Addition and subtraction) Basic Technical Mathematics with Calculus Peter K. F. Kuhfitting

Chapter 5 Factoring and Fractions (Summary ~~September 2018~~

Exercises / Section 5.8 (page 183-185)

- Combine the given fractions and simplify.

Problem # 1. $\frac{1}{2}-\frac{1}{18}+\frac{5}{9}$, Problem # 11. $\frac{2}{xy}-\frac{1}{x}-\frac{y^2+2x-2y}{xy(x-y)}$

Problem # 23. $\frac{a+b}{b}-\frac{a^2}{b(a+2b)}+\frac{a}{(a+2b)}$ Problem # 35. $\frac{1}{2x^2+3xy+y^2}-\frac{1}{x^2+4xy+3y^2}+\frac{1}{2x^2+7xy+3y^2}$

Problem # 9. $\frac{x+3y}{x-y}-\frac{x-3y}{x+y}$ Problem # 15. $\frac{2}{x-3}+\frac{1}{x+2}-\frac{2x-1}{(x-3)(x+2)}$

Problem # 29. $\frac{3}{(x-y)(x+2y)}-\frac{1}{(x+y)(x+2y)}+\frac{1}{(y-x)(x+y)}$

(Problems solved in class # 1, 11, 23, 35) HW: Problem # 9, Problem # 15, Problem # 29,

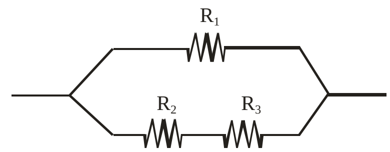
Problem # P1: $\frac{1}{x-y}\left(\frac{x}{y}-\frac{y}{x}\right)$(Answer: $\frac{x+y}{xy}$) Problem # P2: $\frac{2}{w(w+1)}+\frac{3}{w^2}$ (Answer: $\frac{5w+3}{w^2(w+1)}$)

Exercises / Section 5.9 (page 188-189)

- Simplify the complex fractions.

Problem # 7. $\frac{1-\frac{16}{x^2}}{1+\frac{4}{x}}$, Problem # 19. $\frac{\frac{1}{E-1}+\frac{1}{E-2}}{1+\frac{1}{E-2}}$ Problem # 23. $\frac{\frac{x}{x-2}-\frac{2}{(x-1)(x-2)}}{\frac{(x-4)}{(x-1)}}$

Problem # 31 The total resistance of the circuit shown in the figure is given by $R=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2+R_3}}$, simplify the expression for R.



Simplify the complex fractions: Problem # 5. $\frac{3-\frac{1}{x}}{9-\frac{1}{x^2}}$ Problem # 15. $\frac{w-\frac{w}{w-5}}{w-\frac{6}{w-5}}$ Problem # 21

$\frac{\frac{k}{k+1}-\frac{6}{(k+1)^2}}{1-\frac{9}{(k+1)^2}}$

(Problems solved in class # 7, 19, 23, 31) HW: Problem # 5, Problem # 15, Problem # 21,

Problem # P3: $\frac{\frac{2}{x-2}+\frac{1}{x}}{\frac{3x}{x-5}-\frac{2}{x-5}}$ (Answer: $\frac{x-5}{x(x-2)}$) Problem # P4: $2-\frac{m}{1-\frac{1-m}{-m}}$ (Answer: $2-m^2$)

Problem # P5: $\frac{5-\frac{1}{x+2}}{1+\frac{3}{1+\frac{3}{x}}}$ (Answer: $\frac{(5x+9)(x+3)}{(x+2)(4x+3)}$)

7.1 Introduction to partial fractions

In order to resolve an algebraic expression into partial fractions:

(i) The denominator must factorise

(ii) The numerator must be at least one degree less than the denominator. When the degree of numerator is equal to or higher than the degree of the denominator, the numerator must be divided by the denominator (see problem 3 and 4).

Table 7.1

Type	Denominator containing	Expression	Form of partial fraction
	Linear factors (see problem 1 to 4)	$\frac{f(x)}{(x+a)(x-b)(x+c)}$	$\frac{A}{(x+a)}+\frac{B}{(x-b)}+\frac{C}{(x+c)}$
2	Repeated linear factors (see problem 5 to 7)	$\frac{f(x)}{(x+a)^3}$	$\frac{A}{(x+a)}+\frac{B}{(x+a)^2}+\frac{C}{(x+a)^3}$
3	Quadratic factors (see problem 8 and 9)	$\frac{f(x)}{(ax^2+bx+c)(x+d)}$	$\frac{Ax+B}{(ax^2+bx+c)}+\frac{C}{(x+d)}$

7.2 Worked Problems on partial fractions with linear factors

Problem 1. . Resolve $\frac{11-3x}{x^2+2x-3}$ into partial fractions

Problem 2. Convert $\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$ into the sum of three partial fractions

Problem 3. Resolve $\frac{x^2+1}{x^2-3x+2}$ into partial fractions

Problem 4. Express $\frac{x^3-2x^2-4x-4}{x^2+x-2}$ in partial fractions

7.3 Worked Problems on partial fractions with repeated linear factors

Problem 5. Resolve $\frac{2x+3}{(x-2)^2}$ into partial fractions

Problem 6. Express $\frac{5x^2-2x-19}{(x+3)(x-1)^2}$ as the sum of three partial fractions

Problem 7. Resolve $\frac{3x^2+16x+15}{(x+3)^3}$ into partial fractions

7.4 Worked problems on partial fraction with quadratic factors

Problem 8. Express $\frac{7x^2+5x+13}{(x^2+2)(x+1)}$ in partial fractions **Problem 9.** Resolve $\frac{3+6x+4x^2-2x^3}{x^2(x^2+3)}$ into partial fractions

Exercise 5.8 (Page 183-185)

Combine the given fractions and simplify:

Problem #1: $\frac{1}{2} - \frac{1}{18} + \frac{5}{9}$

$$= \frac{9-1+10}{18} = \frac{18}{18} = \boxed{1}$$

Problem #11:

$$\frac{2}{xy} - \frac{1}{x} - \frac{y^2+2x-2y}{xy(x-y)}$$

$$= \frac{2(x-y) - y(x-y) - (y^2+2x-2y)}{xy(x-y)}$$

$$= \frac{2x-2y-xy+y^2-y^2-2x+2y}{xy(x-y)}$$

$$= \frac{-xy}{xy(x-y)} = -\frac{1}{x-y}$$

$$= \boxed{\frac{1}{y-x}}$$

Problem #23

$$\frac{a+b}{b} - \frac{a^2}{b(a+2b)} + \frac{a}{(a+2b)}$$

$$= \frac{(a+b)(a+2b) - a^2 + ab}{b(a+2b)}$$

$$= \frac{a^2+2ab+ab+2b^2-a^2+ab}{b(a+2b)}$$

$$= \frac{4ab+2b^2}{b(a+2b)} = \frac{b(4a+2b)}{b(a+2b)}$$

$$= \frac{4a+2b}{a+2b}$$

Problem #35

$$\frac{1}{2x^2+3xy+y^2} - \frac{1}{x^2+4xy+3y^2} + \frac{1}{2x^2+7xy+3y^2}$$

factorizing

$$= \frac{1}{2x^2+2xy+xy+y^2} - \frac{1}{x^2+3xy+xy+3y^2} + \frac{1}{2x^2+6xy+xy+3y^2}$$

$$= \frac{1}{2x(x+y)+y(x+y)} - \frac{1}{x(x+3y)+y(x+3y)} + \frac{1}{2x(x+3y)+y(x+3y)}$$

$$= \frac{1}{(x+y)(2x+y)} - \frac{1}{(x+3y)(x+y)} + \frac{1}{(x+3y)(2x+y)}$$

$$= \frac{x+3y-(2x+y)+x+y}{(x+y)(2x+y)(x+3y)}$$

$$= \frac{x+3y-2x-y+x+y}{(x+y)(2x+y)(x+3y)}$$

$$= \frac{3y}{(x+y)(2x+y)(x+3y)}$$

Problem # P_1

$$\frac{1}{x-y}\left(\frac{x}{y} - \frac{y}{x}\right)$$

$$= \frac{1}{x-y}\left(\frac{x^2-y^2}{xy}\right)$$

$$= \frac{1}{x-y} \times \frac{(x+y)(x-y)}{xy}$$

$$= \frac{x+y}{xy}$$

Simplify the complex fraction

Problem # 7

$$\frac{1-\frac{16}{x^2}}{1+\frac{4}{x}}$$

$$=\frac{\frac{x^2-16}{x^2}}{\frac{x+4}{x}}$$

$$=\frac{x^2-16}{x^2}\times\frac{x}{x+4}$$

$$=\frac{(x+4)(x-4)}{x}\times\frac{1}{x+4}$$

$$=\frac{x-4}{x}$$

Problem #19

$$\frac{\frac{1}{E-1}+\frac{1}{E-2}}{1+\frac{1}{E-2}}$$

$$=\frac{\frac{E-2+E-1}{(E-1)(E-2)}}{\frac{E-2+1}{E-2}}$$

$$=\frac{2E-3}{(E-1)(E-2)}\times\frac{(E-2)}{E-1}$$

$$=\frac{2E-3}{(E-1)^2}$$

$$=\frac{2E-3}{E^2-2E+1}$$

Problem # 23

$$\frac{\frac{x}{x-2}-\frac{2}{(x-1)(x-2)}}{\frac{x-4}{(x-1)}}$$

$$=\frac{\frac{x(x-1)-2}{(x-1)(x-2)}}{\frac{(x-4)}{(x-1)}}$$

$$=\frac{x^2-x-2}{(x-1)(x-2)}\times\frac{(x-1)}{x-4}$$

$$=\frac{(x^2-x-2)}{(x-2)(x-4)}$$

$$=\frac{x^2-2x+x-2}{(x-2)(x-4)}$$

$$=\frac{x(x-2)+1(x-2)}{(x-2)(x-4)}$$

$$=\frac{(x-2)(x+1)}{(x-2)(x-4)}$$

$$=\frac{x+1}{x-4}$$

Problem #31

$$R=\frac{1}{\frac{1}{R_1}+\frac{1}{R_2+R_3}}$$

$$=\frac{1}{\frac{R_2+R_3+R_1}{R_1(R_2+R_3)}}$$

$$R=\frac{R_1(R_2+R_3)}{R_1+R_2+R_3}$$

Problem # P5

$$\frac{5-\frac{1}{x+2}}{1+\frac{3}{1+\frac{3}{x}}}$$

$$=\frac{\frac{5(x+2)-1}{x+2}}{1+\frac{3}{\frac{x+3}{x}}}=\frac{\frac{5x+10-1}{x+2}}{1+\frac{3x}{x+3}}$$

$$=\frac{\frac{5x+9}{x+2}}{\frac{x+3+3x}{x+3}}=\frac{\frac{5x+9}{x+2}}{\frac{4x+3}{x+3}}=\frac{(5x+9)}{(x+2)}\times\frac{(x+3)}{(4x+3)}$$

$$\therefore\ \frac{(5x+9)(x+3)}{(x+2)(4x+3)}$$

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To add (or subtract) two or more fractions, we find the LCD for the fractions and change each fraction to an equivalent fraction having the LCD for its denominator. Next, we add (or subtract) the numerators of the fractions, placing the result over the LCD. Finally, we simplify the result fraction.

Example #1 Example #1

Combine

$$\frac{5}{6a^2bc} - \frac{4}{15ab^2c} - \frac{3}{20abc^2}$$

Solution. In terms of prime factors, the different denominators are

$$6a^2bc = 2 \cdot 3 \cdot a^2 \cdot b \cdot c$$
$$15ab^2c = 3 \cdot 5 \cdot a \cdot b^2 \cdot c$$
$$20abc^2 = 2^2 \cdot 5 \cdot a \cdot b \cdot c^2$$

To construct the LCD, observe that the factors are 2, 3, 5, a, b, and c. The largest exponent on the factor 2 is 2, the largest exponent on the factor 3 is 1, and so forth. So the LCD is given by

$$\text{LCD} = 2^2 \times 3 \times 5 \times a^2b^2c^2 = 60a^2b^2c^2$$

Now we write the fractions so that they all have the same denominators. For example, since $60a^2b^2c^2 = (6a^2bc)(10bc)$, we get for the first fraction

$$\frac{5}{6a^2bc} = \frac{5}{6a^2bc} \cdot \frac{10bc}{10bc} = \frac{50bc}{60a^2b^2c^2}$$

The other fractions are adjusted similarly:

$$\frac{5(10bc)}{6a^2bc(10bc)} - \frac{4(4ac)}{15ab^2c(4ac)} - \frac{3(3ab)}{20abc^2(3ab)}$$
$$= \frac{50bc}{60a^2b^2c^2} - \frac{16ac}{60a^2b^2c^2} - \frac{9ab}{60a^2b^2c^2}$$
$$= \frac{50bc - 16ac - 9ab}{60a^2b^2c^2}$$

The procedure for adding or subtracting fractions containing polynomials is similar.

Example #2 Example #2

Combine

$$\frac{x}{x - y} - \frac{x^2}{x^2 - y^2}$$

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The acceleration of the system is

$$a = \frac{(w_1 - w_2)g}{w_1 + w_2}$$

Write an expression for $gT(1/a)$ and simplify.

36. In the study of the dispersion of X rays, the expression

$$A = \frac{Ne^2}{\pi m(f_0^2 - f^2)}$$

arises. Multiply A by $m(f_0 + f)$ and simplify.

37. A perfectly flexible cable, suspended from two points at the same height, hangs under its own weight. The tension T_0 at its lowest point is

$$T_0 = \frac{w(s^2 - 4H^2)}{8H}$$

where s is the length of the cable, H the sag, and w the weight per unit length. Find an expression for $T_0[H/(s - 2H)]$.

5.8 Addition and Subtraction of Fractions

In considering the addition and subtraction of algebraic fractions, recall from arithmetic that the fractions $\frac{1}{3}$ and $\frac{1}{6}$ can be added if $\frac{1}{3}$ is changed to $\frac{2}{6}$, so that

$$\frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

The number 6 is called the **lowest common denominator (LCD).**

Since algebraic fractions are added by the same rules, let us first state the definition of the lowest common denominator.

Lowest common denominator: The lowest common denominator (LCD) of two or more fractions is an expression that is divisible by every denominator and does not have any more factors than needed to satisfy this condition.

The LCD can be found by the procedure described next.

To construct the lowest common denominator for a set of algebraic fractions, factor each of the denominators. Then the LCD is the product of the factors of the denominators, each with an exponent equal to the largest of the exponents of any of these factors

Common errors (1) Failing to change all the signs when subtracting the numerator, For example, in step (5.18) of Example 3, both signs in the numerator c middle fraction are changed to become $-6x - 6$.

(2) Forgetting that

$$\frac{1}{x} + \frac{1}{y} \neq \frac{1}{x+y}$$

(3) Forgetting that

$$\frac{1}{x+y} \neq \frac{1}{x} + \frac{1}{y}$$

In case (2) the correct procedure is

$$\frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy} = \frac{x+y}{xy}$$

In case (3) the fraction

$$\frac{1}{x+y}$$

is already in simplest form and cannot be split up.

Example 4 Two perfectly elastic balls collide with a common velocity v. If their re tive masses are m and M, then the velocity of m after the collision is

$$v\left(\frac{M}{M+m} - \frac{m}{M+m}\right) - \frac{2vm}{M+m}$$

Simplify this expression.

Solution.

$$v\left(\frac{M}{M+m} - \frac{m}{M+m}\right) - \frac{2vm}{M+m}$$
$$= \frac{vM}{M+m} - \frac{vm}{M+m} - \frac{2vm}{M+m}$$
$$= \frac{vM - vm - 2vm}{M+m}$$
$$= \frac{vM - 3vm}{M+m}$$

Exercises / Section 5.8

In Exercises 1–35, combine the given fractions and simplify.

1. $\frac{1}{2} - \frac{1}{18} + \frac{5}{9}$
2. $\frac{5}{36} - \frac{3}{108} + \frac{1}{9}$
3. $\frac{2x+1}{9} + \frac{x}{2} - \frac{x}{6}$
4. $\frac{1}{x} - \frac{3}{2x} - \frac{1}{3x}$

5. $\frac{3x+3}{4x} - \frac{1}{2x} - \frac{1}{x}$
6. $\frac{x-1}{5y} + \frac{x}{10y} - \frac{7x}{30y}$
7. $\frac{3a}{4b} - \frac{4a}{9b} + \frac{a-3}{36b}$
8. $\frac{x}{x+y} - \frac{x-y}{x}$
9. $\frac{x+3y}{x-y} - \frac{x-3y}{x+y}$
10. $\frac{x}{2x+y} - \frac{x}{x+y}$
11. $\frac{2}{xy} - \frac{1}{x} - \frac{y^2+2x-2y}{xy(x-y)}$
12. $\frac{x}{x-3y} - \frac{y}{2x+y}$
13. $\frac{x+y}{x+2y} + \frac{x}{x-y}$
14. $\frac{3}{x-2} - \frac{4}{x-3}$
15. $\frac{2}{x-3} + \frac{1}{x+2} - \frac{2x-1}{(x-3)(x+2)}$
16. $\frac{x}{(x+y)(x+2y)} + \frac{1}{x+y}$
17. $\frac{1}{x+3y} + \frac{4y}{(x-3y)(x-y)}$
18. $\frac{x}{2x-y} - \frac{y}{y-3x}$
19. $\frac{2x}{3x-y} - \frac{2y}{3y-2x}$
20. $\frac{1}{x} + \frac{2}{y} - \frac{1}{x+y}$
21. $\frac{1}{x} - \frac{1}{y} - \frac{y}{x(x+y)}$
22. $\frac{y}{x+y} - \frac{2y^2}{x(x+y)} + \frac{x+2y}{x}$
23. $\frac{a+b}{b} - \frac{a^2}{b(a+2b)} + \frac{a}{a+2b}$
24. $\frac{3ab}{a^2-b^2} + \frac{a}{b-a} + \frac{a-b}{a+b}$
25. $\frac{A}{A+B} - \frac{B^2}{A(A-B)} + \frac{2B}{A}$
26. $\frac{3s-t}{s(s-t)} - \frac{1}{s+t} + \frac{1}{s}$
27. $\frac{n}{n-m} - \frac{m}{n+m} - \frac{2m^2}{n^2-m^2}$
28. $\frac{4}{(R+3r)(R+r)} - \frac{1}{(R+3r)(R+2r)} - \frac{1}{(R+2r)(R+r)}$
29. $\frac{3}{(x-y)(x+2y)} - \frac{1}{(x+y)(x+2y)} + \frac{1}{(y-x)(x+y)}$
30. $\frac{2}{x+1} - \frac{2x}{x^2-1} - \frac{1}{x-1}$
31. $\frac{x}{x-2} - \frac{2}{x+2} - \frac{x^2}{x^2-4}$
32. $\frac{T_0}{T_0-2} - \frac{2}{T_0+2} - \frac{2T_0^2}{4-T_0^2}$
33. $\frac{c}{c+2d} - \frac{c}{2c+d}$
34. $\frac{2}{x^2-y^2} - \frac{3}{x^2+xy-2y^2} + \frac{1}{x^2+3xy+2y^2}$
35. $\frac{1}{2x^2+3xy+y^2} - \frac{1}{x^2+4xy+3y^2} + \frac{1}{2x^2+7xy+3y^2}$

36. A light rod of length 1 is clamped at both ends and carries a load W at the center. If x is the distance from one end of the rod, then the deflection is

$$y = \frac{W}{k}\left(\frac{x^3}{12} - \frac{x^2}{16}\right), \quad 0 \le x \le \frac{1}{2}$$

where k is a constant. Write y as a single fraction.

$$\frac{\frac{1}{x+y}\cdot\frac{(x-y)(x+y)}{1}+\frac{1}{x-y}\cdot\frac{(x-y)(x+y)}{1}}{\frac{x}{(x-y)(x+y)}\cdot\frac{(x-y)(x+y)}{1}} = \frac{(x-y)+(x+y)}{x} = \frac{2x}{x} = 2$$

The remaining examples further illustrate the alternate technique discussed in the previous example.

Example #3

Simplify the fraction

$$\frac{\frac{R}{R+3}+\frac{R}{R^2-9}}{\frac{1}{R-3}+1}$$

Solution. Since $R^2 - 9 = (R-3)(R+3)$, it follows that

$$\text{LCD} = (R-3)(R+3)$$

As before, multiplying the numerator and denominator of the complex fraction by the LCD will reduce the fraction directly.

$$\frac{\frac{R}{R+3}\cdot\frac{(R-3)(R+3)}{1}+\frac{R}{R^2-9}\cdot\frac{(R-3)(R+3)}{1}}{\frac{1}{R-3}\cdot\frac{(R-3)(R+3)}{1}+1\cdot(R-3)(R+3)}$$

$$= \frac{R(R-3)+R}{(R+3)+(R-3)(R+3)}$$

$$= \frac{R^2-3R+R}{R+3+R^2-9}$$

$$= \frac{R^2-2R}{R^2+R-6}$$

$$= \frac{R(R-2)}{(R+3)(R-2)}$$

$$= \frac{R}{R+3}$$

Example #4

Just as electrical components can be connected in parallel, blood vessels that branch out and come together again are said to be connected in parallel.

Each such vessel offers a certain resistance to the flow of blood. If r_1, r_2, and r_3 are the respective resistance forces of the blood vessels in parallel, the combined resistance is given by

$$\frac{1}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}$$

Simplify this expression.

Solution.

$$\frac{1}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}} = \frac{1}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}\cdot\frac{r_1r_2r_3}{r_1r_2r_3}$$

$$= \frac{r_1r_2r_3}{r_2r_3+r_1r_3+r_1r_2}$$

Exercises / Section 5.9

In Exercises 1–26, simplify the complex fractions.

1. $\dfrac{1+\frac{1}{3}}{2+\frac{2}{3}}$

2. $\dfrac{\frac{1}{7}+\frac{2}{7}}{1+\frac{2}{7}}$

3. $\dfrac{1+\frac{1}{3}}{2-\frac{1}{6}}$

4. $\dfrac{\frac{1}{3}+\frac{1}{2}}{1-\frac{5}{6}}$

5. $\dfrac{3-\frac{1}{x}}{9-\frac{1}{x^2}}$

6. $\dfrac{\frac{1}{x}-2}{4-\frac{1}{x^2}}$

7. $\dfrac{1-\frac{16}{x^2}}{1+\frac{4}{x}}$

8. $\dfrac{1-\frac{9}{x^2}}{1-\frac{3}{x}}$

9. $\dfrac{\frac{1}{C_1}+\frac{1}{C_2}}{\frac{1}{C_1C_2}}$

10. $\dfrac{v_1-\frac{v_2^2}{v_1}}{1-\frac{v_2}{v_1}}$

11. $\dfrac{1+\frac{3}{x}-\frac{10}{x^2}}{1-\frac{4}{x}+\frac{4}{x^2}}$

12. $\dfrac{2-\frac{11}{y}-\frac{6}{y^2}}{2-\frac{5}{y}-\frac{3}{y^2}}$

13. $\dfrac{3-\frac{17}{h}+\frac{10}{h^2}}{3+\frac{10}{h}-\frac{8}{h^2}}$

14. $\dfrac{s-\frac{3s}{s+6}}{s+\frac{9}{s+6}}$

15. $\dfrac{w-\frac{w}{w-5}}{w-\frac{6}{w-5}}$

16. $\dfrac{z+\frac{4z}{z-3}}{z-\frac{4}{z-3}}$

17. $\dfrac{\frac{1}{\beta+1}+\frac{1}{\beta^2-1}}{\frac{\beta}{\beta-1}}$

18. $\dfrac{a-\frac{3a+2}{a+2}}{a-\frac{2a+1}{a+2}}$

Section 5.4 (page 166)

1. $(x + 2y)^2$ **3.** $(3x - 4y)^2$ **5.** $(x - 1)(x - 3)$ **7.** $(x - 4)(x + 3)$ **9.** $2(a - 2b)^2$
11. $2(x + 6)(x + 1)$ **13.** $(x - 6y)(x + y)$ **15.** $(D + 7)(D - 2)$ **17.** $(2x - y)(x - y)$
19. $(5x - y)(x - 2y)$ **21.** $(4x + y)(x + 3y)$ **23.** $(2x - 3y)(3x + 4y)$ **25.** $(5w_1 - 2w_2)(w_1 - 4w_2)$
27. $8(L - 3C)(L + 2C)$ **29.** $2(3f - 4g)^2$ **31.** $x^2(x - 2)^2$ **33.** not factorable
35. $(a + b - 3)(a + b + 2)$ **37.** $(n + m - 2)(n + m - 1)$ **39.** $(2a + 2b - 1)(a + b - 4)$
41. $(f_1 + 2f_2)^2(f_1 + 2f_2 - 1)$ **43.** $(1 - x + y)(1 + x - y + x^2 - 2xy + y^2)$ **45.** $(7a - 2b)(4a + b)$
47. $(5x - y)(8x + 3y)$ **49.** $(3\alpha - 2\beta)(4\alpha - 5\beta)$ **51.** $t = 3$ sec **53.** $t = 1.33$ sec

Section 5.5 (page 169)

1. $(x - y)(a + b)$ **3.** $(x + 3y)(2x + 1)$ **5.** $(a - b)(4c + 1)$ **7.** $(x - y)(5b - 1)$
9. $2(x + y)(a - c)$ **11.** $3(R - r)(a - 2b)$ **13.** $(x + y)(x - y - z)$ **15.** $(x + y)(a - x + y)$
17. $(x - y)(x + y - 2z)$ **19.** $(x - y)(x + 4y - 1)$ **21.** $(2x - y - z)(2x - y + z)$
23. $(x + 2y - z)(x + 2y + z)$ **25.** $(3a - 2b - c)(3a + 2b + c)$ **27.** $(x - y)(3 - x - 4y)$
29. $(x + 2y)(a - x + 3y)$ **31.** $a(A - 1)(Aa - 4)$

Section 5.6 (page 174)

1. $\frac{x}{2}$ **3.** $\frac{a}{2x}$ **5.** x **7.** $\frac{2}{x}$ **9.** $x + 4$ **11.** $x^2 - xy - y^2$ **13.** $\frac{x - y}{3}$
15. $R^2 + 2R + 4$ **17.** $x - 4$ **19.** $i + 5$ **21.** $\frac{3x - 4}{x - 3}$ **23.** $\frac{7x + 2}{2x + 1}$ **25.** $\frac{m_1 + m_2}{m_1 - 2m_2}$
27. -1 **29.** $\frac{2L + C}{3L - 4C}$ **31.** $\frac{x - 4}{x - 6}$ **33.** $\frac{a - 4b}{2a - b}$ **35.** $\frac{1}{2x + 1}$ **37.** $\frac{1}{2t + 1}$ **39.** $\frac{1}{3t + 1}$
41. $\frac{1}{3E - 2}$ **43.** $P - Q$ **45.** $\frac{M^2 + m^2}{(M + m)^2}$ **47.** $2t - 6$ (amperes)

Section 5.7 (page 177)

1. $\frac{3xy}{2zw^2}$ **3.** $\frac{8x^2y}{3a^2b}$ **5.** $\frac{6}{5}bcxy$ **7.** $\frac{dx}{ac^2}$ **9.** $2x(x - y)$ **11.** $\frac{x + y}{x - y}$ **13.** $-\frac{a + 2b}{3}$
15. $-(1 + 3a)(x^2 - 3xy + 9y^2)$ **17.** $\frac{4(2a - b)}{(3x - 1)(a + b)}$ **19.** $\frac{(3v_0 - 4)(v_1 + 6)}{(2v_0 - 3)(2v_1 - 1)}$
21. $\frac{(3T + J)(5K - 6)}{(4T + J)(K - 7)}$ **23.** $\frac{x - 3y}{(x + y)(x^2 - 2xy + 4y^2)}$ **25.** $\frac{x - y}{x - 2y}$ **27.** $\frac{1}{(R + 2r)(R + r - 1)}$
29. $\frac{2(2L - 7)}{(L + 4)(C - 5)}$ **31.** $\frac{2}{c - d}$ **33.** 2 **35.** $\frac{2w_1w_2}{w_1 - w_2}$ **37.** $\frac{1}{8}w(s - 2H)$

Section 5.8 (page 183)

1. 1 **3.** $\frac{5x + 1}{9}$ **5.** $\frac{3x - 3}{4x}$ **7.** $\frac{4a - 1}{12b}$ **9.** $\frac{8xy}{x^2 - y^2}$ **11.** $\frac{1}{y - x}$ **13.** $\frac{2x^2 + 2xy - y^2}{(x + 2y)(x - y)}$
15. $\frac{1}{x - 3}$ **17.** $\frac{1}{x - y}$ **19.** $\frac{4x^2 - 2y^2}{(3x - y)(2x - 3y)}$ **21.** $-\frac{x}{y(x + y)}$ **23.** $\frac{4a + 2b}{a + 2b}$ **25.** $\frac{A + B}{A}$
27. 1 **29.** $\frac{1}{x^2 - y^2}$ **31.** $\frac{4}{x^2 - 4}$ **33.** $\frac{c^2 - cd}{(c - 2d)(2c - d)}$ **35.** $\frac{3y}{(x + y)(2x + y)(x + 3y)}$
37. $\frac{k(2np - p^2)}{n^2(n - p)^2}$ **39.** $\frac{k^2}{k^2 + L^2}$

Chapter 7

Partial fractions

7.1 Introduction to partial fractions

By algebraic addition,

$$\frac{1}{x-2}+\frac{3}{x+1}=\frac{(x+1)+3(x-2)}{(x-2)(x+1)}$$
$$=\frac{4x-5}{x^2-x-2}$$

The reverse process of moving from $\frac{4x-5}{x^2-x-2}$ to $\frac{1}{x-2}+\frac{3}{x+1}$ is called resolving into **partial fractions**.

In order to resolve an algebraic expression into partial fractions:

(i) the denominator must factorise (in the above example, x^2-x-2 factorises as $(x-2)(x+1)$, and

(ii) the numerator must be at least one degree less than the denominator (in the above example $(4x-5)$ is of degree 1 since the highest powered x term is x^1 and (x^2-x-2) is of degree 2)

When the degree of the numerator is equal to or higher than the degree of the denominator, the numerator must be divided by the denominator (see Problems 3 and 4).

There are basically three types of partial fraction and the form of partial fraction used is summarised in Table 7.1 where $f(x)$ is assumed to be of less degree than the relevant denominator and A, B and C are constants to be determined.

(In the latter type in Table 7.1, ax^2+bx+c is a quadratic expression which does not factorise without containing surds or imaginary terms.)

Resolving an algebraic expression into partial fractions is used as a preliminary to integrating certain functions (see Chapter 51).

7.2 Worked problems on partial fractions with linear factors

Problem 1. Resolve $\frac{11-3x}{x^2+2x-3}$ into partial fractions

The denominator factorises as $(x-1)(x+3)$ and the numerator is of less degree than the denominator.

Table 7.1

Type	Denominator containing	Expression	Form of partial fraction
1	Linear factors (see Problems 1 to 4)	$\frac{f(x)}{(x+a)(x-b)(x+c)}$	$\frac{A}{(x+a)}+\frac{B}{(x-b)}+\frac{C}{(x+c)}$
2	Repeated linear factors (see Problems 5 to 7)	$\frac{f(x)}{(x+a)^3}$	$\frac{A}{(x+a)}+\frac{B}{(x+a)^2}+\frac{C}{(x+a)^3}$
3	Quadratic factors (see Problems 8 and 9)	$\frac{f(x)}{(ax^2+bx+c)(x+d)}$	$\frac{Ax+B}{(ax^2+bx+c)}+\frac{C}{(x+d)}$

Thus $\frac{11-3x}{x^2+2x-3}$ may be resolved into partial fractions. Let

$$\frac{11-3x}{x^2+2x-3} \equiv \frac{11-3x}{(x-1)(x+3)} \equiv \frac{A}{(x-1)} + \frac{B}{(x+3)},$$

where A and B are constants to be determined,

i.e. $$\frac{11-3x}{(x-1)(x+3)} \equiv \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}$$

by algebraic addition.

Since the denominators are the same on each side of the identity then the numerators are equal to each other.

Thus, $11-3x \equiv A(x+3)+B(x-1)$

To determine constants A and B, values of x are chosen to make the term in A or B equal to zero.

When $x=1$, then $11-3(1) \equiv A(1+3)+B(0)$

i.e. $8=4A$

i.e. $\mathbf{A=2}$

When $x=-3$, then $11-3(-3) \equiv A(0)+B(-3-1)$

i.e. $20=-4B$

i.e. $\mathbf{B=-5}$

Thus $$\mathbf{\frac{11-3x}{x^2+2x-3}} \equiv \frac{2}{(x-1)} + \frac{-5}{(x+3)}$$

$$\equiv \mathbf{\frac{2}{(x-1)} - \frac{5}{(x+3)}}$$

[Check: $$\frac{2}{(x-1)} - \frac{5}{(x+3)}$$

$$= \frac{2(x+3)-5(x-1)}{(x-1)(x+3)}$$

$$= \frac{11-3x}{x^2+2x-3}$$]

Problem 2. Convert $\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$ into the sum of three partial fractions

Let $$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$$

$$\equiv \frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x+3)}$$

$$\equiv \frac{A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)}{(x+1)(x-2)(x+3)}$$

by algebraic addition

Equating the numerators gives:

$$2x^2-9x-35 \equiv A(x-2)(x+3)+B(x+1)(x+3)+C(x+1)(x-2)$$

Let $x=-1$. Then

$$2(-1)^2-9(-1)-35 \equiv A(-3)(2)+B(0)(2)+C(0)(-3)$$

i.e. $-24=-6A$

i.e. $\mathbf{A} = \frac{-24}{-6} = \mathbf{4}$

Let $x=2$. Then

$$2(2)^2-9(2)-35 \equiv A(0)(5)+B(3)(5)+C(3)(0)$$

i.e. $-45=15B$

i.e. $\mathbf{B} = \frac{-45}{15} = \mathbf{-3}$

Let $x=-3$. Then

$$2(-3)^2-9(-3)-35 \equiv A(-5)(0)+B(-2)(0)+C(-2)(-5)$$

i.e. $10=10C$

i.e. $\mathbf{C=1}$

Thus $$\frac{2x^2-9x-35}{(x+1)(x-2)(x+3)}$$

$$\equiv \mathbf{\frac{4}{(x+1)} - \frac{3}{(x-2)} + \frac{1}{(x+3)}}$$

Problem 3. Resolve $\frac{x^2+1}{x^2-3x+2}$ into partial fractions

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The denominator is of the same degree as the numerator. Thus dividing out gives:

$$\begin{array}{r} 1 \\ x^2-3x+2\,\overline{)\,x^2 \qquad +1} \\ \underline{x^2-3x+2} \\ 3x-1 \end{array}$$

For more on polynomial division, see Section 6.1, page 48.

Hence $\dfrac{x^2+1}{x^2-3x+2} \equiv 1+\dfrac{3x-1}{x^2-3x+2}$

$$\equiv 1+\frac{3x-1}{(x-1)(x-2)}$$

Let $\dfrac{3x-1}{(x-1)(x-2)} \equiv \dfrac{A}{(x-1)}+\dfrac{B}{(x-2)}$

$$\equiv \frac{A(x-2)+B(x-1)}{(x-1)(x-2)}$$

Equating numerators gives:

$$3x-1 \equiv A(x-2)+B(x-1)$$

Let $x=1$. Then $2=-A$

i.e. $\mathbf{A=-2}$

Let $x=2$. Then $\mathbf{5=B}$

Hence $\dfrac{3x-1}{(x-1)(x-2)} \equiv \dfrac{-2}{(x-1)}+\dfrac{5}{(x-2)}$

Thus $\boldsymbol{\dfrac{x^2+1}{x^2-3x+2} \equiv 1-\dfrac{2}{(x-1)}+\dfrac{5}{(x-2)}}$

Problem 4. Express $\dfrac{x^3-2x^2-4x-4}{x^2+x-2}$ in partial fractions

The numerator is of higher degree than the denominator. Thus dividing out gives:

$$\begin{array}{r} x-3 \\ x^2+x-2\,\overline{)\,x^3-2x^2-4x-4} \\ \underline{x^3+x^2-2x} \\ -3x^2-2x-4 \\ \underline{-3x^2-3x+6} \\ x-10 \end{array}$$

Thus $\dfrac{x^3-2x^2-4x-4}{x^2+x-2} \equiv x-3+\dfrac{x-10}{x^2+x-2}$

$$\equiv x-3+\frac{x-10}{(x+2)(x-1)}$$

Let $\dfrac{x-10}{(x+2)(x-1)} \equiv \dfrac{A}{(x+2)}+\dfrac{B}{(x-1)}$

$$\equiv \frac{A(x-1)+B(x+2)}{(x+2)(x-1)}$$

Equating the numerators gives:

$$x-10 \equiv A(x-1)+B(x+2)$$

Let $x=-2$. Then $-12=-3A$

i.e. $\mathbf{A=4}$

Let $x=1$. Then $-9=3B$

i.e. $\mathbf{B=-3}$

Hence $\dfrac{x-10}{(x+2)(x-1)} \equiv \dfrac{4}{(x+2)}-\dfrac{3}{(x-1)}$

Thus $\boldsymbol{\dfrac{x^3-2x^2-4x-4}{x^2+x-2}}$

$$\boldsymbol{\equiv x-3+\frac{4}{(x+2)}-\frac{3}{(x-1)}}$$

Now try the following exercise

Exercise 26 Further problems on partial fractions with linear factors

Resolve the following into partial fractions:

1. $\dfrac{12}{x^2-9}$ $\left[\dfrac{2}{(x-3)}-\dfrac{2}{(x+3)}\right]$
2. $\dfrac{4(x-4)}{x^2-2x-3}$ $\left[\dfrac{5}{(x+1)}-\dfrac{1}{(x-3)}\right]$
3. $\dfrac{x^2-3x+6}{x(x-2)(x-1)}$ $\left[\dfrac{3}{x}+\dfrac{2}{(x-2)}-\dfrac{4}{(x-1)}\right]$
4. $\dfrac{3(2x^2-8x-1)}{(x+4)(x+1)(2x-1)}$ $\left[\dfrac{7}{(x+4)}-\dfrac{3}{(x+1)}-\dfrac{2}{(2x-1)}\right]$
5. $\dfrac{x^2+9x+8}{x^2+x-6}$ $\left[1+\dfrac{2}{(x+3)}+\dfrac{6}{(x-2)}\right]$

6. $\dfrac{x^2 - x - 14}{x^2 - 2x - 3}$ $\left[1 - \dfrac{2}{(x-3)} + \dfrac{3}{(x+1)}\right]$

7. $\dfrac{3x^3 - 2x^2 - 16x + 20}{(x-2)(x+2)}$

$\left[3x - 2 + \dfrac{1}{(x-2)} - \dfrac{5}{(x+2)}\right]$

7.3 Worked problems on partial fractions with repeated linear factors

Problem 5. Resolve $\dfrac{2x+3}{(x-2)^2}$ into partial fractions

The denominator contains a repeated linear factor, $(x-2)^2$

$$\text{Let } \frac{2x+3}{(x-2)^2} \equiv \frac{A}{(x-2)} + \frac{B}{(x-2)^2} \equiv \frac{A(x-2)+B}{(x-2)^2}$$

Equating the numerators gives:

$$2x + 3 \equiv A(x-2) + B$$

Let $x = 2$. Then $7 = A(0) + B$

i.e. $\mathbf{B = 7}$

$$2x + 3 \equiv A(x-2) + B \equiv Ax - 2A + B$$

Since an identity is true for all values of the unknown, the coefficients of similar terms may be equated.

Hence, equating the coefficients of x gives: $\mathbf{2 = A}$
[Also, as a check, equating the constant terms gives: $3 = -2A + B$. When $A = 2$ and $B = 7$,
$\text{RHS} = -2(2) + 7 = 3 = \text{LHS}$]

$$\text{Hence } \mathbf{\frac{2x+3}{(x-2)^2} \equiv \frac{2}{(x-2)} + \frac{7}{(x-2)^2}}$$

Problem 6. Express $\dfrac{5x^2 - 2x - 19}{(x+3)(x-1)^2}$ as the sum of three partial fractions

The denominator is a combination of a linear factor and a repeated linear factor.

$$\text{Let } \frac{5x^2 - 2x - 19}{(x+3)(x-1)^2} \equiv \frac{A}{(x+3)} + \frac{B}{(x-1)} + \frac{C}{x-1)^2} \equiv \frac{A(x-1)^2 + B(x+3)(x-1) + C(x+3)}{(x+3)(x-1)^2}$$

by algebraic addition

Equating the numerators gives:

$$5x^2 - 2x - 19 \equiv A(x-1)^2 + B(x+3)(x-1) + C(x+3) \quad (1)$$

Let $x = -3$. Then

$$5(-3)^2 - 2(-3) - 19 \equiv A(-4)^2 + B(0)(-4) + C(0)$$

i.e. $32 = 16A$

i.e. $\mathbf{A = 2}$

Let $x = 1$. Then

$$5(1)^2 - 2(1) - 19 \equiv A(0)^2 + B(4)(0) + C(4)$$

i.e. $-16 = 4C$

i.e. $\mathbf{C = -4}$

Without expanding the RHS of equation (1) it can be seen that equating the coefficients of x^2 gives: $5 = A + B$, and since $A = 2$, $\mathbf{B = 3}$
[Check: Identity (1) may be expressed as:

$$5x^2 - 2x - 19 \equiv A(x^2 - 2x + 1) + B(x^2 + 2x - 3) + C(x + 3)$$

i.e. $5x^2 - 2x - 19 \equiv Ax^2 - 2Ax + A + Bx^2 + 2Bx - 3B + Cx + 3C$

Equating the x term coefficients gives:

$$-2 \equiv -2A + 2B + C$$

When $A = 2$, $B = 3$ and $C = -4$ then $-2A + 2B + C = -2(2) + 2(3) - 4 = -2 = \text{LHS}$
Equating the constant term gives:

$$-19 \equiv A - 3B + 3C$$

$$\text{RHS} = 2 - 3(3) + 3(-4) = 2 - 9 - 12 = -19 = \text{LHS}]$$

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Hence $\dfrac{\mathbf{5x^2 - 2x - 19}}{\mathbf{(x+3)(x-1)^2}}$

$$\equiv \frac{\mathbf{2}}{\mathbf{(x+3)}} + \frac{\mathbf{3}}{\mathbf{(x-1)}} - \frac{\mathbf{4}}{\mathbf{(x-1)^2}}$$

Problem 7. Resolve $\dfrac{3x^2+16x+15}{(x+3)^3}$ into partial fractions

Let

$$\frac{3x^2+16x+15}{(x+3)^3} \equiv \frac{A}{(x+3)} + \frac{B}{(x+3)^2} + \frac{C}{(x+3)^3}$$
$$\equiv \frac{A(x+3)^2 + B(x+3) + C}{(x+3)^3}$$

Equating the numerators gives:

$$3x^2 + 16x + 15 \equiv A(x+3)^2 + B(x+3) + C \quad (1)$$

Let $x = -3$. Then

$$3(-3)^2 + 16(-3) + 15 \equiv A(0)^2 + B(0) + C$$

i.e. $\mathbf{-6 = C}$

Identity (1) may be expanded as:

$$3x^2 + 16x + 15 \equiv A(x^2 + 6x + 9) + B(x+3) + C$$

i.e. $3x^2 + 16x + 15 \equiv Ax^2 + 6Ax + 9A + Bx + 3B + C$

Equating the coefficients of x^2 terms gives:

$$\mathbf{3 = A}$$

Equating the coefficients of x terms gives:

$$16 = 6A + B$$

Since $A = 3$, $\mathbf{B = -2}$

[Check: equating the constant terms gives:

$$15 = 9A + 3B + C$$

When $A = 3$, $B = -2$ and $C = -6$,

$$9A + 3B + C = 9(3) + 3(-2) + (-6)$$
$$= 27 - 6 - 6 = 15 = \text{LHS}]$$

Thus $\dfrac{\mathbf{3x^2 + 16x + 15}}{\mathbf{(x+3)^3}}$

$$\equiv \frac{\mathbf{3}}{\mathbf{(x+3)}} - \frac{\mathbf{2}}{\mathbf{(x+3)^2}} - \frac{\mathbf{6}}{\mathbf{(x+3)^3}}$$

Now try the following exercise

Exercise 27 **Further problems on partial fractions with repeated linear factors**

1. $\dfrac{4x-3}{(x+1)^2}$ $\left[\dfrac{4}{(x+1)} - \dfrac{7}{(x+1)^2}\right]$

2. $\dfrac{x^2+7x+3}{x^2(x+3)}$ $\left[\dfrac{1}{x^2} + \dfrac{2}{x} - \dfrac{1}{(x+3)}\right]$

3. $\dfrac{5x^2-30x+44}{(x-2)^3}$ $\left[\dfrac{5}{(x-2)} - \dfrac{10}{(x-2)^2} + \dfrac{4}{(x-2)^3}\right]$

4. $\dfrac{18+21x-x^2}{(x-5)(x+2)^2}$ $\left[\dfrac{2}{(x-5)} - \dfrac{3}{(x+2)} + \dfrac{4}{(x+2)^2}\right]$

7.4 Worked problems on partial fractions with quadratic factors

Problem 8. Express $\dfrac{7x^2+5x+13}{(x^2+2)(x+1)}$ in partial fractions

The denominator is a combination of a quadratic factor, (x^2+2), which does not factorise without introducing imaginary surd terms, and a linear factor, $(x+1)$. Let

$$\frac{7x^2+5x+13}{(x^2+2)(x+1)} \equiv \frac{Ax+B}{(x^2+2)} + \frac{C}{(x+1)}$$
$$\equiv \frac{(Ax+B)(x+1) + C(x^2+2)}{(x^2+2)(x+1)}$$

Equating numerators gives:

$$7x^2 + 5x + 13 \equiv (Ax+B)(x+1) + C(x^2+2) \quad (1)$$

Let $x = -1$. Then

$$7(-1)^2 + 5(-1) + 13 \equiv (Ax+B)(0) + C(1+2)$$

i.e. $15 = 3C$

i.e. $\mathbf{C = 5}$

Identity (1) may be expanded as:

$$7x^2 + 5x + 13 \equiv Ax^2 + Ax + Bx + B + Cx^2 + 2C$$

Equating the coefficients of x^2 terms gives:

$$7 = A + C, \text{ and since } C = 5, \ \mathbf{A = 2}$$

Equating the coefficients of x terms gives:

$$5 = A + B, \text{ and since } A = 2, \ \mathbf{B = 3}$$

[Check: equating the constant terms gives:

$$13 = B + 2C$$

When B = 3 and C = 5, B + 2C = 3 + 10 = 13 = LHS]

Hence $\displaystyle \mathbf{\frac{7x^2 + 5x + 13}{(x^2 + 2)(x + 1)} \equiv \frac{2x + 3}{(x^2 + 2)} + \frac{5}{(x + 1)}}$

Problem 9. Resolve $\dfrac{3 + 6x + 4x^2 - 2x^3}{x^2(x^2 + 3)}$ into partial fractions

Terms such as x^2 may be treated as $(x + 0)^2$, i.e. they are repeated linear factors

Let $\displaystyle \frac{3 + 6x + 4x^2 - 2x^3}{x^2(x^2 + 3)}$

$$\equiv \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{(x^2 + 3)}$$

$$\equiv \frac{Ax(x^2 + 3) + B(x^2 + 3) + (Cx + D)x^2}{x^2(x^2 + 3)}$$

Equating the numerators gives:

$$3 + 6x + 4x^2 - 2x^3 \equiv Ax(x^2 + 3) + B(x^2 + 3) + (Cx + D)x^2$$
$$\equiv Ax^3 + 3Ax + Bx^2 + 3B + Cx^3 + Dx^2$$

Let $x = 0$. Then $3 = 3B$

i.e. $\mathbf{B = 1}$

Equating the coefficients of x^3 terms gives:

$$-2 = A + C \qquad (1)$$

Equating the coefficients of x^2 terms gives:

$$4 = B + D$$

Since B = 1, **D = 3**

Equating the coefficients of x terms gives:

$$6 = 3A$$

i.e. $\mathbf{A = 2}$

From equation (1), since A = 2, **C = −4**

Hence $\displaystyle \mathbf{\frac{3 + 6x + 4x^2 - 2x^3}{x^2(x^2 + 3)}}$

$$\equiv \frac{2}{x} + \frac{1}{x^2} + \frac{-4x + 3}{x^2 + 3}$$

$$\mathbf{\equiv \frac{2}{x} + \frac{1}{x^2} + \frac{3 - 4x}{x^2 + 3}}$$

Now try the following exercise

Exercise 28 Further problems on partial fractions with quadratic factors

1. $\dfrac{x^2 - x - 13}{(x^2 + 7)(x - 2)}$ $\quad\left[\dfrac{2x + 3}{(x^2 + 7)} - \dfrac{1}{(x - 2)}\right]$

2. $\dfrac{6x - 5}{(x - 4)(x^2 + 3)}$ $\quad\left[\dfrac{1}{(x - 4)} + \dfrac{2 - x}{(x^2 + 3)}\right]$

3. $\dfrac{15 + 5x + 5x^2 - 4x^3}{x^2(x^2 + 5)}$ $\quad\left[\dfrac{1}{x} + \dfrac{3}{x^2} + \dfrac{2 - 5x}{(x^2 + 5)}\right]$

4. $\dfrac{x^3 + 4x^2 + 20x - 7}{(x - 1)^2(x^2 + 8)}$
$$\left[\frac{3}{(x - 1)} + \frac{2}{(x - 1)^2} + \frac{1 - 2x}{(x^2 + 8)}\right]$$

5. When solving the differential equation $\dfrac{d^2\theta}{dt^2} - 6\dfrac{d\theta}{dt} - 10\theta = 20 - e^{2t}$ by Laplace transforms, for given boundary conditions, the following expression for $\mathcal{L}\{\theta\}$ results:
$$\mathcal{L}\{\theta\} = \frac{4s^3 - \frac{39}{2}s^2 + 42s - 40}{s(s - 2)(s^2 - 6s + 10)}$$
Show that the expression can be resolved into partial fractions to give:
$$\mathcal{L}\{\theta\} = \frac{2}{s} - \frac{1}{2(s - 2)} + \frac{5s - 3}{2(s^2 - 6s + 10)}$$