Chapter 13

Universal Gravitation

13.1: Newton's Law of Universal Gravitation

13.2: Free-Fall Acceleration and the Gravitational Force

Planetary Motion

A large amount of data had been collected by 1687.

- There was no clear understanding of the forces related to these motions.
- Isaac Newton provided the answer.

Newton's First Law

- A net force had to be acting on the Moon because the Moon does not move in a straight line.
- Newton reasoned the force was the gravitational attraction between the

Earth and the Moon.

Newton recognized this attraction was a special case of a general and universal attraction between objects.

Newton's Law of Universal Gravitation

Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the distance between them.

$$F_g = G\frac{m_1 m_2}{r^2}$$

G is the universal gravitational constant and equals 6.673 x 10^{-11} $N.m^2/kg^2$.

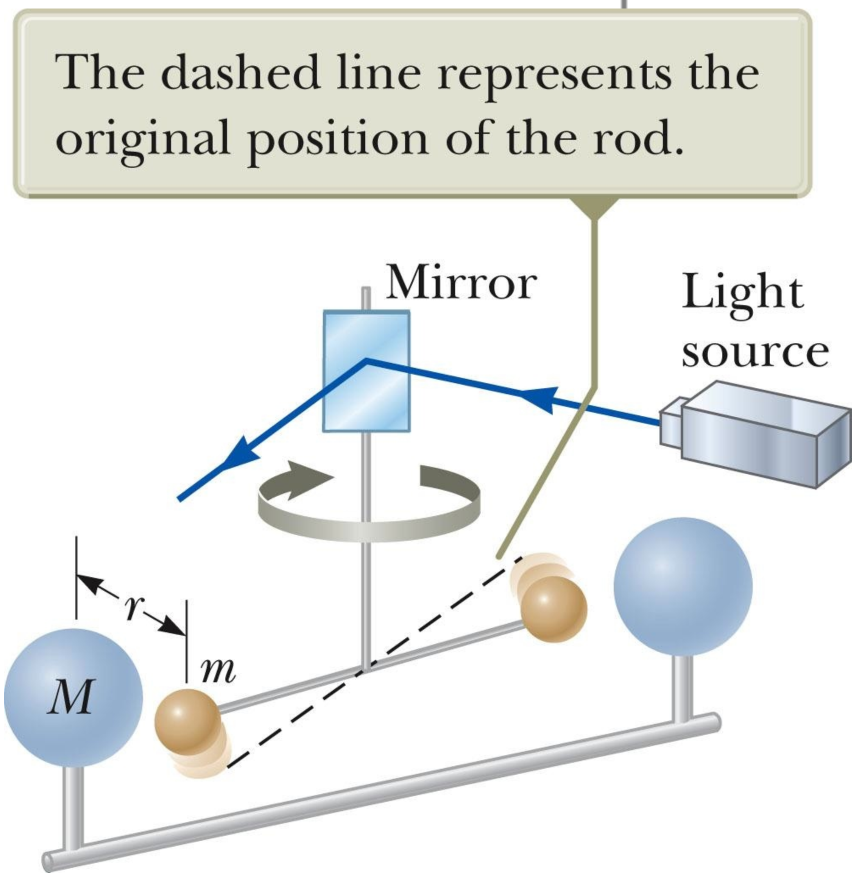
Classes of Forces

In 1789 Henry Cavendish measured G.

The two small spheres are fixed at the ends of a light horizontal rod.

Two large masses were placed near the small ones.

The angle of rotation was measured by the deflection of a light beam reflected from a mirror attached to the vertical suspension.



Law of Gravitation, cont

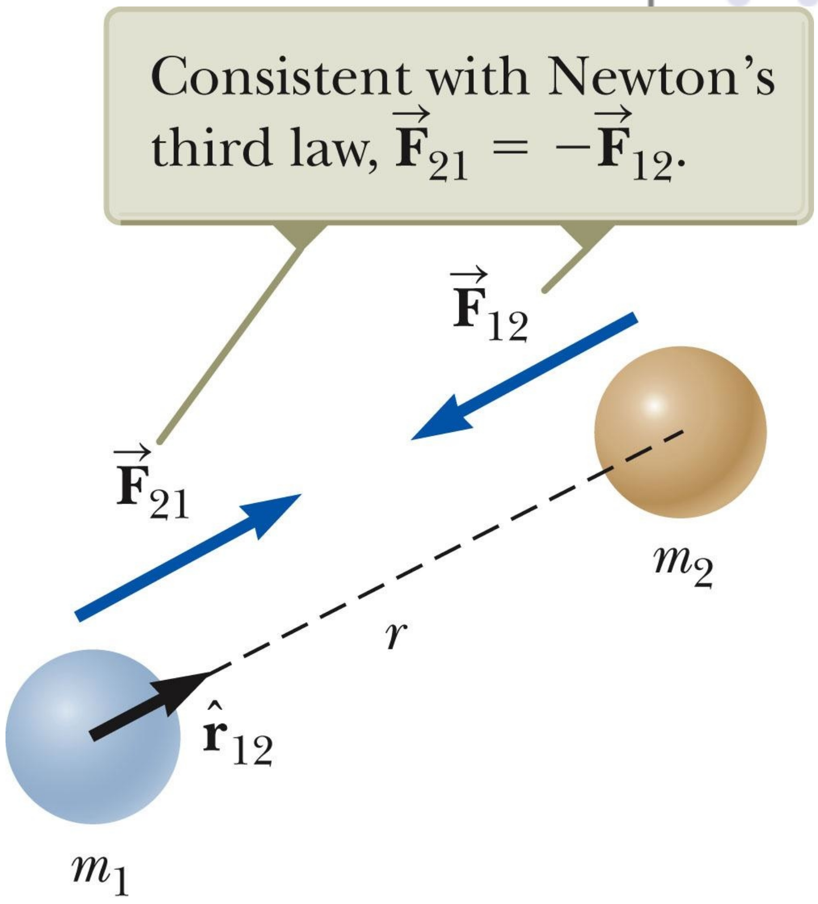
This is an example of an ***inverse square law.***

- The magnitude of the force varies as the inverse square of the separation of the particles.

The law can also be expressed in vector form

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12}$$

The negative sign indicates an attractive force.



Notation

$\mathbf{F}_{12}$ is the force exerted by particle 1 on particle 2. The negative sign in the vector form of the equation indicates that particle 2 is attracted toward particle 1.

$\mathbf{F}_{21}$ is the force exerted by particle 2 on particle 1.

$F_{12}=-F_{21}$

- The forces form a Newton's Third Law action-reaction pair.

More About Forces

Gravitation is a field force that always exists between two particles, regardless of the medium between them.

The force decreases rapidly as distance increases.

- A consequence of the inverse square law.

Gravitational Force Due to a Distribution of Mass

- The gravitational force exerted by a finite-size, spherically symmetric mass distribution on a particle outside the distribution is the same as if the entire mass of the distribution were concentrated at the center.

For example, the force exerted by the Earth on a particle of mass m near the surface of the Earth is

$$F_g = G\frac{M_E m}{R_E^2}$$

G vs. g

Always distinguish between G and g.

G is the universal gravitational constant.

- It is the same everywhere.

g is the acceleration due to gravity.

- $g = 9.80$ m/s at the surface of the Earth.
- g will vary by location.

Example 13.1 Billiards, Anyone?

Three 0.300-kg billiard balls are placed on a table at the corners of a right triangle as shown in Figure 13.3. The sides of the triangle are of lengths $a = 0.400$ m, $b = 0.300$ m, and $c = 0.500$ m. Calculate the gravitational force vector on the cue ball (designated m_1) resulting from the other two balls as well as the magnitude and direction of this force.

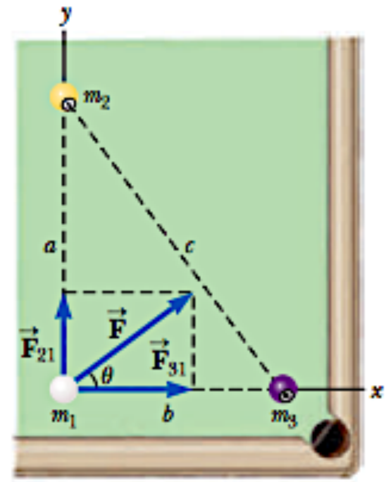


Figure 13.3 (Example 13.1) The resultant gravitational force acting on the cue ball is the vector sum $\vec{F}_{21} + \vec{F}_{31}$.

SOLUTION

Conceptualize Notice in Figure 13.3 that the cue ball is attracted to both other balls by the gravitational force. We can see graphically that the net force should point upward and toward the right. We locate our coordinate axes as shown in Figure 13.3, placing our origin at the position of the cue ball.

Categorize This problem involves evaluating the gravitational forces on the cue ball using Equation 13.3. Once these forces are evaluated, it becomes a vector addition problem to find the net force.

Analyze Find the force exerted by m_2 on the cue ball:

$$\vec{F}_{21} = G\frac{m_2 m_1}{a^2}\hat{j}$$

$$= (6.674 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(0.300\ \text{kg})(0.300\ \text{kg})}{(0.400\ \text{m})^2}\hat{j}$$

$$= 3.75 \times 10^{-11}\,\hat{j}\ \text{N}$$

Find the force exerted by m_3 on the cue ball:

$$\vec{F}_{31} = G\frac{m_3 m_1}{b^2}\hat{i}$$

$$= (6.674 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(0.300\ \text{kg})(0.300\ \text{kg})}{(0.300\ \text{m})^2}\hat{i}$$

$$= 6.67 \times 10^{-11}\,\hat{i}\ \text{N}$$

13.1 continued

Find the net gravitational force on the cue ball by adding these force vectors:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{21} = (6.67\,\hat{\mathbf{i}} + 3.75\,\hat{\mathbf{j}}) \times 10^{-11}\text{ N}$$

Find the magnitude of this force:

$$F = \sqrt{F_{31}^2 + F_{21}^2} = \sqrt{(6.67)^2 + (3.75)^2} \times 10^{-11}\text{ N}$$

$$= 7.66 \times 10^{-11}\text{ N}$$

Find the tangent of the angle θ for the net force vector:

$$\tan\theta = \frac{F_y}{F_x} = \frac{F_{21}}{F_{31}} = \frac{3.75 \times 10^{-11}\text{ N}}{6.67 \times 10^{-11}\text{ N}} = 0.562$$

Evaluate the angle θ:

$$\theta = \tan^{-1}(0.562) = 29.4^\circ$$

Finalize The result for F shows that the gravitational forces between everyday objects have extremely small magnitudes.

Free-Fall Acceleration and the Gravitational Force

Finding g from G

The magnitude of the force acting on an object of mass m in free-fall near the Earth's surface is mg.

This can be set equal to the force of universal gravitation acting on the object.

$$mg = G\frac{M_E m}{R_E^2}$$

$$g = G\frac{M_E}{R_E^2}$$

If an object is some distance h above the Earth's surface, r becomes $R + h$.

$$F_g = G\frac{M_E m}{r^2} = G\frac{M_E m}{(R_E + h)^2}$$

This shows that g decreases with increasing altitude.

As $r \rightarrow \alpha$, the weight of the object approaches zero.

Variation of *g* with Height

TABLE 13.1

Free-Fall Acceleration g at Various Altitudes Above the Earth's Surface

Altitude *h* (km)	*g* (m/s^2)
1 000	7.33
2 000	5.68
3 000	4.53
4 000	3.70
5 000	3.08
6 000	2.60
7 000	2.23
8 000	1.93
9 000	1.69
10 000	1.49
50 000	0.13
∞	0

Example 13.2 **The Density of the Earth**

Using the known radius of the Earth and that $g = 9.80\ \text{m/s}^2$ at the Earth's surface, find the average density of the Earth

SOLUTION

Conceptualize Assume the Earth is a perfect sphere. The density of material in the Earth varies, but let's adopt a simplified model in which we assume the density to be uniform throughout the Earth. The resulting density is the average density of the Earth.

Categorize This example is a relatively simple substitution problem.

Using Equation 13.5, solve for the mass of the Earth:

$$M_E = \frac{gR_E^2}{G}$$

Substitute this mass and the volume of a sphere into the definition of density (Eq. 1.1):

$$\rho_E = \frac{M_E}{V_E} = \frac{gR_E^2/G}{\frac{4}{3}\pi R_E^3} = \frac{3}{4}\frac{g}{\pi G R_E}$$

$$= \frac{3}{4}\frac{9.80\ \text{m/s}^2}{\pi(6.674\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.37\times10^6\ \text{m})} = \boxed{5.50\times10^3\ \text{kg/m}^3}$$

WHAT IF? What if you were told that a typical density of granite at the Earth's surface is $2.75 \times 10^3\ \text{kg/m}^3$? What would you conclude about the density of the material in the Earth's interior?

Answer Because this value is about half the density we calculated as an average for the entire Earth, we would conclude that the inner core of the Earth has a density much higher than the average value. It is most amazing that the Cavendish experiment—which can be used to determine G and can be done today on a tabletop—combined with simple free-fall measurements of g provides information about the core of the Earth!

Homework

1- A 200-kg object and a 500-kg object are separated by 4.00 m. (a) Find the net gravitational force exerted by these objects on a 50.0-kg object placed midway between them. (b) At what position (other than an infinitely remote one) can the 50.0-kg object be placed so as to experience a net force of zero from the other two objects?

2- During a solar eclipse, the Moon, the Earth, and the Sun all lie on the same line, with the Moon between the Earth and the Sun. (a) What force is exerted by the Sun on the Moon? (b) What force is exerted by the Earth on the Moon? (c) What force is exerted by the Sun on the Earth? (d) Compare the answers to parts (a) and (b). Why doesn't the Sun capture the Moon away from the Earth?

3- Three uniform spheres of masses m_1 = 2.00 kg, m_2 = 4.00 kg, and m_3 = 6.00 kg are placed at the corners of a right triangle as shown in Figure P13.6. Calculate the resultant gravitational force on the object of mass m_2, assuming the spheres are isolated from the rest of the Universe.

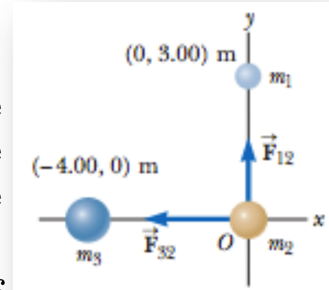


Figure P13.6

4-Two identical isolated particles, each of mass 2.00 kg, are separated by a distance of 30.0 cm. What is the magnitude of the gravitational force exerted by one particle on the other?

5- When a falling meteoroid is at a distance above the Earth's surface of 3.00 times the Earth's radius, what is its acceleration due to the Earth's gravitation?

6-. The free-fall acceleration on the surface of the Moon is about one-sixth that on the surface of the Earth. The radius of the Moon is about $0.250R_E$ (R_E = Earth's radius = 6.37×10^6 m). Find the ratio of their average densities, r_{Moon}/r_{Earth}.