Math-254

Numerical Methods

Recommended Textbook:

INTRODUCTION TO NUMERICAL ANALYSIS USING MATLAB

AUTHOR: DR. RIZWAN BUTT

REFERENCE TEXTBOOK:

Numerical Analysis (Seventh Edition)

AUTHORS: RICHARD L. BURDEN AND J. DOUGLAS FAIRES

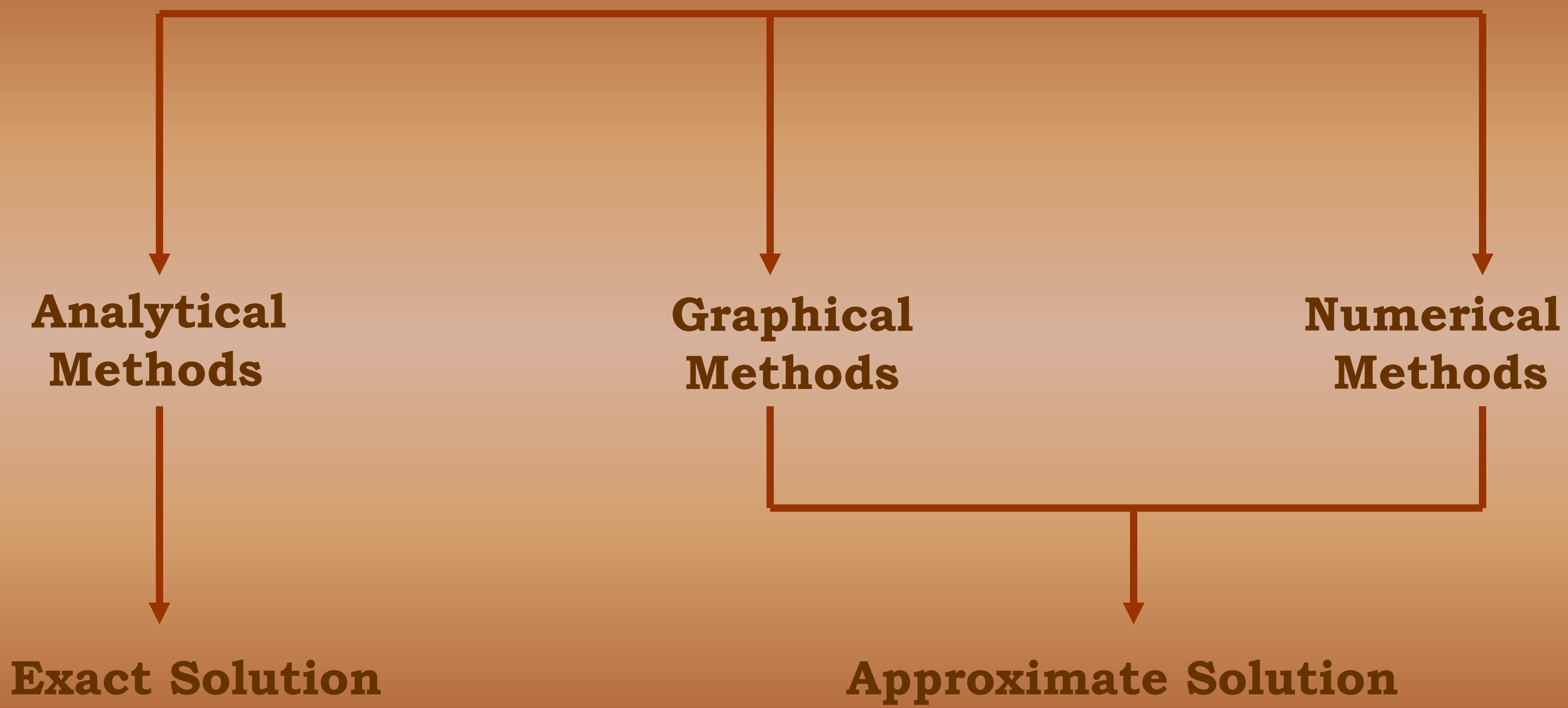
PDF SLIDES OF THE COURSE:

HTTPS://FAC.KSU.EDU.SA/MABDELWAHED/COURSE/308640

TOPICS FOR THE COURSE

CHAPTER 1	NUMBER SYSTEMS AND ERROR	(1 LECTURE)
CHAPTER 2	SOLUTIONS OF NONLINEAR EQUATIONS	(13 LECTURES)
CHAPTER 3	SYSTEMS OF LINEAR EQUATIONS	(13 LECTURES)
CHAPTER 4	APPROXIMATING FUNCTIONS	(7 LECTURES)
CHAPTER 5	DIFFERENTIATION AND INTEGRATION	(7 LECTURES)
CHAPTER 6	ORDINARY DIFFERENTIAL EQUATIONS	(4 LECTURES)

Means of Solving Problems



What is Numerical Analysis ?

1- Numerical analysis can be defined as the development and implementation of techniques to find numerical solutions to mathematical problems.

2- It is an increasingly important link between pure mathematics and its application in science and technology.

3- With the accessibility of computers, it is possible now to get rapid and accurate solutions to many complex problems that give difficulties to the mathematician, engineer and scientist.

4. Frequently, numerical analysis is called the *mathematics of scientific computing.*

Why Numerical methods ?

There are many problems which have no exact solutions; like:

1. Polynomial equations of degree greater than four:
2. Simple equation such as

$$x = \cos x$$

3. Integrals of type

$$\int_a^b e^{x^2} dx \quad \text{and} \quad \int_a^b \frac{\sin x}{x} dx; \qquad \text{etc.}$$

A numerical method is an algorithm produces an approximate solution to within any prescribed accuracy.

A numerical method can be used even exact solution of a problem exists (because easy to use).

The numerical methods deal with only numbers.

Types of Numerical Methods

Direct Methods	Iterative Methods
Need finite number of steps.	Need infinite number of steps.
No need for initial approximation.	Need for initial approximation.
Good for small problems.	Good for large problems.
Chance of getting exact information about problems.	No chance of getting exact information about problems.

Stopping Criteria of Numerical Methods

An iterative process may converge or diverge. If the divergence occurs, the procedure should be terminated because there may be no solution. We can restart the procedure by changing the initial approximation if necessary.

In case of convergence we have to apply some stopping procedures to end the computations. By selecting a tolerance $\epsilon > 0$ and generate approximate solutions $x_1, x_2, \ldots, x_n$ until one of the following conditions is satisfied:

1- $|x_n - x_{n-1}| < \epsilon.$

2- $\frac{|x_n - x_{n-1}|}{|x_n|} < \epsilon.$

3- $f(x_n) \approx 0.$

4- Fixed number of iterations n, then x_n may be considered as the value of root.

Practical Justification Through MATLAB

1- In our lectures, practical justification of the methods is presented through computer examples through the use of MATLAB.

2- In recent years, the number of MATLAB users has dramatically increased.

3- The surge of popularity in MATLAB is related to the increasing popularity of UNIX and computer graphics.

4- To what extend numerical computations in the future will be programmed in MATLAB is uncertain.

Error Analyzing

Exact Error

An approximate number p is a number that differs but slightly from an exact number α. We write

$$p \approx \alpha$$

By error E of an approximate number p, we mean difference between exact number α and its computed approximation p.

Thus we define

$$E = \alpha - p \tag{1}$$

If $\alpha > p$, the error E is positive, and if $\alpha < p$, then it is negative.

Absolute Error

In many situations, the sign of the error may not be known and might even be irrelevant. Therefore, we define *absolute error* as

$$|E| = |\alpha - p| \tag{2}$$

Relative Error

The *relative error* RE of an approximate number p is the ratio of the absolute error of the number to the absolute value of the corresponding exact number α. Thus

$$RE = \frac{|\alpha - p|}{|\alpha|}, \qquad \alpha \neq 0 \quad (3)$$

For example, if we approximate $\frac{1}{3}$ by 0.333, we have

$$E = \frac{1}{3} \times 10^{-3} \qquad \textbf{and} \qquad RE = 10^{-3}$$

Note that relative error is generally a better measure of the extend of error than the actual error. But one should also note that relative error is undefined if the exact answer is equal to zero.

Percentage Error

Sometimes the quantity

$$\frac{|\alpha - p|}{|\alpha|} \times 100\% \qquad (4)$$

is defined as *percentage error*. For example, if we approximate $\frac{1}{3}$ by 0.333, we have

$$E = \frac{1}{3} \times 10^{-3} \qquad \text{and} \qquad RE = 10^{-3}$$

$$PE = 0.001 \times 100 = 0.1\%$$

Error Bound

In investigating the effect of the total error in various methods, we shall often mathematically derive an error, called, *error bound* and which is a limit on how large the error can be.

$$Lowerbound(Min.\ error) \leq Error \leq Upperbound(Max.\ error)$$

We will discuss only maximum error. The error bound can be much larger than exact error.

Sources of Errors

In analyzing the accuracy of numerical result, one should be aware of the possible sources of error in each stage of the computational process and of the extend to which these errors can affect the final answer.

Human Error

These types of errors arise due to sources such as the idealistic assumptions made to simplify the model, inaccurate measurements of data, miscopying of figures, the inaccurate representation of mathematical constants, etc.

Truncation Error

This type of error is caused when we are forced to use mathematical techniques which give approximate, rather than exact, answer. For example, suppose that we use the Maclaurin's series expansion to represent $\sin x$, so that

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

If we want a number that approximates $\sin(\frac{\pi}{2})$, we must terminate the expansion in order to obtain

$$\sin(\frac{\pi}{2}) = \frac{\pi}{2} - \frac{(\pi/2)^3}{3!} + \frac{(\pi/2)^5}{5!} - \frac{(\pi/2)^7}{7!} + E$$

where E is the truncation error introduced in the calculation.

Round-off Error

This type of errors are associated with the limited number of digits numbers in the computer. For example, by rounding off 1.32463672 to six decimal places to give 1.324637.