- P=1
- $X \sim N(\mu, \sigma^2)$
- $Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$
- $Z^2 = \left(\frac{X-\mu}{\sigma}\right)^2 \sim \boldsymbol{\chi_1^2}$
- $\bar{X} \sim N\left(\mu, \frac{1}{n}\sigma^2\right)$

$$(n-1)\frac{S^2}{\sigma} \sim \boldsymbol{\chi_{n-1}^2}$$

$$(n-1)S^2 \sim \sigma^2 \boldsymbol{\chi_{n-1}^2}$$

- P>1
- $X \sim \boldsymbol{N_p}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$
- $\mathbf{Z} = \boldsymbol{\Sigma}^{-1/2}(\boldsymbol{X} - \boldsymbol{\mu}) \sim \boldsymbol{N}(\mathbf{0}, \boldsymbol{I})$
- $\boldsymbol{Z^2} = \mathbf{Z}'\,\mathbf{Z} = (\boldsymbol{X} - \boldsymbol{\mu})' \Sigma^{-1} (\boldsymbol{X} - \boldsymbol{\mu}) \sim \boldsymbol{\chi_p^2}$
- $\bar{\boldsymbol{X}} \sim \boldsymbol{N_p}\left(\boldsymbol{\mu}, \frac{\mathbf{1}}{\boldsymbol{n}}\boldsymbol{\Sigma}\right)$
- $(n-1)S \sim W(.|\boldsymbol{\Sigma})\ with\ df = (n-1).$

- $Hotelling's\ Statistic:\quad T^2 = n(\bar{\boldsymbol{X}} - \boldsymbol{\mu_0})'\boldsymbol{S}^{-1}(\bar{\boldsymbol{X}} - \boldsymbol{\mu_0})$ (5.4)
- $T^2 \sim \frac{(n-1)p}{n-p} F_{p,n-p}$ (5.5)
- $H_0: \boldsymbol{\mu} = \boldsymbol{\mu}_{0\,P\times 1} = \begin{bmatrix} \mu_{10} \\ \mu_{20} \\ \cdots \\ \mu_{p0} \end{bmatrix}$
- Reject H_0 if $T^2 > \frac{n-1}{n-p} F_{p,n-p}(1-\alpha)$ (5.7)

Result 5.3: Simultaneous Confidence Intervals (T^2 intervals.)

- $\boldsymbol{\mu_i} \in \left[\bar{x}_i \pm \sqrt{\frac{p(n-1)}{(n-p)} F_{p,n-p}(\alpha)} \sqrt{\frac{s_{ii}}{n}}\right]$ **with probability** $(1-\alpha)$ (5.24)
- $\boldsymbol{a'\mu} \in \left[a'\bar{X} \pm \sqrt{\frac{p(n-1)}{n(n-p)} F_{p,n-p}(\alpha)\ a'Sa}\right]$ **with probability** $(1-\alpha)$

Simultaneous Confidence Intervals One-at-a-Time Intervals

- $\boldsymbol{\mu_i} \in \left[\bar{x}_i \pm t_{n-1,\frac{\alpha}{2}} \sqrt{\frac{s_{ii}}{n}}\right]$ **with probability** $(1-\alpha)$
- $\boldsymbol{a'\mu} \in \left[a'\bar{X} \pm t_{n-1,\frac{\alpha}{2}} \sqrt{\frac{a'Sa}{n}}\right]$ **with probability** $(1-\alpha)$

The Bonferroni Confidence Intervals

- $\boldsymbol{\mu_i} \in \left[\bar{x}_i \pm t_{n-1,\frac{\alpha}{2p}} \sqrt{\frac{s_{ii}}{n}}\right]$ **with probability** $(1-\alpha)$ (5.29)
- $\boldsymbol{a'\mu} \in \left[a'\bar{X} \pm t_{n-1,\frac{\alpha}{2p}} \sqrt{\frac{a'Sa}{n}}\right]$ **with probability** $(1-\alpha)$

Bonferroni intervals provide more precise estimates than the T^2 intervals.

Large Sample Inferences:

Result 5.4:

When $\boldsymbol{n - p}$ is large, the hypothesis $H_0: \mu = \mu_0$ is rejected in favor of $H_1: \mu \neq \mu_0$, at a level of significance approximately α, if the observed

$$n(\bar{\boldsymbol{X}} - \boldsymbol{\mu_0})' \boldsymbol{S}^{-1} (\bar{\boldsymbol{X}} - \boldsymbol{\mu_0}) > \chi_p^2(\alpha)$$

Result 5.5:

the large sample $100(1-\alpha)\%$ simultaneous confidence intervals for the population

mean .

$$\boldsymbol{a}' \bar{\boldsymbol{X}} \pm \sqrt{\chi_p^2(\alpha)} \sqrt{\frac{\boldsymbol{a}' \boldsymbol{S} \boldsymbol{a}}{\boldsymbol{n}}}$$

Result 5.3: Simultaneous Confidence Intervals (T^2 intervals.)

- $\boldsymbol{\mu_i} \in \left[\overline{\boldsymbol{x}}_{\boldsymbol{i}} \pm \sqrt{\chi_p^2(\alpha)}\sqrt{\frac{\boldsymbol{s_{ii}}}{\boldsymbol{n}}}\right]$ $\boldsymbol{with\ probability\ (1-\alpha)}$ $\boldsymbol{(5.24)}$
- $\boldsymbol{a'\mu} \in \left[\boldsymbol{a'\overline{X}} \pm \sqrt{\chi_p^2(\alpha)}\sqrt{\frac{\boldsymbol{a'Sa}}{\boldsymbol{n}}}\right]$ $\boldsymbol{with\ probability\ (1-\alpha)}$

Simultaneous Confidence Intervals One-at-a-Time Intervals

- $\boldsymbol{\mu_i} \in \left[\overline{\boldsymbol{x}}_{\boldsymbol{i}} \pm \boldsymbol{Z}_{\frac{\boldsymbol{\alpha}}{\boldsymbol{2}}} \sqrt{\frac{\boldsymbol{s_{ii}}}{\boldsymbol{n}}}\right]$ $\boldsymbol{with\ probability\ (1-\alpha)}$
- $\boldsymbol{a'\mu} \in \left[\boldsymbol{a'\overline{X}} \pm \boldsymbol{Z}_{\frac{\boldsymbol{\alpha}}{\boldsymbol{2}}}\sqrt{\frac{\boldsymbol{a'Sa}}{\boldsymbol{n}}}\right]$ $\boldsymbol{with\ probability\ (1-\alpha)}$

The Bonferroni Confidence Intervals

- $\boldsymbol{\mu_i} \in \left[\overline{\boldsymbol{x}}_{\boldsymbol{i}} \pm \boldsymbol{Z}_{\frac{\boldsymbol{\alpha}}{\boldsymbol{2p}}} \sqrt{\frac{\boldsymbol{s_{ii}}}{\boldsymbol{n}}}\right]$ $\boldsymbol{with\ probability\ (1-\alpha)}$ $\boldsymbol{(5.29)}$
- $\boldsymbol{a'\mu} \in \left[\boldsymbol{a'\overline{X}} \pm \boldsymbol{Z}_{\frac{\boldsymbol{\alpha}}{\boldsymbol{2p}}}\sqrt{\frac{\boldsymbol{a'Sa}}{\boldsymbol{n}}}\right]$ $\boldsymbol{with\ probability\ (1-\alpha)}$