Chapter 15

Oscillatory Motion

15.1 Motion of an Object Attached to a Spring

15.2 Analysis Model: Particle in Simple Harmonic Motion

15.5 The Pendulum

Oscillations and Mechanical Waves

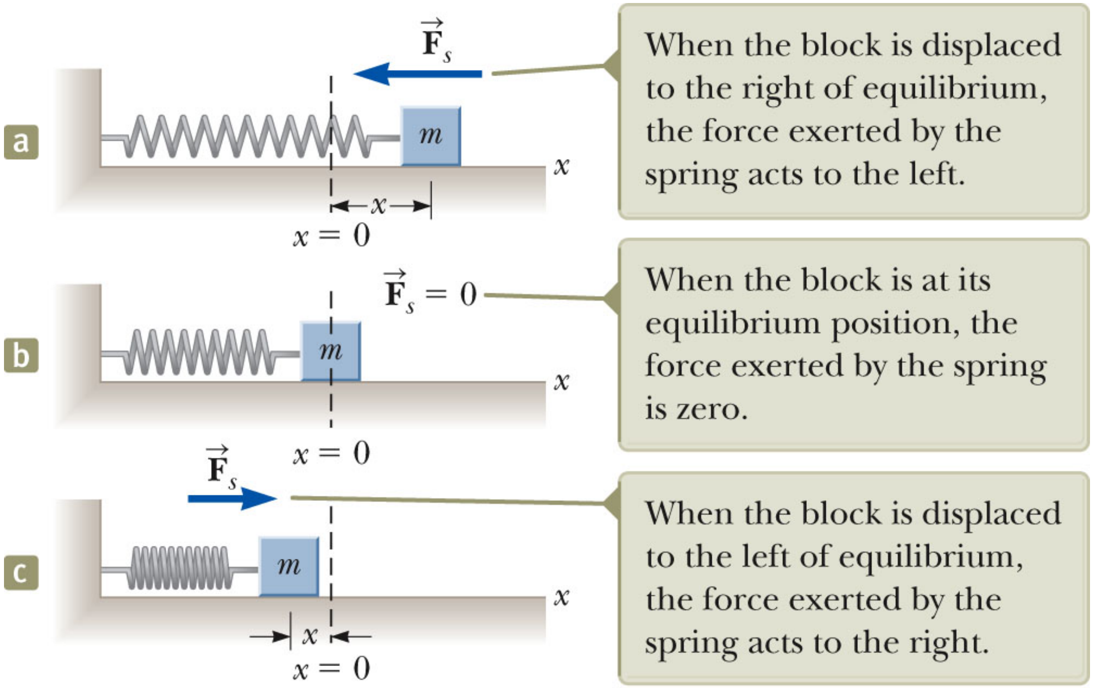
- *Periodic motion* is the repeating motion of an object in which it continues to return to a given position after a fixed time interval.
- The repetitive movements are called *oscillations.*
- A special case of periodic motion called *simple harmonic motion* will be the focus.
  - Simple harmonic motion also forms the basis for understanding mechanical waves.
- Oscillations and waves also explain many other phenomena quantity.
  - Oscillations of bridges and skyscrapers
  - Radio and television
  - Understanding atomic theory

Periodic Motion

- ***Periodic motion*** is motion of an object that regularly returns to a given position after a fixed time interval.
- A special kind of periodic motion occurs in mechanical systems when the force acting on the object is proportional to the position of the object relative to some equilibrium position.
  - If the force is always directed toward the equilibrium position, the motion is called ***simple harmonic motion.***

Motion of a Spring-Mass System

- A block of mass *m* is attached to a spring, the block is free to move on a frictionless horizontal surface.
- When the spring is neither stretched nor compressed, the block is at the ***equilibrium position.***
  - $x = 0$
- Such a system will oscillate back and forth if disturbed from its equilibrium position.



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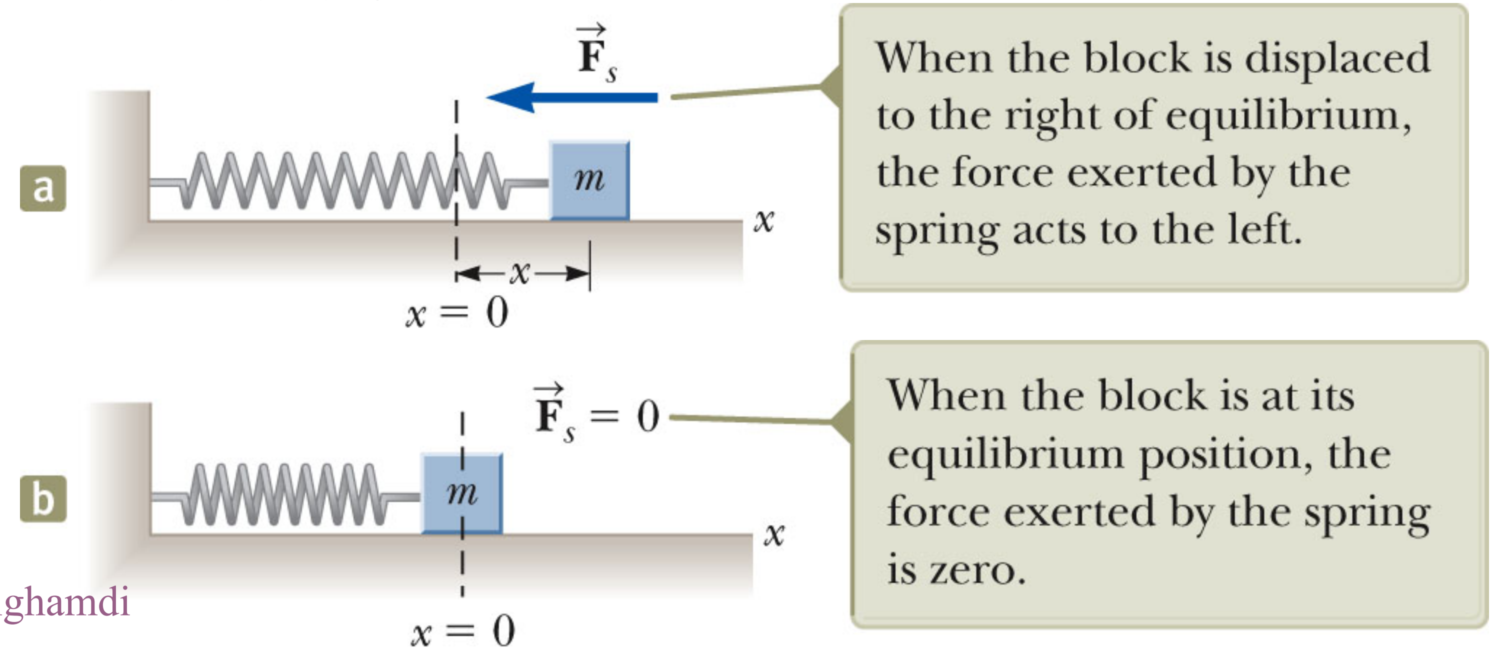
Hooke's Law

- Hooke's Law states $F_s = -kx$
  - F_s is the restoring force.
    - It is always directed toward the equilibrium position.
    - Therefore, it is always opposite the displacement from equilibrium.
  - k is the force (spring) constant.
  - x is the displacement.

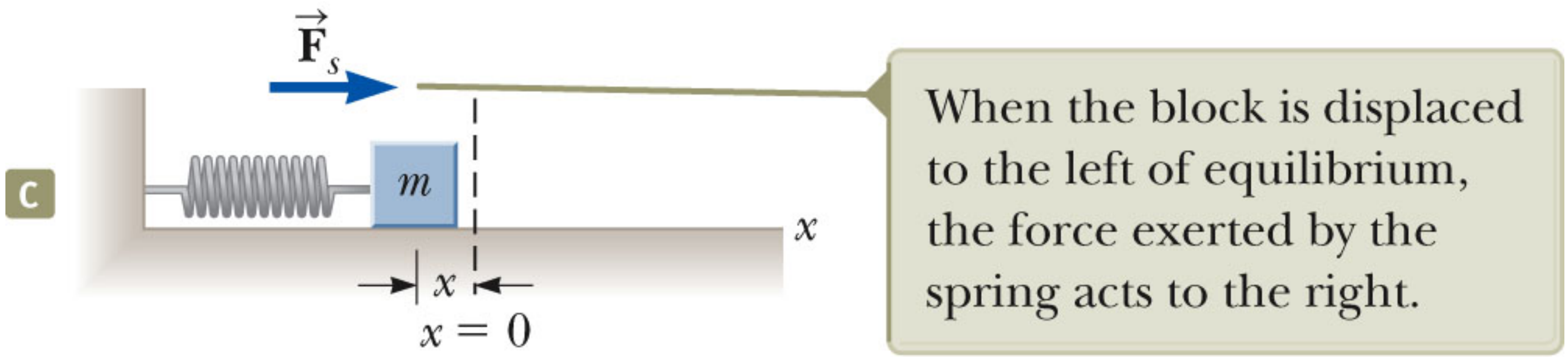
Restoring Force and the Spring Mass System

- In a, the block is displaced to the right of $x = 0$.
  - The position is positive; The restoring force is directed to the left.
- In b, the block is at the equilibrium position.
  - $x = 0$; The spring is neither stretched nor compressed; The force is 0.



Restoring Force, cont.

- The block is displaced to the left of $x = 0$.
  - The position is negative.
  - The restoring force is directed to the right.



Acceleration

- When the block is displaced from the equilibrium point and released, it is a particle under a net force and therefore has an acceleration.
- The force described by Hooke's Law is the net force in Newton's Second Law.

$$-kx = ma_x$$

$$a_x = -\frac{k}{m}x$$

- The acceleration is proportional to the displacement of the block.
- The direction of the acceleration is opposite the direction of the displacement from equilibrium.
- An object moves with simple harmonic motion whenever its acceleration is proportional to its position and is oppositely directed to the displacement from equilibrium.

Acceleration, cont.

- The acceleration is ***not*** constant.
  - Therefore, the kinematic equations cannot be applied.
  - If the block is released from some position $x = A$, then the initial acceleration is $-kA/m$.
  - When the block passes through the equilibrium position, $a = 0$.
  - The block continues to $x = -A$ where its acceleration is $+kA/m$.

Motion of the Block

- The block continues to oscillate between –A and +A.
  - These are turning points of the motion.
- The force is conservative.
- In the absence of friction, the motion will continue forever.
  - Real systems are generally subject to friction, so they do not actually oscillate forever.

Analysis Model: A Particle in Simple Harmonic Motion

- Model the block as a particle.
  - The representation will be **particle in simple harmonic motion model.**
- Choose x as the axis along which the oscillation occurs.
- Acceleration
- We let

$$a = \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

- Then $a = -\omega^2 x$

$$\omega^2 = \frac{k}{m}$$

A Particle in Simple Harmonic Motion, 2

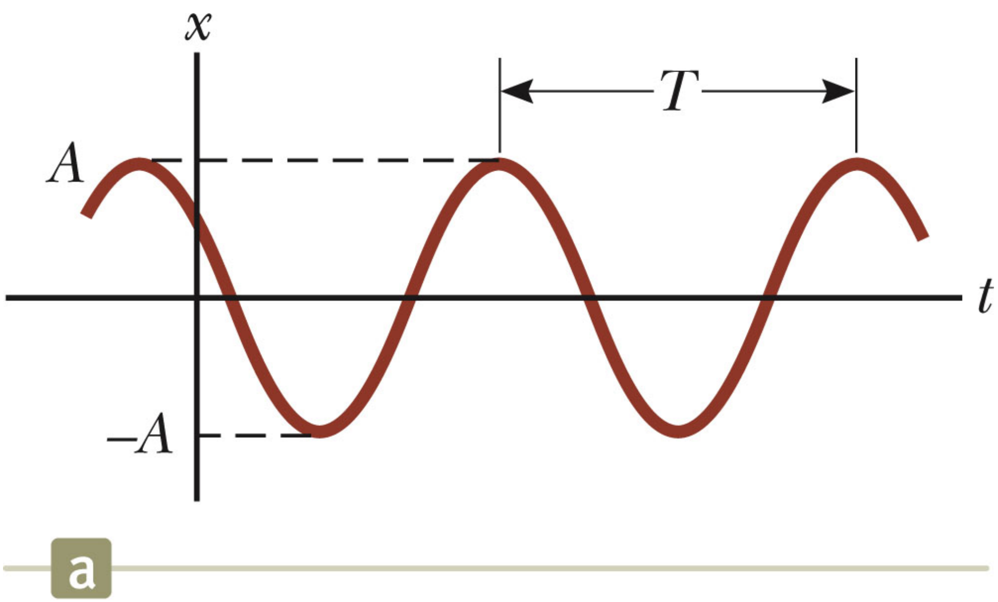
- A function that satisfies the equation is needed.
  - Need a function *x*(*t*) whose second derivative is the same as the original function with a negative sign and multiplied by ω^2.
  - The sine and cosine functions meet these requirements.

Simple Harmonic Motion – Graphical Representation

- A solution is

$$x(t) = A\cos(\omega t + \phi)$$

- A, ω, ϕ are all constants
- A cosine curve can be used to give physical significance to these constants.



Simple Harmonic Motion – Definitions

- A is the amplitude of the motion.
  - This is the maximum position of the particle in either the positive or negative x direction.
- ω is called the angular frequency.
  - Units are rad/s

$$\omega = \sqrt{\frac{k}{m}}$$

- ϕ is the phase constant or the initial phase angle.

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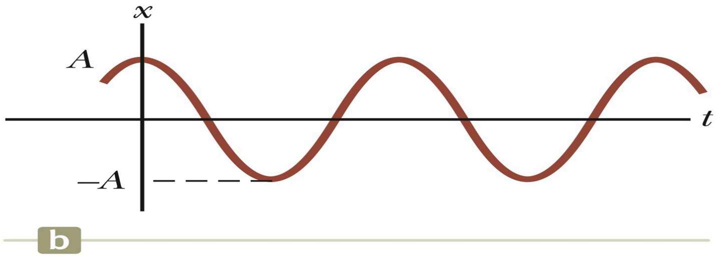
Simple Harmonic Motion, cont.

- A and ϕ are determined uniquely by the position and velocity of the particle at $t = 0$.
  - If the particle is at $x = A$ at $t = 0$, then $\phi = 0$
- The **phase** of the motion is the quantity $(\omega t + \phi)$.
- x (t) is periodic and its value is the same each time ωt increases by 2π radians.



Section 15.2

Period

- The ***period***, *T*, of the motion is the time interval required for the particle to go through one full cycle of its motion.
  - The values of *x* and *v* for the particle at time *t* equal the values of *x* and *v* at *t* + *T*.

$$T = \frac{2\pi}{\omega}$$

Quick Quiz 15.4 An object of mass m is hung from a spring and set into oscillation. The period of the oscillation is measured and recorded as T. The object of mass m is removed and replaced with an object of mass $2m$. When this object is set into oscillation, what is the period of the motion? **(a)** $2T$ **(b)** $\sqrt{2}\,T$ **(c)** T **(d)** $T/\sqrt{2}$ **(e)** $T/2$

Frequency

- The inverse of the period is called the ***frequency.***
- The frequency represents the number of oscillations that the particle undergoes per unit time interval.

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

- Units are cycles per second = hertz (Hz).

Summary Equations – Period and Frequency

- The frequency and period equations can be rewritten to solve for ω.

$$\omega = 2\pi f = \frac{2\pi}{T}$$

- The period and frequency can also be expressed as:

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- The frequency and the period depend only on the mass of the particle and the force constant of the spring.
- The frequency is larger for a stiffer spring (large values of *k*) and decreases with increasing mass of the particle.

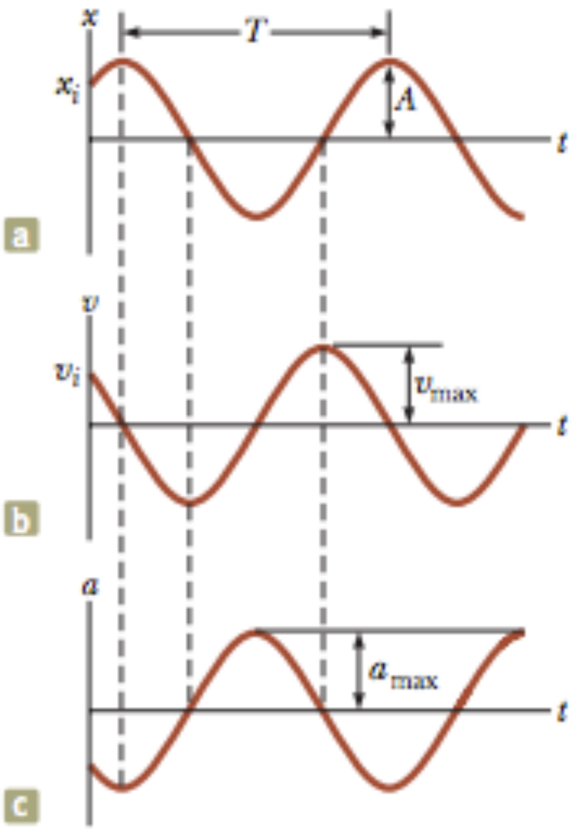
Motion Equations for Simple Harmonic Motion

$$x(t) = A\cos(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

- Simple harmonic motion is one-dimensional and so directions can be denoted by + or - sign.
- Remember, simple harmonic motion is **not** uniformly accelerated motion.



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Maximum Values of v and a

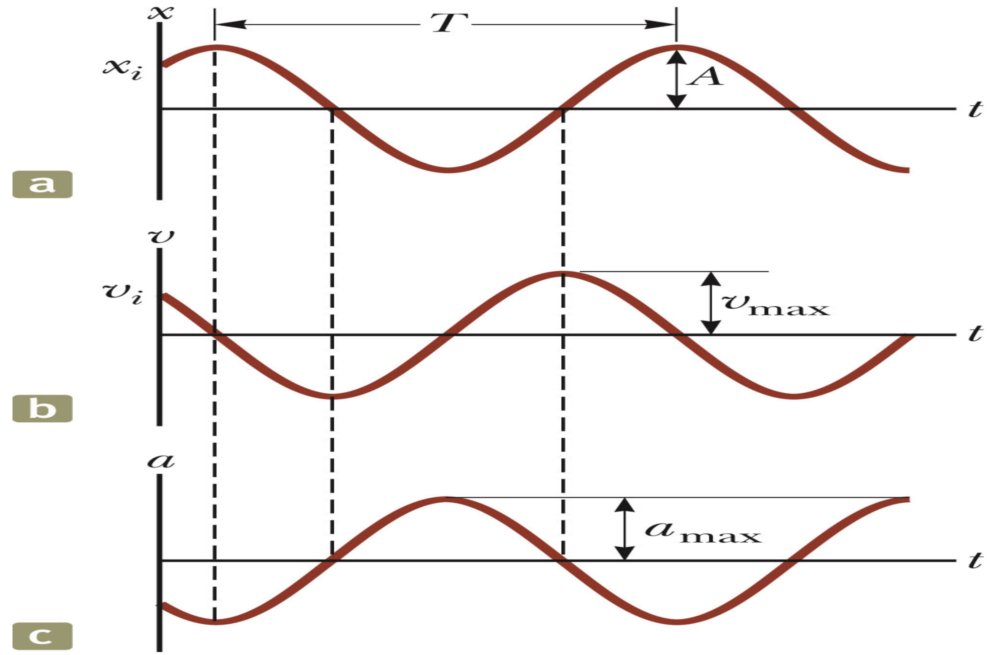
- Because the sine and cosine functions oscillate between ±1, we can easily find the maximum values of velocity and acceleration for an object in SHM.

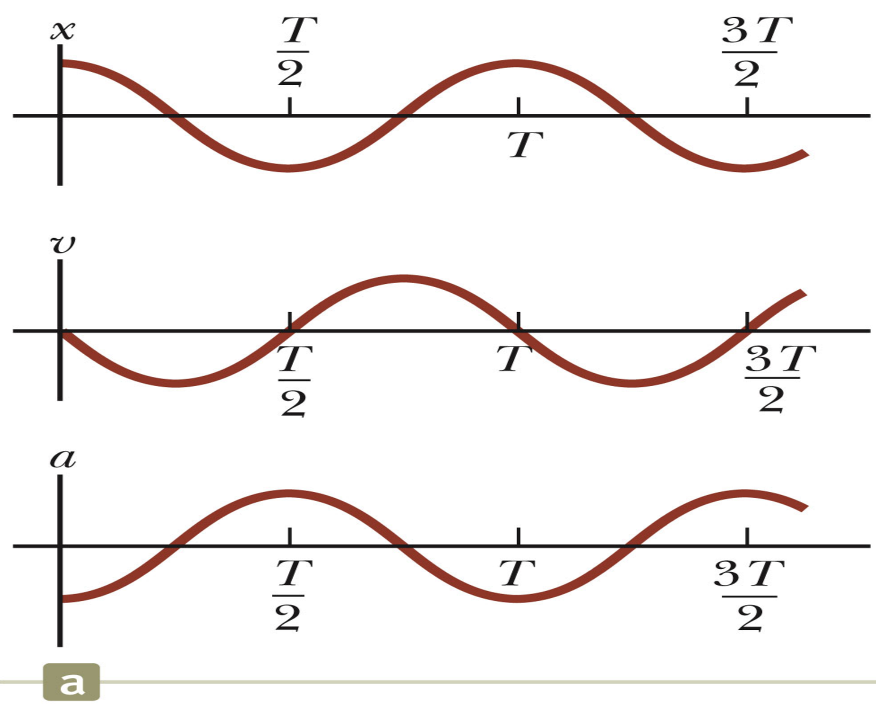
$$v_{max} = \omega A = \sqrt{\frac{k}{m}}\,A$$

$$a_{max} = \omega^2 A = \frac{k}{m}\,A$$

Graphs

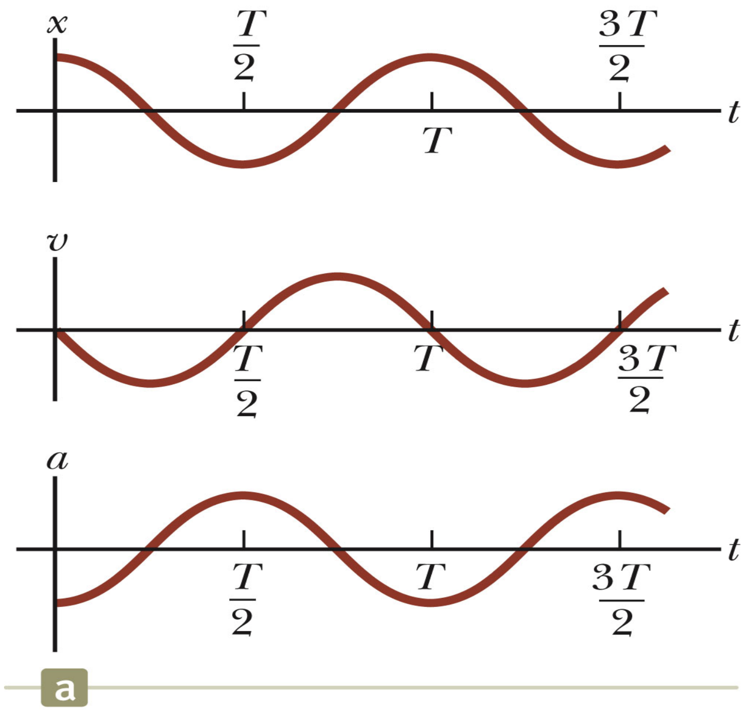
- The graphs show:
  - (a) displacement as a function of time
  - (b) velocity as a function of time
  - (c) acceleration as a function of time
- The velocity is 90° out of phase with the displacement and the acceleration is 180° out of phase with the displacement.





SHM Example 1

- Initial conditions at $t = 0$ are
  - $x(0) = A$
  - $v(0) = 0$
- This means $\phi = 0$
- The acceleration reaches extremes of $\pm \omega^2 A$ at ±A.
- The velocity reaches extremes of $\pm \omega A$ at x = 0.



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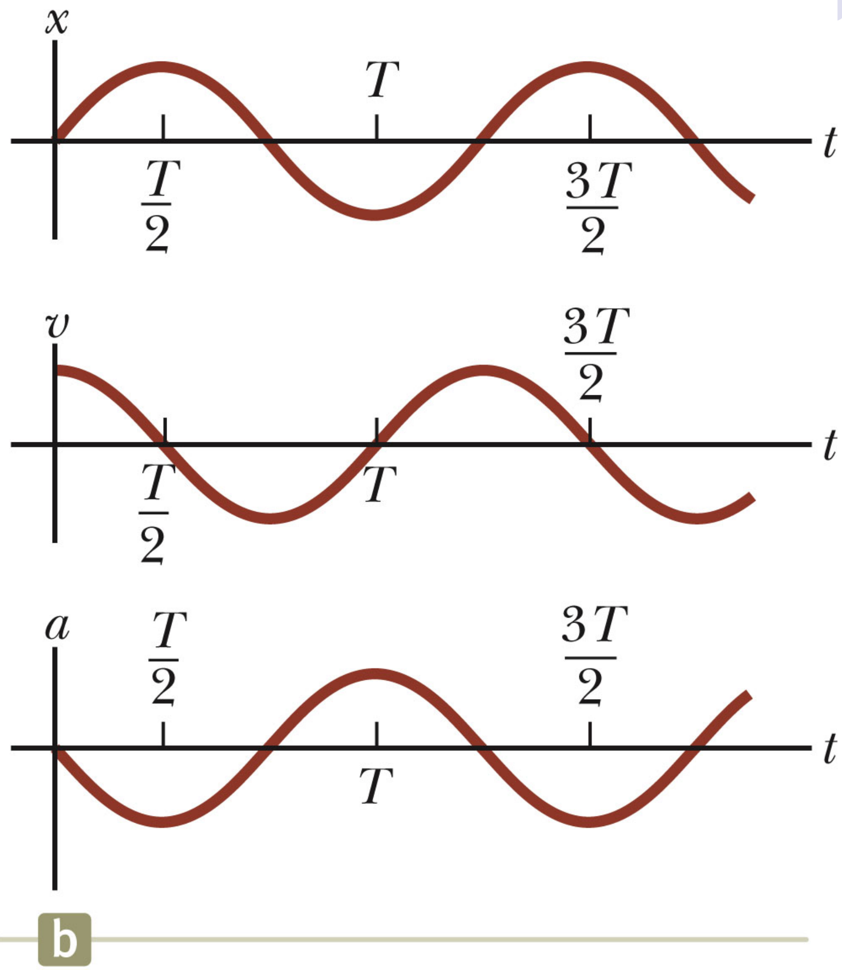
SHM Example 2

- Initial conditions at $t = 0$ are
  - $x\ (0)=0$
  - $v\ (0) = v_i$
- This means $\phi = -\pi / 2$
- The graph is shifted one-quarter cycle to the right compared to the graph of $x\ (0) = A$.



Section 15.2

Example 15.1 A Block–Spring System AM

A 200-g block connected to a light spring for which the force constant is 5.00 N/m is free to oscillate on a frictionless, horizontal surface. The block is displaced 5.00 cm from equilibrium and released from rest as in Figure 15.6.

(A) Find the period of its motion.

SOLUTION

Conceptualize Study Figure 15.6 and imagine the block moving back and forth in simple harmonic motion once it is released. Set up an experimental model in the vertical direction by hanging a heavy object such as a stapler from a strong rubber band.

Categorize The block is modeled as a *particle in simple harmonic motion*.

Analyze

Use Equation 15.9 to find the angular frequency of the block–spring system:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00\ \text{N/m}}{200 \times 10^{-3}\ \text{kg}}} = 5.00\ \text{rad/s}$$

Use Equation 15.13 to find the period of the system:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00\ \text{rad/s}} = \boxed{1.26\ \text{s}}$$

(B) Determine the maximum speed of the block.

SOLUTION

Use Equation 15.17 to find v_{max}:

$$v_{max} = \omega A = (5.00\ \text{rad/s})(5.00 \times 10^{-2}\ \text{m}) = \boxed{0.250\ \text{m/s}}$$

(C) What is the maximum acceleration of the block?

Use Equation 15.18 to find a_{max}: $a_{max} = \omega^2 A = (5.00 \text{ rad/s})^2(5.00 \times 10^{-2} \text{ m}) = \boxed{1.25 \text{ m/s}^2}$

(D) Express the position, velocity, and acceleration as functions of time in SI units.

SOLUTION

Find the phase constant from the initial condition that $x = A$ at $t = 0$: $x(0) = A \cos \phi = A \rightarrow \phi = 0$

Use Equation 15.6 to write an expression for $x(t)$: $x = A \cos (\omega t + \phi) = \boxed{0.0500 \cos 5.00t}$

Use Equation 15.15 to write an expression for $v(t)$: $v = -\omega A \sin (\omega t + \phi) = \boxed{-0.250 \sin 5.00t}$

Use Equation 15.16 to write an expression for $a(t)$: $a = -\omega^2 A \cos (\omega t + \phi) = \boxed{-1.25 \cos 5.00t}$

Finalize Consider part (a) of Figure 15.7, which shows the graphical representations of the motion of the block in this problem. Make sure that the mathematical representations found above in part (D) are consistent with these graphical representations.

WHAT IF? What if the block were released from the same initial position, $x_i = 5.00$ cm, but with an initial velocity of $v_i = -0.100$ m/s? Which parts of the solution change, and what are the new answers for those that do change?

Answers Part (A) does not change because the period is independent of how the oscillator is set into motion. Parts (B), (C), and (D) will change.

Write position and velocity expressions for the initial conditions:

(1) $x(0) = A \cos \phi = x_i$

(2) $v(0) = -\omega A \sin \phi = v_i$

Divide Equation (2) by Equation (1) to find the phase constant:

$$\frac{-\omega A \sin \phi}{A \cos \phi} = \frac{v_i}{x_i}$$

$$\tan \phi = -\frac{v_i}{\omega x_i} = -\frac{-0.100 \text{ m/s}}{(5.00 \text{ rad/s})(0.0500 \text{ m})} = 0.400$$

$$\phi = \tan^{-1} (0.400) = 0.121\pi$$

Use Equation (1) to find A:

$$A = \frac{x_i}{\cos \phi} = \frac{0.0500 \text{ m}}{\cos (0.121\pi)} = 0.0539 \text{ m}$$

Find the new maximum speed: $v_{max} = \omega A = (5.00 \text{ rad/s})(5.39 \times 10^{-2} \text{ m}) = 0.269 \text{ m/s}$

Find the new magnitude of the maximum acceleration: $a_{max} = \omega^2 A = (5.00 \text{ rad/s})^2(5.39 \times 10^{-2} \text{ m}) = 1.35 \text{ m/s}^2$

Find new expressions for position, velocity, and acceleration in SI units:

$$x = 0.0539 \cos (5.00t + 0.121\pi)$$
$$v = -0.269 \sin (5.00t + 0.121\pi)$$
$$a = -1.35 \cos (5.00t + 0.121\pi)$$

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Example 15.2 Watch Out for Potholes! AM

A car with a mass of 1 300 kg is constructed so that its frame is supported by four springs. Each spring has a force constant of 20 000 N/m. Two people riding in the car have a combined mass of 160 kg. Find the frequency of vibration of the car after it is driven over a pothole in the road. *continued*

Analyze First, let's determine the effective spring constant of the four springs combined. For a given extension x of the springs, the combined force on the car is the sum of the forces from the individual springs.

Find an expression for the total force on the car:

$$F_{\text{total}} = \sum(-kx) = -\left(\sum k\right)x$$

In this expression, x has been factored from the sum because it is the same for all four springs. The effective spring constant for the combined springs is the sum of the individual spring constants.

Evaluate the effective spring constant:

$$k_{\text{eff}} = \sum k = 4 \times 20\,000 \text{ N/m} = 80\,000 \text{ N/m}$$

Use Equation 15.14 to find the frequency of vibration:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{80\,000 \text{ N/m}}{1\,460 \text{ kg}}} = \boxed{1.18 \text{ Hz}}$$

Finalize The mass we used here is that of the car plus the people because that is the total mass that is oscillating. Also notice that we have explored only up-and-down motion of the car. If an oscillation is established in which the car rocks back and forth such that the front end goes up when the back end goes down, the frequency will be different.

WHAT IF? Suppose the car stops on the side of the road and the two people exit the car. One of them pushes downward on the car and releases it so that it oscillates vertically. Is the frequency of the oscillation the same as the value we just calculated?

Answer The suspension system of the car is the same, but the mass that is oscillating is smaller: it no longer includes the mass of the two people. Therefore, the frequency should be higher. Let's calculate the new frequency, taking the mass to be 1 300 kg:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{80\,000 \text{ N/m}}{1\,300 \text{ kg}}} = 1.25 \text{ Hz}$$

Simple Pendulum

- A simple pendulum also exhibits periodic motion.
- It consists of a particle-like bob of mass *m* suspended by a light string of length *L.*
- The motion occurs in the vertical plane and is driven by gravitational force.
- The motion is very close to that of the SHM oscillator.
  - If the angle is $<10^{o}$

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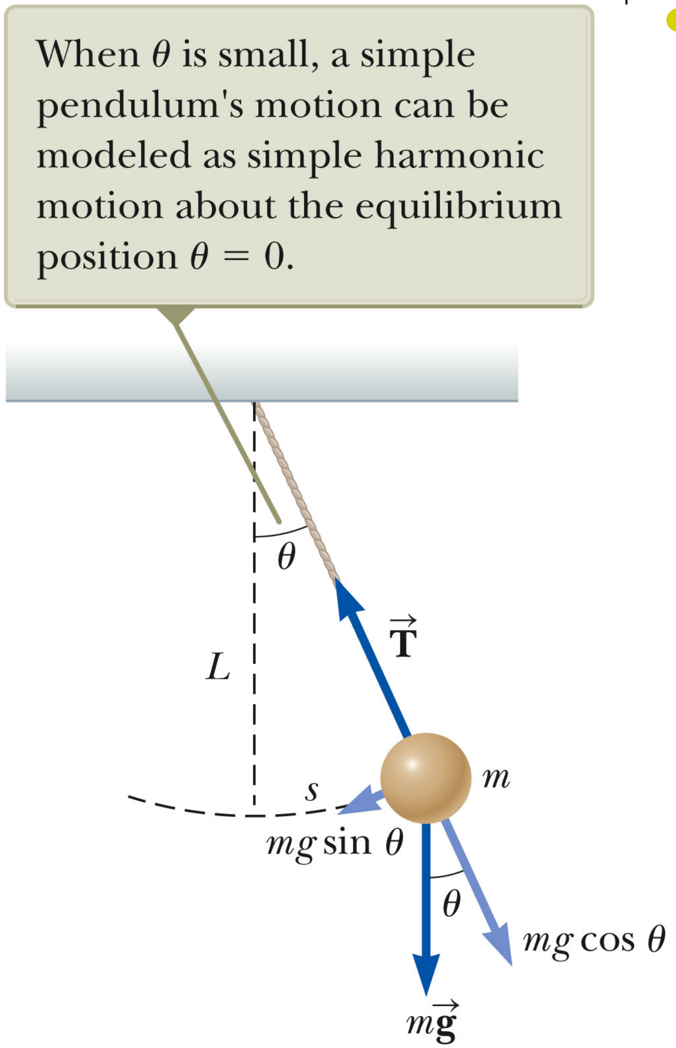
Simple Pendulum

- The forces acting on the bob are the tension and the weight.
  - $\vec{\mathbf{T}}$ is the force exerted on the bob by the string.
  - $m\vec{\mathbf{g}}$ is the gravitational force.
- The tangential component of the gravitational force is a restoring force.



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Simple Pendulum

- In the tangential direction,

$$F_t = ma_t \rightarrow -mg\sin\theta = m\frac{d^2s}{dt^2}$$

- The length, L, of the pendulum is constant, and for small values of θ.

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta = -\frac{g}{L}\theta$$

- This confirms the mathematical form of the motion is the same as for SHM.

Simple Pendulum

- The function θ can be written as $\theta = \theta_{max} \cos (\omega t + \phi)$.
- The angular frequency is

$$\omega = \sqrt{\frac{g}{L}}$$

- The period is

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}}$$

Simple Pendulum, Summary

- The period and frequency of a simple pendulum depend only on the length of the string and the acceleration due to gravity.
- The period is independent of the mass.
- All simple pendula that are of equal length and are at the same location oscillate with the same period.

A swinging rod

- A uniform rod of mass M and length L is pivoted about one end and oscillates in a vertical plane. Find the period of oscillation if the amplitude of the motion is small.

Example 15.5 A Connection Between Length and Time

Christian Huygens (1629–1695), the greatest clockmaker in history, suggested that an international unit of length could be defined as the length of a simple pendulum having a period of exactly 1 s. How much shorter would our length unit be if his suggestion had been followed?

SOLUTION

Conceptualize Imagine a pendulum that swings back and forth in exactly 1 second. Based on your experience in observing swinging objects, can you make an estimate of the required length? Hang a small object from a string and simulate the 1-s pendulum.

Categorize This example involves a simple pendulum, so we categorize it as a substitution problem that applies the concepts introduced in this section.

Solve Equation 15.26 for the length and substitute the known values:

$$L = \frac{T^2 g}{4\pi^2} = \frac{(1.00\ \text{s})^2(9.80\ \text{m/s}^2)}{4\pi^2} = \boxed{0.248\ \text{m}}$$

The meter's length would be slightly less than one-fourth of its current length. Also, the number of significant digits depends only on how precisely we know g because the time has been defined to be exactly 1 s.

WHAT IF? What if Huygens had been born on another planet? What would the value for g have to be on that planet such that the meter based on Huygens's pendulum would have the same value as our meter?

Answer Solve Equation 15.26 for g:

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(1.00\ \text{m})}{(1.00\ \text{s})^2} = 4\pi^2\ \text{m/s}^2 = 39.5\ \text{m/s}^2$$

HomeWork

1. In an engine, a piston oscillates with simple harmonic motion so that its position varies according to the expression

$$x = 5.00 \cos\left(2t + \frac{\pi}{6}\right)$$

where x is in centimeters and t is in seconds. At $t = 0$, find (a) the position of the particle, (b) its velocity, and (c) its acceleration. Find (d) the period and (e) the amplitude of the motion.

(a) $x = (5.00\text{ cm})\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $x = (5.00\text{ cm})\cos\left(\frac{\pi}{6}\right) = \boxed{4.33\text{ cm}}$

(b) $v = \frac{dx}{dt} = -(10.0\text{ cm/s})\sin\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $v = \boxed{-5.00\text{ cm/s}}$

(c) $a = \frac{dv}{dt} = -\left(20.0\text{ cm/s}^2\right)\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $a = \boxed{-17.3\text{ cm/s}^2}$

(d) $A = \boxed{5.00\text{ cm}}$ and $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \boxed{3.14\text{ s}}$

2\. The position of a particle is given by the expression $x = 4.00 \cos(3.00\pi t + \pi)$, where x is in meters and t is in seconds. Determine (a) the frequency and (b) period of the motion, (c) the amplitude of the motion, (d) the phase constant, and (e) the position of the particle at $t = 0.250$ s.

$x = (4.00 \text{ m})\cos(3.00\pi t + \pi)$ Compare this with $x = A\cos(\omega t + \phi)$ to find

(a) $\omega = 2\pi f = 3.00\pi$

or $\boxed{f = 1.50 \text{ Hz}}$ $T = \frac{1}{f} = \boxed{0.667 \text{ s}}$

(b) $A = \boxed{4.00 \text{ m}}$

(c) $\phi = \boxed{\pi \text{ rad}}$

(d) $x(t = 0.250 \text{ s}) = (4.00 \text{ m})\cos(1.75\pi) = \boxed{2.83 \text{ m}}$

3.

For the motion shown in Fig. 11-3, what are the amplitude, period, and frequency?

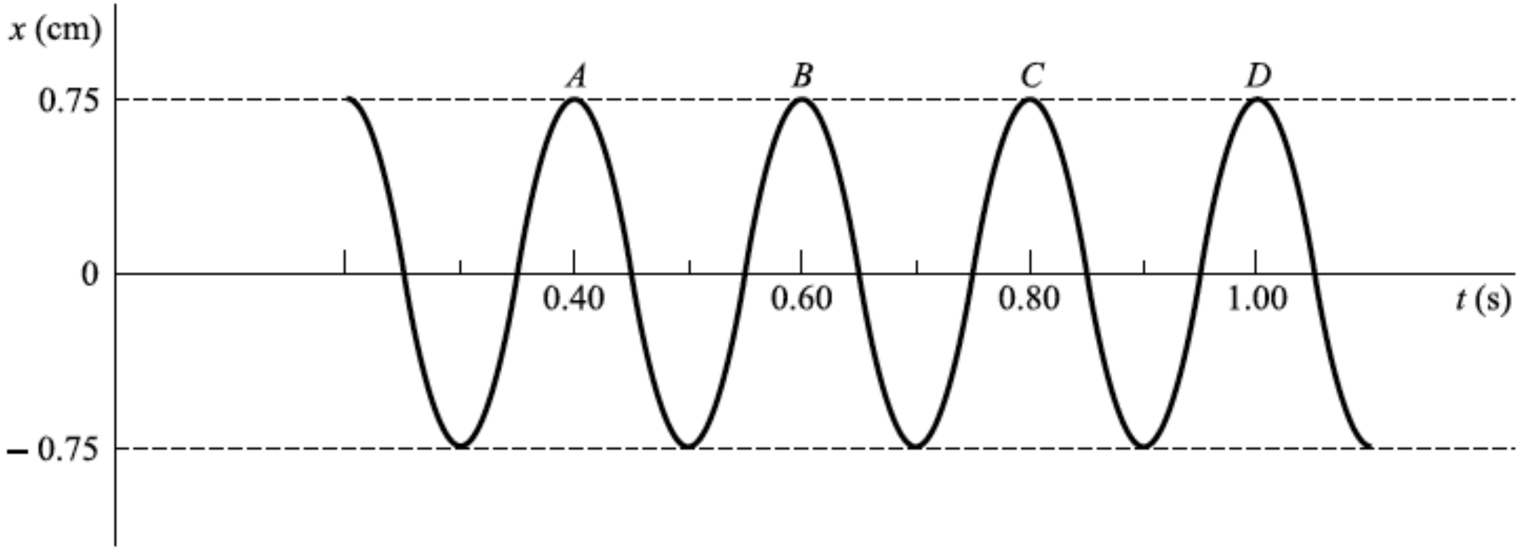

Fig. 11-3

The amplitude is the maximum displacement from the equilibrium position and so is 0.75 cm. The period is the time for one complete cycle, the time from *A* to *B*, for example. Therefore the period is 0.20 s. The frequency is

$$f = \frac{1}{T} = \frac{1}{0.20\ \text{s}} = 5.0\ \text{cycles/s} = 5.0\ \text{Hz}$$

4.

A spring makes 12 vibrations in 40 s. Find the period and frequency of the vibration.

$$T = \frac{\text{elapsed time}}{\text{vibrations made}} = \frac{40\ \text{s}}{12} = 3.3\ \text{s} \qquad f = \frac{\text{vibrations made}}{\text{elapsed time}} = \frac{12}{40\ \text{s}} = 0.30\ \text{Hz}$$

When a mass m is hung on a spring, the spring stretches 6.0 cm. Determine its period of vibration if it is then pulled down a little and released.

Since

$$k = \frac{F_{\text{ext}}}{x} = \frac{mg}{0.060\ \text{m}}$$

we have

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.060\ \text{m}}{g}} = 0.49\ \text{s}$$

5. A simple pendulum makes 120 complete oscillations in 3.00 min at a location where $g = 9.80\ m/s^2$. Find (a) the period of the pendulum and (b) its length.

$t=3\ min=3\times60=180\ sec$

a) $T=180/120=1.5\ sec$

b) $T=2\pi\sqrt{L/g}$

$L=T^2g/(4\pi^2)=(1.5)^2\ 9.8/(4\times3.14^2)=0.559\ m$

حالات خاصة		الحركة التوافقية لكتلة نابض		الوحدة	Simple Harmonic Motion ص الحركة التوافقية
		قانون هوك : F=-kX (k constant force)		N	القوة المؤثرة (قوة الارجاع) F
$X_0=0, V=V_0, t=0,$ $\phi=\pi/2$ فإن $X=A\cos(\omega t-\pi/2)$ $X=(V_0/\omega)\sin\omega t$	$X_0=A, V_0=0, t=0$ $\phi=0$ فإن $X=A\cos\omega t$	مسار الحركة باتجاه المحور Y $Y=A\sin(kx-\omega t+\Phi)$	مسار الحركة باتجاه المحور X $X=A\cos(\omega t+\Phi)$ $X_0=A\cos(\Phi)$ (t=0)	m	سعة الإزاحة
$V=-V_0\cos\omega t$	$V=-\omega A\sin\omega t$	$V=-\omega A\cos(kx-\omega t+\Phi)$ $V_0=-\omega A\cos(\Phi)$ (t=0) $V_{max}=\lvert\omega A\rvert=\sqrt{(k/m)}$	$V=-\omega A\sin(\omega t+\Phi)$ $V_0=-\omega A\sin(\Phi)$ (t=0) $V_{max}=\lvert\omega A\rvert=A\sqrt{(k/m)}$	m/s	السرعة $dx/dt=v$
$a=-\omega^2 v_0\sin\omega t$	$a=-\omega^2 A$	$a=\omega^2 A\cos(kx-\omega t+\Phi)$ $a_{max}=\lvert\omega^2 A\rvert=(k/m)A$	$a=-\omega^2 A\cos(\omega t+\Phi)$ $a_{max}=\lvert\omega^2 A\rvert=(k/m)A$	m/s^2	التسارع $a=dv/dt$
$\Phi=-\pi/2$	$\Phi=0$	$(kx-\omega t+\Phi)$	$(\omega t+\Phi)$	درجة	الطور
		$\Phi=\tan^{-1}(-V_0/\omega X_0)$		درجة	phase constant ثابت الطور Φ
$A=v0/\omega$	$A=X$	$A=\sqrt{(X_0^2+(V_0/\omega)^2)}$		m	اقصى ازاحة(سعة الحركة)
		$\omega=\sqrt{(k/m)}$		Rad/sec	التردد الزاوي) the angular (frequency $\omega=2\pi f$
		$f=\sqrt{(k/m)}/(2\pi)$		Hz Or 1/sec	frequencyد $f=\omega/2\pi$, $f=1/T$
		$T=(2\pi)\sqrt{(m/k)}$		sec	***Periodi*** الزمن الدوري $T=2\pi/\omega$, $T=1/f$
				Rad/m	العدد الموجي $K=2\pi/\lambda$
				m	الطول الموجي $\lambda=VT$, $\lambda=V/f$

Abeer Alghamdi

الحركة الاهتزازية البسيطة في منظومة البندول	الوحدة	خصائص الحركة التوافقية
قوة مماسة: $F=-mg\sin\theta$	N	القوة المؤثرة (قوة الارجاع) F
$S=L\theta$	m	سعة الإزاحة
$\theta=\theta_0\cos(\omega t+\Phi)$	درجة (O)	السعة الزاوية Θ
	درجة (O)	السعة الزاوية القصوى θ_0
$d\theta/dt=-\theta_0\omega\sin(\omega t+\Phi)$ $V_{max}=\|\omega A\|=A\sqrt{(g/L)}$	rad/s	السرعة $d\theta/dt=v$
$d^2\theta/dt^2=\theta_0\omega^2\cos(\omega t+\Phi)$ $a_{max}=\|\omega^2 A\|=Ag/L$	rad/s^2	التسارع $a=d^2\theta/dt^2$
$\omega=\sqrt{(g/L)}$	rad/sec	التردد الزاوي $\omega=2\pi f$
$f=\sqrt{(g/L)}/(2\pi)$	Hz Or 1/sec	التردد $f=\omega/2\pi$, $f=1/T$
$T=(2\pi)\sqrt{(L/g)}$	sec	الزمن الدوري $T=2\pi/\omega$, $T=1/f$