated as in Equation 7.7 or 7.8.

Work Done By A Spring

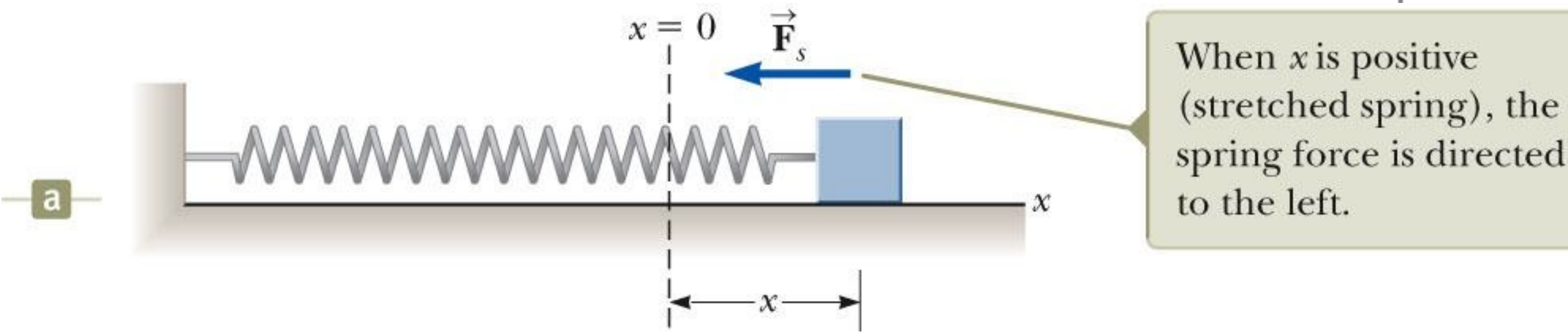
A model of a common physical system for which the force varies with position.

The block is on a horizontal, frictionless surface.

Observe the motion of the block with various values of the spring constant.



Spring Force (Hooke's Law)

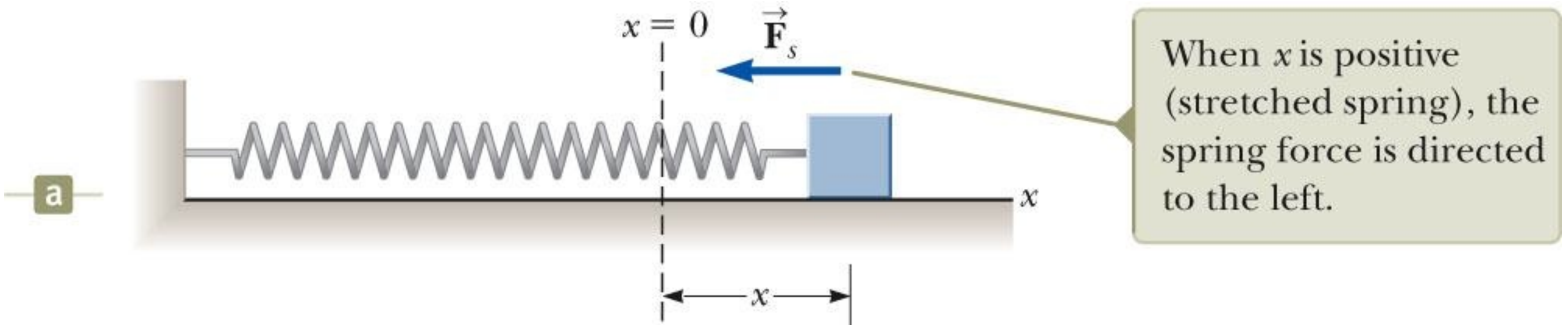


The force exerted by the spring is $F_s = -kx$

- x is the position of the block with respect to the equilibrium position (x = 0).
- k is called the spring constant or force constant
- k measures the stiffness of the spring.

This is called Hooke's Law.

Hooke's Law, cont.



The vector form of Hooke's Law is

$$\mathbf{F}_s = F_x \mathbf{i} = -kx\,\mathbf{i}$$

- When x is positive (spring is stretched), F is negative
- When x is 0 (at the equilibrium position), F is 0
- When x is negative (spring is compressed), F is positive

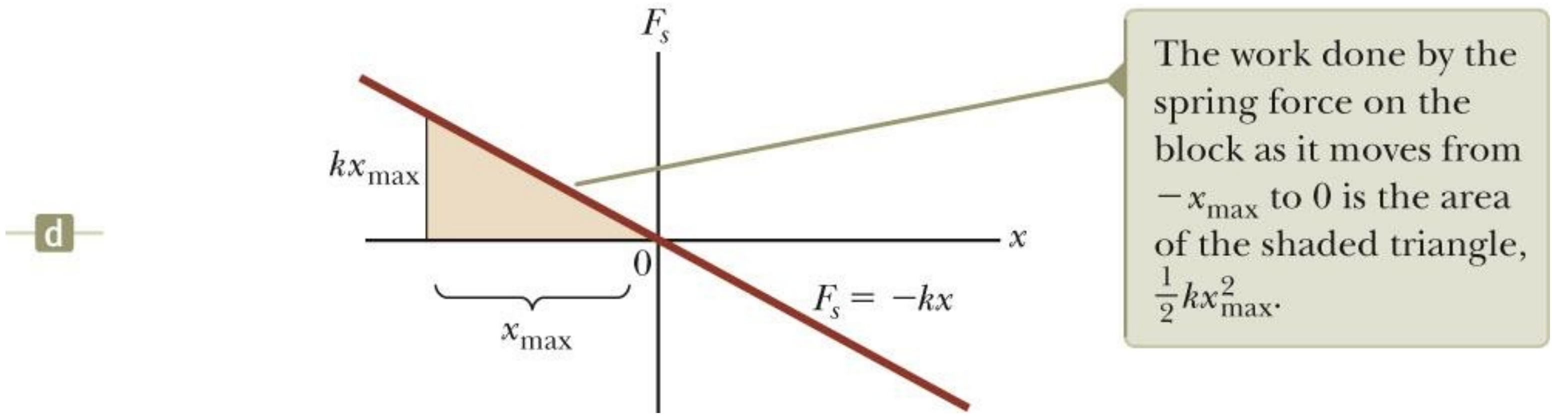
Hooke's Law, final

The force exerted by the spring is always directed opposite to the displacement from equilibrium.

The spring force is sometimes called the restoring force.

If the block is released it will oscillate back and forth between $-x$ and x.

Work Done by a Spring



Identify the block as the system.

Calculate the work as the block moves from $x_i = -x_{max}$ to $x_f = 0$.

$$W_s = \int F_s . dr = \int_{x_i}^{x_f} (-kxi)(dxi)$$

$$\int_{-x_{max}}^{0} (-kx)dx = \frac{1}{2} k\, x^2{}_{max}$$

The net work done as the block moves from $-x_{max}$ to x_{max} is zero

Work Done by a Spring, cont.

Assume the block undergoes an arbitrary displacement from $x = x_i$ to $x = x_f$.

The work done(spring is stretched) by the spring on the block is

$$W_s = \int_{x_i}^{x_f} (-kx)dx = \frac{1}{2}k\,x^2{}_i - \frac{1}{2}kx^2{}_f$$

- If the motion ends where it begins, W = 0

Spring with an Applied Force

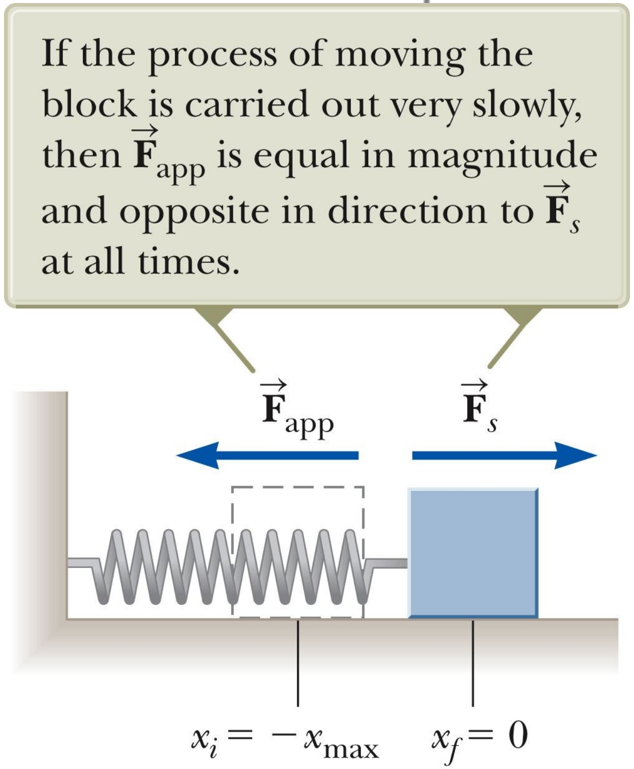
Suppose an external agent, F_{app}, stretches the spring.

The applied force is equal and opposite to the spring force.

$F_{app}=F_{app}i=-F_s=-(-kxi)=kxi$

Work done by F_{app} as the block moves from $-x_{max}$ to $x = 0$ is equal to $-\frac{1}{2} kx^2_{max}$

For any displacement, the work done (spring is compressed)by the applied force is



$$W_s = \int_{x_i}^{x_f} (kx)dx = \frac{1}{2}k\,x^2{}_f - \frac{1}{2}kx^2{}_i$$

Example 7.5 Measuring k for a Spring AM

A common technique used to measure the force constant of a spring is demonstrated by the setup in Figure 7.11. The spring is hung vertically (Fig. 7.11a), and an object of mass m is attached to its lower end. Under the action of the "load" mg, the spring stretches a distance d from its equilibrium position (Fig. 7.11b).

(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?

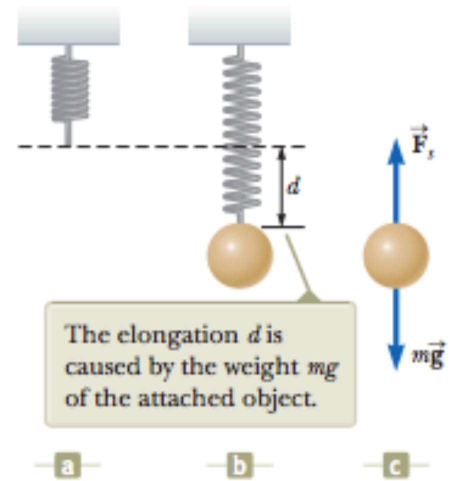


Figure 7.11 (Example 7.5) Determining the force constant k of a spring.

SOLUTION

Conceptualize Figure 7.11b shows what happens to the spring when the object is attached to it. Simulate this situation by hanging an object on a rubber band.

Categorize The object in Figure 7.11b is at rest and not accelerating, so it is modeled as a *particle in equilibrium*.

Analyze Because the object is in equilibrium, the net force on it is zero and the upward spring force balances the downward gravitational force $m\vec{g}$ (Fig. 7.11c).

Apply the particle in equilibrium model to the object:

$$\vec{F}_s + m\vec{g} = 0 \quad \rightarrow \quad F_s - mg = 0 \quad \rightarrow \quad F_s = mg$$

Apply Hooke's law to give $F_s = kd$ and solve for k:

$$k = \frac{mg}{d} = \frac{(0.55\text{ kg})(9.80\text{ m/s}^2)}{2.0 \times 10^{-2}\text{ m}} = \boxed{2.7 \times 10^2\text{ N/m}}$$

(B) How much work is done by the spring on the object as it stretches through this distance?

SOLUTION

Use Equation 7.12 to find the work done by the spring on the object:

$$W_s = 0 - \tfrac{1}{2}kd^2 = -\tfrac{1}{2}(2.7 \times 10^2\text{ N/m})(2.0 \times 10^{-2}\text{ m})^2$$
$$= \boxed{-5.4 \times 10^{-2}\text{ J}}$$

Finalize This work is negative because the spring force acts upward on the object, but its point of application (where the spring attaches to the object) moves downward. As the object moves through the 2.0-cm distance, the gravitational force also does work on it. This work is positive because the gravitational force is downward and so is the displacement of the point of application of this force. Would we expect the work done by the gravitational force, as the applied force in a direction opposite to the spring force, to be the negative of the answer above? Let's find out.

Evaluate the work done by the gravitational force on the object:

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = (mg)(d)\cos 0 = mgd$$

$$= (0.55\ \text{kg})(9.80\ \text{m/s}^2)(2.0 \times 10^{-2}\ \text{m}) = 1.1 \times 10^{-1}\ \text{J}$$

If you expected the work done by gravity simply to be that done by the spring with a positive sign, you may be surprised by this result! To understand why that is not the case, we need to explore further, as we do in the next section.

Kinetic Energy

One possible result of work acting as an influence on a system is that the system changes its speed.

The system could possess kinetic energy.

Kinetic Energy is the energy of a particle due to its motion.

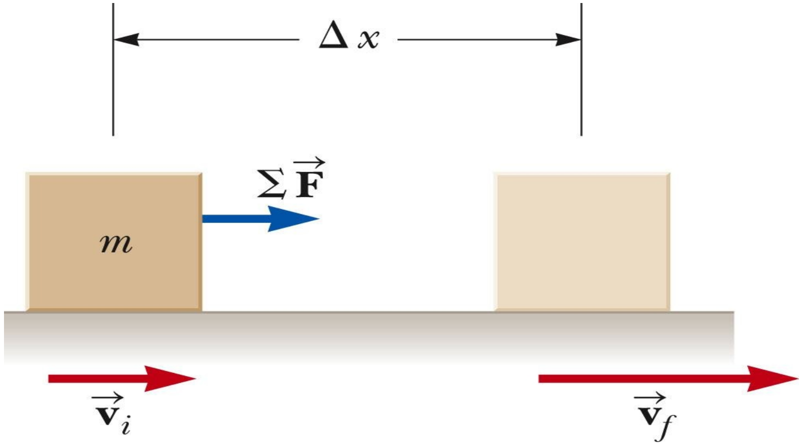
K = ½ mv2

- K is the kinetic energy
- m is the mass of the particle
- v is the speed of the particle

A change in kinetic energy is one possible result of doing work to transfer energy into a system. **$\Sigma W = \Delta K = k_f - k_i$**

Kinetic Energy, cont

Calculating the work:



$$W_{ext} = \int F.dx = \int_{x_i}^{x_f} ma\,dx$$

$$W_{ext} = \int_{x_i}^{x_f} m\frac{dv}{dt}\,dx = \int_{x_i}^{x_f} m\frac{dx}{dt}\,dv = \int_{v_i}^{v_f} mv\,dv$$

$$W_{ext} = \frac{1}{2}m{v_f}^2 - \frac{1}{2}m{v_i}^2$$

$$W_{ext} = K_f - K_i = \Delta K$$

Work-Kinetic Energy Theorem

The Work-Kinetic Energy Theorem states $W_{ext} = K_f - K_i = \Delta K$

When work is done on a system and the only change in the system is in its speed, the net work done on the system equals the change in kinetic energy of the system.

- The speed of the system increases if the work done on it is positive.
- The speed of the system decreases if the net work is negative.
- Also valid for changes in rotational speed

The work-kinetic energy theorem is not valid if other changes (besides its speed) occur in the system or if there are other interactions with the environment besides work.

The work-kinetic energy theorem applies to the speed of the system, not its velocity.

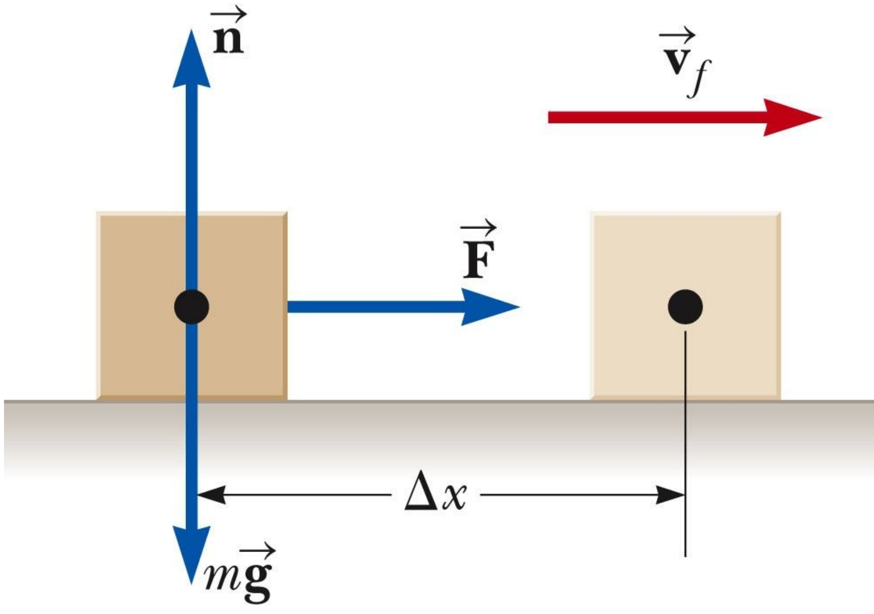
Work-Kinetic Energy Theorem - Example

The block is the system and three external forces act on it.

The normal and gravitational forces do no work since they are perpendicular to the direction of the displacement.

$W_{ext} = \Delta K = ½ \, mv_f^2 - 0$

The answer could be checked by modeling the block as a particle and using the kinematic equations.



Example 7.6 A Block Pulled on a Frictionless Surface AM

A 6.0-kg block initially at rest is pulled to the right along a frictionless, horizontal surface by a constant horizontal force of magnitude 12 N. Find the block's speed after it has moved through a horizontal distance of 3.0 m.

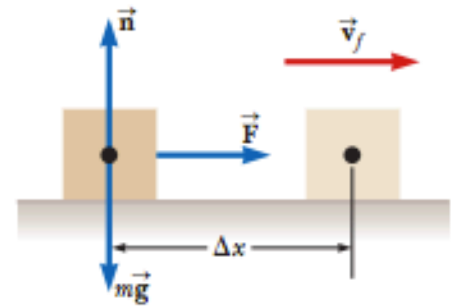


Figure 7.13 (Example 7.6) A block pulled to the right on a frictionless surface by a constant horizontal force.

SOLUTION

Conceptualize Figure 7.13 illustrates this situation. Imagine pulling a toy car across a table with a horizontal rubber band attached to the front of the car. The force is maintained constant by ensuring that the stretched rubber band always has the same length.

Categorize We could apply the equations of kinematics to determine the answer, but let us practice the energy approach. The block is the system, and three external forces act on the system. The normal force balances the gravitational force on the block, and neither of these vertically acting forces does work on the block because their points of application are horizontally displaced.

Analyze The net external force acting on the block is the horizontal 12-N force.

Use the work–kinetic energy theorem for the block, noting that its initial kinetic energy is zero:

$$W_{ext} = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \frac{1}{2}mv_f^2$$

Solve for v_f and use Equation 7.1 for the work done on the block by $\vec{F}$:

$$v_f = \sqrt{\frac{2W_{ext}}{m}} = \sqrt{\frac{2F\,\Delta x}{m}}$$

Substitute numerical values:

$$v_f = \sqrt{\frac{2(12\text{ N})(3.0\text{ m})}{6.0\text{ kg}}} = 3.5\text{ m/s}$$

Finalize You should solve this problem again by modeling the block as a *particle under a net force* to find its acceleration and then as a *particle under constant acceleration* to find its final velocity. In Chapter 8, we will see that the energy procedure followed above is an example of the analysis model of the *nonisolated system*.

WHAT IF? Suppose the magnitude of the force in this example is doubled to $F' = 2F$. The 6.0-kg block accelerates to 3.5 m/s due to this applied force while moving through a displacement $\Delta x'$. How does the displacement $\Delta x'$ compare with the original displacement Δx?

Conceptual Example 7.7 Does the Ramp Lessen the Work Required?

A man wishes to load a refrigerator onto a truck using a ramp at angle θ as shown in Figure 7.14. He claims that less work would be required to load the truck if the length L of the ramp were increased. Is his claim valid?

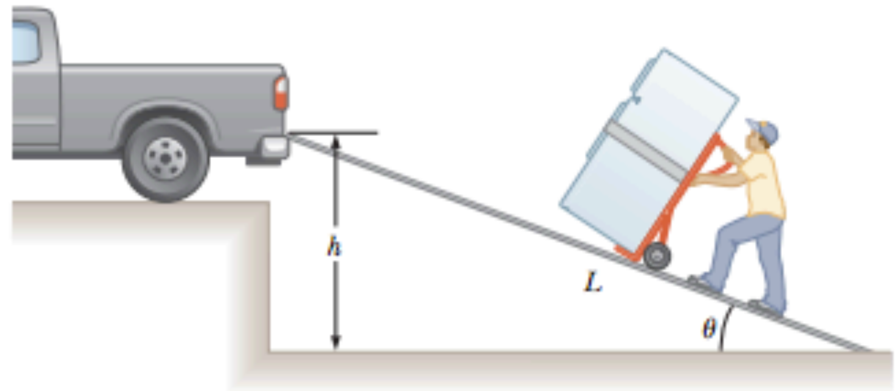


Figure 7.14 (Conceptual Example 7.7) A refrigerator attached to a frictionless, wheeled hand truck is moved up a ramp at constant speed.

SOLUTION

No. Suppose the refrigerator is wheeled on a hand truck up the ramp at constant speed. In this case, for the system of the refrigerator and the hand truck, $\Delta K = 0$. The normal force exerted by the ramp on the system is directed at 90° to the displacement of its point of application and so does no work on the system. Because $\Delta K = 0$, the work–kinetic energy theorem gives

$$W_{\text{ext}} = W_{\text{by man}} + W_{\text{by gravity}} = 0$$

The work done by the gravitational force equals the product of the weight mg of the system, the distance L through which the refrigerator is displaced, and $\cos(\theta + 90°)$. Therefore,

$$W_{\text{by man}} = -W_{\text{by gravity}} = -(mg)(L)[\cos(\theta + 90°)]$$
$$= mgL \sin\theta = mgh$$

where $h = L \sin\theta$ is the height of the ramp. Therefore, the man must do the same amount of work mgh on the system *regardless* of the length of the ramp. The work depends only on the height of the ramp. Although less force is required with a longer ramp, the point of application of that force moves through a greater displacement.

Potential Energy

Potential energy is energy determined by the configuration of a system in which the components of the system interact by forces.

- The forces are internal to the system.
- Can be associated with only specific types of forces acting between members of a system

Gravitational Potential Energy

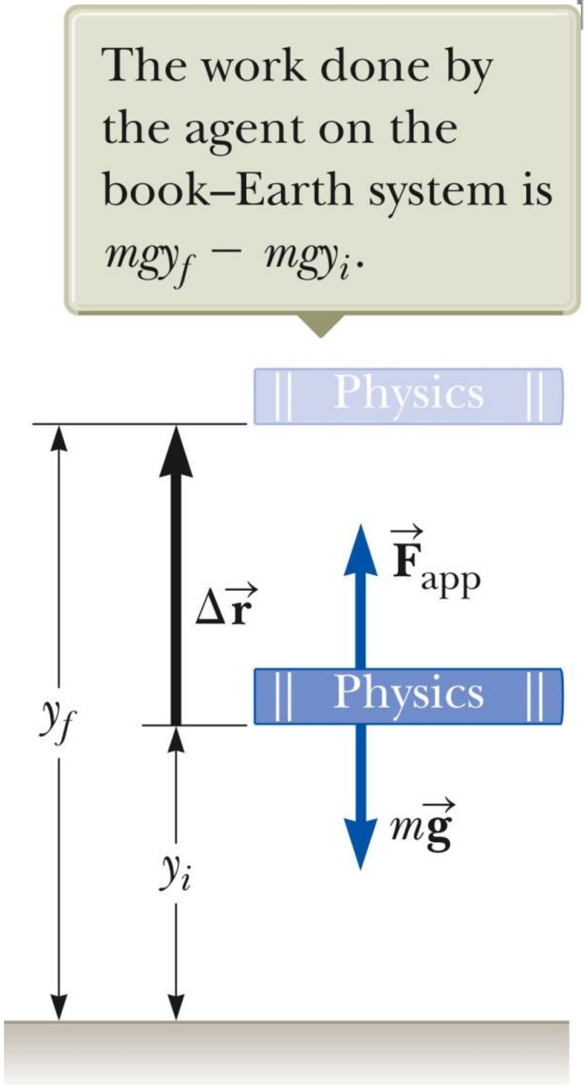
The system is the Earth and the book.

Do work on the book by lifting it slowly through a vertical displacement.

$\Delta r=(y_f-y_i)j$

The work done on the system must appear as an increase in the energy of the system.

The energy storage mechanism is called potential energy.



Gravitational Potential Energy, cont

Assume the book in fig. 7.15 is allowed to fall.

There is no change in kinetic energy since the book starts and ends at rest.

Gravitational potential energy is the energy associated with an object at a given location above the surface of the Earth.

$$W_{ext} = (F_{app}).\Delta r$$
$$W_{ext} = (mg)j.(y_f - y_i)j$$
$$W_{ext} = (mgy_f - mgy_i)$$

Gravitational Potential Energy, final

The quantity mgy is identified as the gravitational potential energy, U_g.

- $U_g = mgy$
- Units are joules (J)
- Is a scalar

Work may change the gravitational potential energy of the system.

- $W_{ext} = \Delta U_g$

Potential energy is always associated with a system of two or more interacting objects.

Gravitational Potential Energy, Problem Solving

- The gravitational potential energy depends only on the vertical height of the object above Earth's surface.
- In solving problems, you must choose a reference configuration for which the gravitational potential energy is set equal to some reference value, normally zero.

Example 7.8 The Proud Athlete and the Sore Toe

A trophy being shown off by a careless athlete slips from the athlete's hands and drops on his foot. Choosing floor level as the $y = 0$ point of your coordinate system, estimate the change in gravitational potential energy of the trophy–Earth system as the trophy falls. Repeat the calculation, using the top of the athlete's head as the origin of coordinates.

SOLUTION

Conceptualize The trophy changes its vertical position with respect to the surface of the Earth. Associated with this change in position is a change in the gravitational potential energy of the trophy–Earth system.

Categorize We evaluate a change in gravitational potential energy defined in this section, so we categorize this example as a substitution problem. Because there are no numbers provided in the problem statement, it is also an estimation problem.

The problem statement tells us that the reference configuration of the trophy–Earth system corresponding to zero potential energy is when the bottom of the trophy is at the floor. To find the change in potential energy for the system, we need to estimate a few values. Let's say the trophy has a mass of approximately 2 kg, and the top of a person's foot is about 0.05 m above the floor. Also, let's assume the trophy falls from a height of 1.4 m.

Calculate the gravitational potential energy of the trophy–Earth system just before the trophy is released:

$$U_i = mgy_i = (2\text{ kg})(9.80\text{ m/s}^2)(1.4\text{ m}) = 27.4\text{ J}$$

Calculate the gravitational potential energy of the trophy–Earth system when the trophy reaches the athlete's foot:

$$U_f = mgy_f = (2\text{ kg})(9.80\text{ m/s}^2)(0.05\text{ m}) = 0.98\text{ J}$$

Evaluate the change in gravitational potential energy of the trophy–Earth system:

$$\Delta U_g = 0.98\text{ J} - 27.4\text{ J} = -26.4\text{ J}$$

7.8 continued

We should probably keep only two digits because of the roughness of our estimates; therefore, we estimate that the change in gravitational potential energy is $-26\ \text{J}$. The system had about 27 J of gravitational potential energy before the trophy began its fall and approximately 1 J of potential energy as the trophy reaches the top of the foot.

The second case presented indicates that the reference configuration of the system for zero potential energy is chosen to be when the trophy is on the athlete's head (even though the trophy is never at this position in its motion). We estimate this position to be 2.0 m above the floor).

Calculate the gravitational potential energy of the trophy–Earth system just before the trophy is released from its position 0.6 m below the athlete's head:

$$U_i = mgy_i = (2\ \text{kg})(9.80\ \text{m/s}^2)(-0.6\ \text{m}) = -11.8\ \text{J}$$

Calculate the gravitational potential energy of the trophy–Earth system when the trophy reaches the athlete's foot located 1.95 m below its initial position:

$$U_f = mgy_f = (2\ \text{kg})(9.80\ \text{m/s}^2)(-1.95\ \text{m}) = -38.2\ \text{J}$$

Evaluate the change in gravitational potential energy of the trophy–Earth system:

$$\Delta U_g = -38.2\ \text{J} - (-11.8\ \text{J}) = -26.4\ \text{J} \approx -26\ \text{J}$$

This value is the same as before, as it must be. The change in potential energy is independent of the choice of configuration of the system representing the zero of potential energy. If we wanted to keep only one digit in our estimates, we could write the final result as 3×10^1 J.

Elastic Potential Energy

Elastic Potential Energy is associated with a spring.

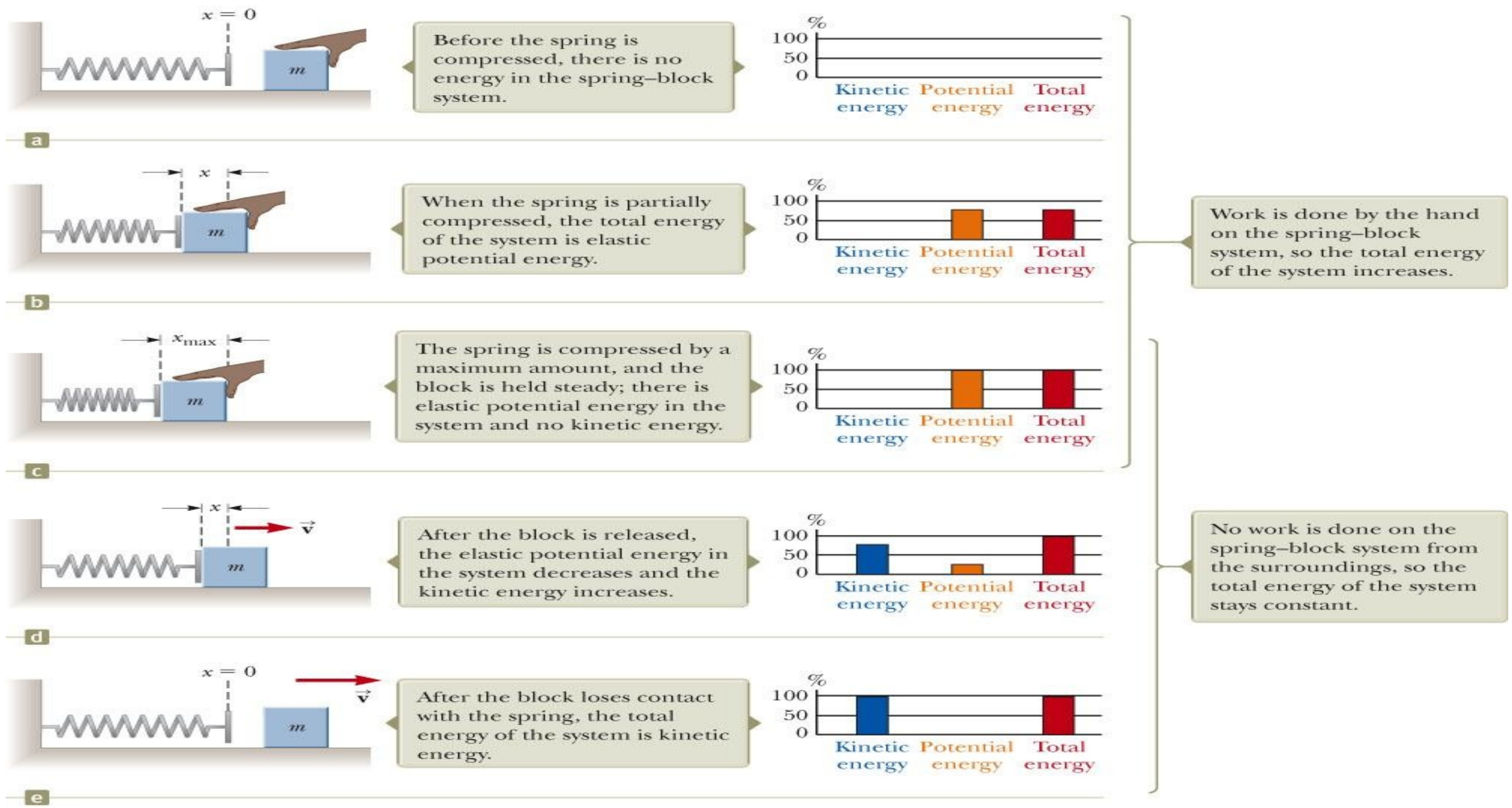
The force the spring exerts (on a block, for example) is $F_s = - kx$

The work done by an external applied force on a spring-block system is

- $W = ½\ kx_f – ½\ kx_i$

- The work is equal to the difference between the initial and final values of an expression related to the configuration of the system.

Elastic Potential Energy, cont.



This expression is the elastic potential energy:

$U_s = \frac{1}{2} kx^2$

The elastic potential energy can be thought of as the energy stored in the deformed spring.

The stored potential energy can be converted into kinetic energy.

Observe the effects of different amounts of compression of the spring.

Elastic Potential Energy, final

The elastic potential energy stored in a spring is zero whenever the spring is not deformed ($U = 0$ when $x = 0$).

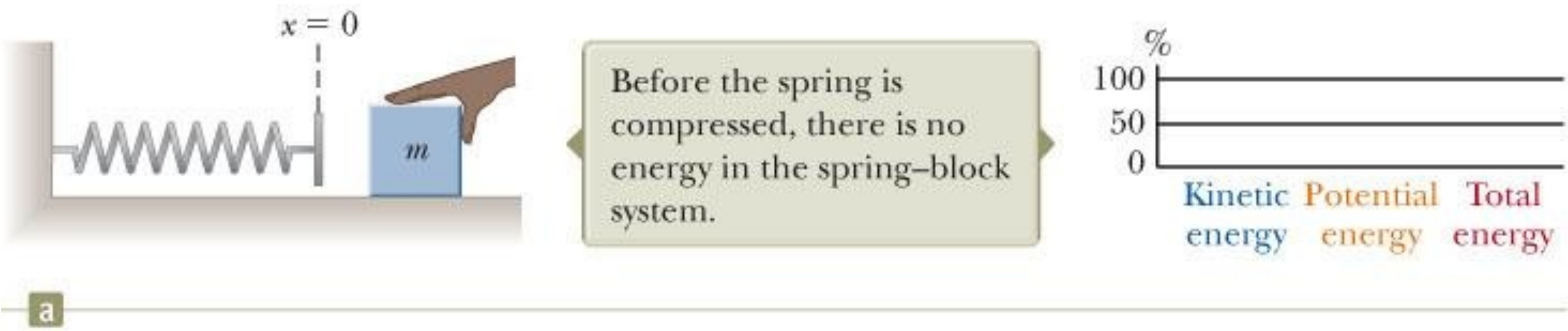
- The energy is stored in the spring only when the spring is stretched or compressed.

The elastic potential energy is a maximum when the spring has reached its maximum extension or compression.

The elastic potential energy is always positive.

- x^2 will always be positive.

Energy Bar Chart Example



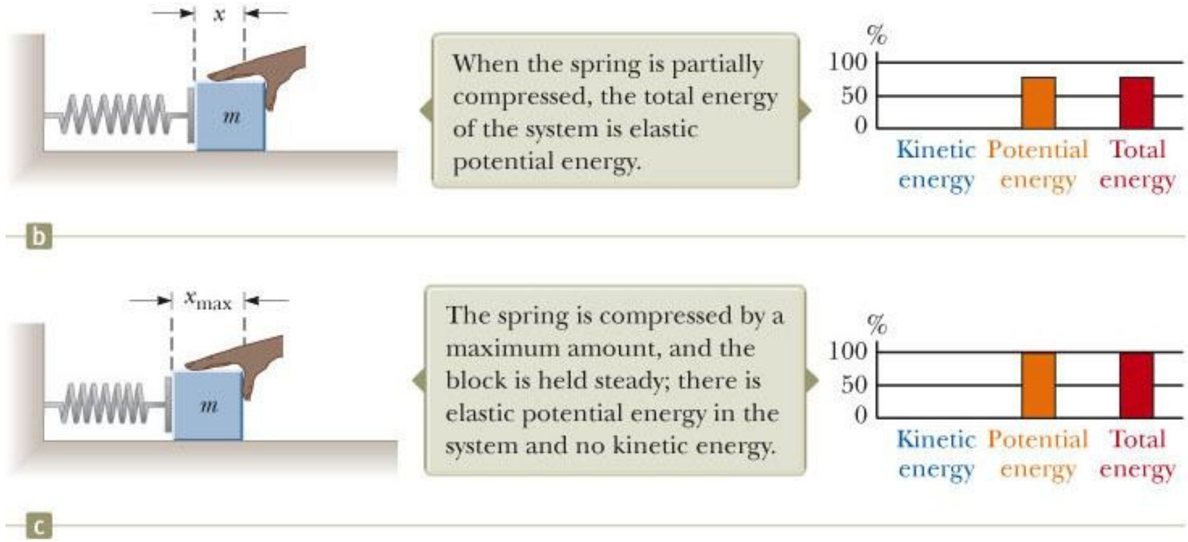
An energy bar chart is an important graphical representation of information related to the energy of a system.

- The vertical axis represents the amount of energy of a given type in the system.
- The horizontal axis shows the types of energy in the system.

In a, there is no energy.

- The spring is relaxed, the block is not moving.

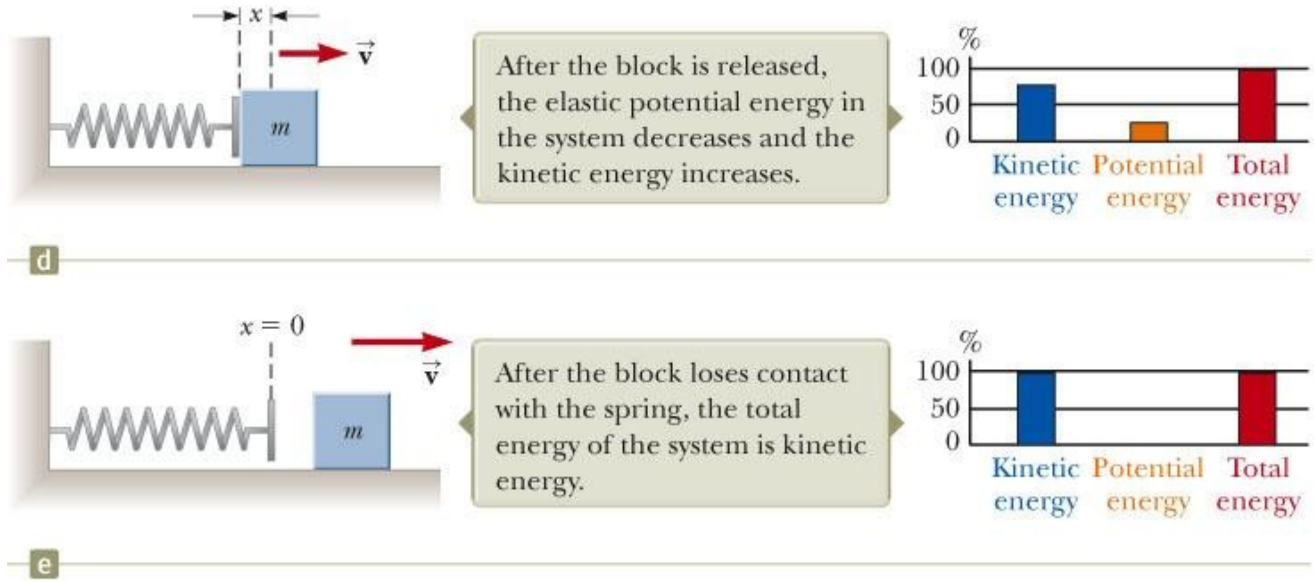
Energy Bar Chart Example, cont.



Between b and c, the hand has done work on the system.

- The spring is compressed.
- There is elastic potential energy in the system.
- There is no kinetic energy since the block is held steady.

Energy Bar Chart Example, final



In d, the block has been released and is moving to the right while still in contact with the spring.

- The elastic potential energy of the system decreases while the kinetic energy increases.

In e, the spring has returned to its relaxed length and the system contains only kinetic energy associated with the moving block.

Internal Energy

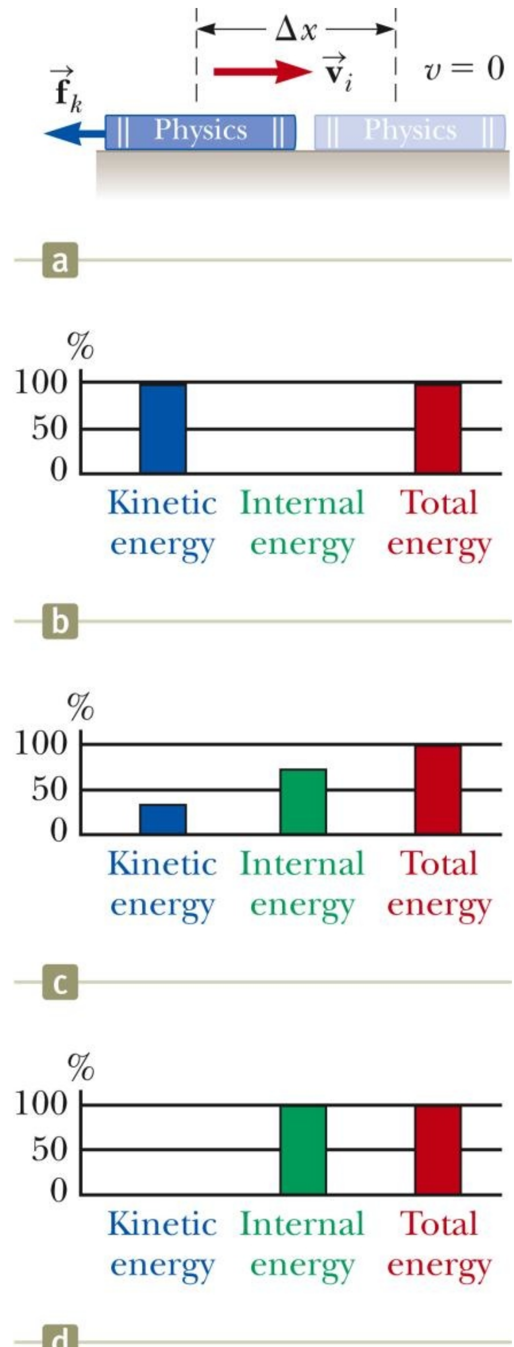
The energy associated with an object's temperature is called its *internal energy*, E_{int}.

In this example, the surface is the system.

The friction does work and increases the internal energy of the surface.

When the book stops, all of its kinetic energy has been transformed to internal energy.

The total energy remains the same.



Conservative Forces

The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.

The work done by a conservative force on a particle moving through any closed path is zero.

- A closed path is one in which the beginning and ending points are the same.

Examples of conservative forces:

- Gravity(W_g is zero when the object moves over any closed path (where yi= yf))

- Spring force(W_s depends only on the initial and final x coordinates of the object and is zero for any closed path)

Conservative Forces

We can associate a potential energy for a system with any conservative force acting between members of the system.

- This can be done only for conservative forces.
- In general: $W_{int} = - \Delta U$
- W_{int} is used as a reminder that the work is done by one member of the system on another member and is internal to the system.
- Positive work done by an outside agent on a system causes an increase in the potential energy of the system.
- Work done on a component of a system by a conservative force internal to an isolated system causes a decrease in the potential energy of the system.

Non-conservative Forces

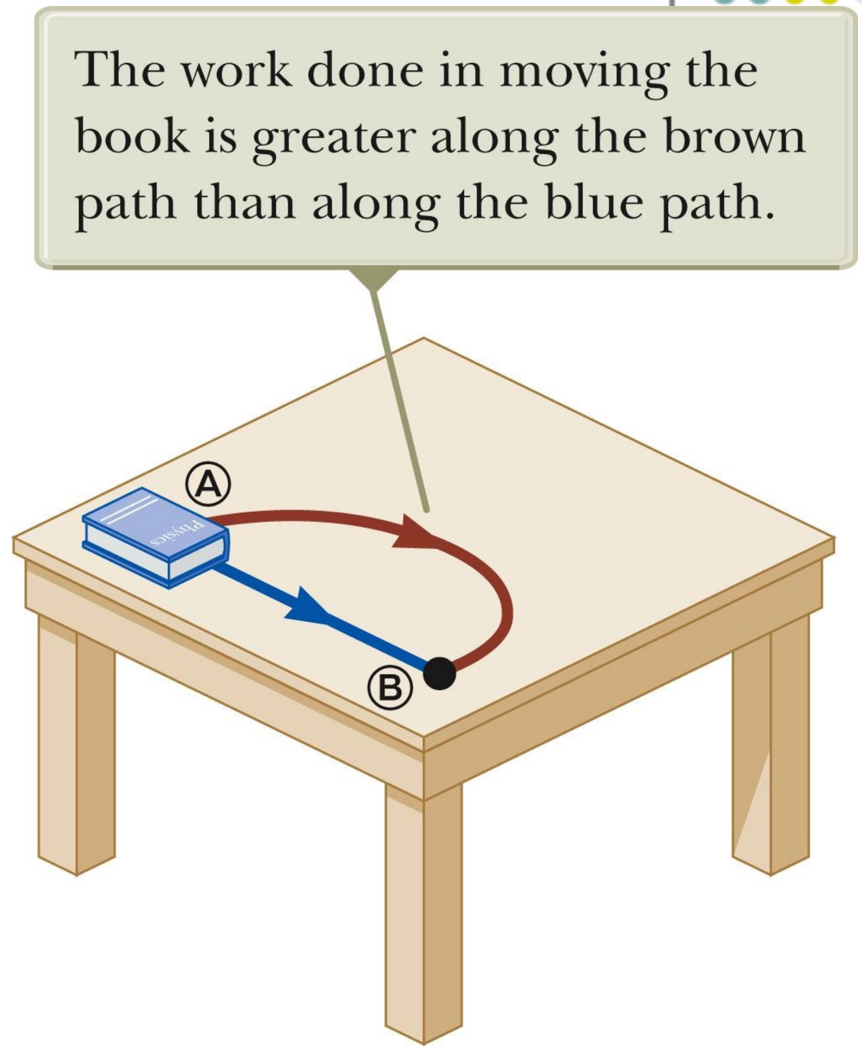
A non-conservative force does not satisfy the conditions of conservative forces.

Non-conservative forces acting in a system cause a *change* in the mechanical energy of the system.

$E_{mech} = K + U$

- K includes the kinetic energy of all moving members of the system.
- U includes all types of potential energy in the system.

Non-conservative Forces, cont



Non-conservative Forces, cont.

The work done against friction is greater along the brown path than along the blue path.

Because the work done depends on the path, friction is a non-conservative force.

Conservative Forces and Potential Energy

Define a potential energy function, U, such that the work done by a conservative force equals the decrease in the potential energy of the system.

The work done by such a force, F, is

$$W_{int} = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$

- ΔU is negative when F and x are in the same direction

Conservative Forces and Potential Energy

The conservative force is related to the potential energy function through.

$$F_x = -\frac{dU}{dx}$$

The x component of a conservative force acting on an object within a system equals the negative of the potential energy of the system with respect to x.

- Can be extended to three dimensions

Conservative Forces and Potential Energy - Check

Look at the case of a deformed spring:

$$F_s = -\frac{dU_s}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

- This is Hooke's Law and confirms the equation for U

U is an important function because a conservative force can be derived from it.

Homework

Section 7.2 Work Done by a Constant Force

1- If the net work done by external forces on a particle is zero, which of the following statements about the particle must be true? (a) Its velocity is zero. (b) Its velocity is decreased. (c) Its velocity is unchanged. (d) Its speed is unchanged. (e) There is no displacement for the object.

2- . If the speed of a particle is doubled, what happens to its kinetic energy? (a) It becomes four times larger. (b) It becomes two times larger. (c) It becomes !2 times larger. (d) It is unchanged. (e) It becomes half as large.

3- Can a normal force do work? If not, why not? If so, give an example.

4- Explain why the total energy of a system can be either positive or negative, whereas the kinetic energy is always positive.

5- A block of mass m =2.50 kg is pushed a distance d = 2.20 m along a frictionless, horizontal table by a constant applied force of magnitude F =16.0 N directed at an angle θ = 25.0 below the horizontal as shown in Figure P7.5. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, (c) the gravitational force, and (d) the net force on the block.

6- A shopper in a supermarket pushes a cart with a force of 35.0 N directed at an angle of 25.0° downward from the horizontal. Find the work done by the shopper on the cart as he moves down an aisle 50.0 m long.

Section 7.3 The Scalar Product of Two Vectors

7- Vector A has a magnitude of 5.00 units, and B has a magnitude of 9.00 units. The two vectors make an angle of 50.0° with each other. Find A.B

8- A force F = (6i − 2j) N acts on a particle that undergoes a displacement Δr =(3i + j) m. Find (a) the work done by the force on the particle and (b) the angle between F and Δr.

9- Find the scalar product of the vectors in Figure P7.10

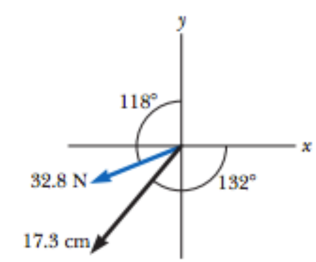


Figure P7.10

10-For any two vectors A and B , show that A . B = $A_xB_x + A_yB_y + A_zB_z$

11- . For A = 3i +j - k, B = -i + 2j + 5 k, and C = 2j - 3k,find C.(A - B)

Section 7.4 Work Done by a Varying Force

12- The force acting on a particle varies as shown in Figure P7.14. Find the work done by the force on the particle as it moves (a) from x = 0 to x = 8.00 m, (b) from x =8.00 m to x=10.0 m, and (c) from x = 0 to x=10.0 m

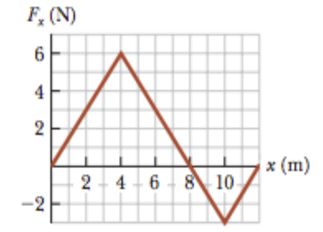


Figure P7.14

13-A particle is subject to a force F_x that varies with position as shown in Figure P7.15. Find the work done by the force on the particle as it moves (a) from x = 0 to x 5.00 m, (b) from x 5 5.00 m to x 5 10.0 m, and (c) from x 5 10.0 m to x 5 15.0 m. (d) What is the total work done by the force over the distance x =0 to x = 15.0 m?

14- A force F =(4xi +3yj) N acts on an object as the object moves in the x direction from the origin to x = 5.00 m. Find the*W=F.dr* work done on the object by the force.

15-An archer pulls her bowstring back 0.400 m by exerting a force that increases uniformly from zero to 230 N. (a) What is the equivalent spring constant of the bow? (b) How much work does the archer do in pulling the bow?

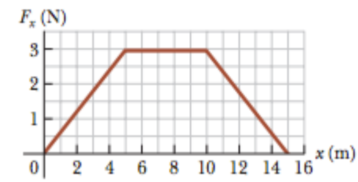


Figure P7.15 Problems 15 and 34.

Section 7.5 Kinetic Energy and the Work–Kinetic Energy Theorem

16- If it takes 4.00 J of work to stretch a Hooke's-law spring 10.0 cm from its unstressed length, determine the extra work required to stretch it an additional 10.0 cm.

17- Express the units of the force constant of a spring in SI base units.

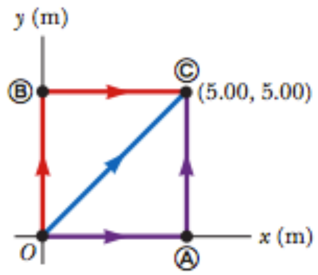
18- A 0.600-kg particle has a speed of 2.00 m/s at point A and kinetic energy of 7.50 J at point B . What is (a) its kinetic energy at A? (b) its speed at B? (c) the total work done on the particle as it moves from A to B?

19. A 0.300-kg ball has a speed of 15.0 m/s. (a) What is its kinetic energy? (b) What If? If its speed were doubled, what would be its kinetic energy?

20. A 3.00-kg object has a velocity (6.00i - 2.00j) m/s. (a) What is its kinetic energy at this time? (b) Find the total work done on the object if its velocity changes to (8.00i +4.00j) m/s. (Note: From the definition of the dot product, v^2 =v . v

Section 7.7 Conservative and Nonconservative Forces

21- A 4.00-kg particle moves from the origin to position C, having coordinates x " 5.00 m and y " 5.00 m. One force on the particle is the gravitational force acting in the negative y direction (Fig. P8.21). Using Equation 7.3, calculate the work done by the gravitational force in going from O to C along (a) OAC. (b) OBC. (c) OC. Your results should all be identical. Why?



Section 7.8 Relationship Between Conservative Forces and Potential Energy

22-. A single conservative force acting on a particle varies as $F=(-Ax+Bx^2)i$, where A and B are constants and x is in meters. (a) Calculate the potential-energy function U(x) associated with this force, taking U =0 at x = 0. (b) Find the change in potential energy and the change in kinetic energy as the particle moves from x = 2.00 m to x = 3.00 m.

23- A single conservative force acts on a 5.00-kg particle. The equation $F_x = (2x + 4)$ N describes the force, where x is in meters. As the particle moves along the x axis from x = 1.00 m to x = 5.00 m, calculate (a) the work done by this force, (b) the change in the potential energy of the system, and (c) the kinetic energy of the particle at x = 5.00 m if its speed is 3.00 m/s at x = 1.00 m.